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BLACK–MERTON–SCHOLES OPTION PRICING: A 50-YEAR CELEBRATION—LOOKING AHEAD

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It is a great honor to be invited to participate and contribute to the 50-year celebration of the path-breaking option pricing theory of Fischer Black, Robert Merton, and Myron Scholes (Black and Scholes (1973) and Merton (1973)). My focus is on financial intermediation and looking ahead on future challenges.

The precursor of the modern theory of the pricing of options is Bachelier's (1900) doctoral thesis at the Sorbonne. Bachelier introduced the key assumptions that underlie the Black–Scholes–Merton model and derived the first results outlined by Pliska (2010): assumed that price fluctuations over small time intervals are independent of the present and past price levels; applied the central limit theorem to deduce that the price increments are independent and normally distributed, so that the price process is a Brownian motion as the diffusion limit of a random walk; used the lack-of-memory (Markov) property to derive the Chapman–Kolmogorov equation; established the connection with the heat equation; derived a simple formula for the price of at-the-money calls; and recognized the concept of arbitrage.

Prior to the discovery in the 1950s of Bachelier's thesis by Jimmy Savage and Paul Samuelson, economists, including Samuelson (1965), struggled to determine the appropriate stock appreciation rate and the rate at which the expected payoff of an option should be discounted.

Thorp and Kassouf (1967) set the stock's appreciation rate equal to the riskless interest rate and discounted the expected payoff of an option at the same rate, arriving at the BSM formula. They used the formula for profitable trading but could not prove why it was correct. And there comes the Black, Merton, and Scholes seminal theory that established solid foundations for the pricing

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of options and set off the 50-year revolution that we witness today.

The risk-neutral valuation of options was first introduced by Cox and Ross (1976) and further developed by Cox *et al.* (1979). John Cox and Stephen Ross hypothesized (but did not prove) that, for a wide variety of processes, the price of an option can be computed such that expected returns for both the stock and the option are equal to the returns under the riskless rate. Harrison and Kreps (1979) formalized the theory of risk-neutral valuation implied by the absence of arbitrage. This is one of the most profound result in asset pricing. Nowadays, the theory is applied in the pricing of a wide spectrum of securities, including fixed income.

At the beginning, traders enthusiastically adopted the BMS model. They traded options according to the model and, not surprisingly, empirical studies found that option prices nearly perfectly conform to the model. The first clouds appeared following the October 1987 stock market crash and the jump of the implied volatility of the S&P 500 Index options. Most traders continued using the BMS model and simply increased the volatility input but perceptive traders made a lot of money by realizing that the BMS model needed to be modified, particularly in pricing out-of-the-money puts. Black (1989) highlighted the problem with the BMS model in "The Holes in Black-Scholes".

A robust prediction of the BMS model is that the volatility implied by option prices is constant across strike prices. Rubinstein (1994) tested this prediction on the S&P 500 Index options (SPX) traded on the Chicago Board Options Exchange, an exchange that comes close to the dynamically complete and perfect market assumptions underlying the BMS model. From the start of the exchange-based trading in April 1986 up until the October 1987 stock market crash, the implied volatility is a moderately downward-sloping or U-shaped function of the strike price, a pattern referred to as the "volatility smile," also observed in international markets and to a lesser extent in the prices of individual stock options. Following the crash, the volatility smile is typically more pronounced and downward sloping, often referred to as the "volatility skew."

Whereas downward-sloping or U-shaped implied volatility curves are inconsistent with the BMS model, it is well understood that this pattern is not necessarily inconsistent with economic theory. The observed volatility smile or skew stimulated a voluminous empirical and theoretical research on alternative models that are consistent with the data, notably by Heston (1993). This research focused on stochastic volatility, price jumps, and volatility jumps. It is beyond the scope of the present paper to review this literature. Instead, I focus on my research on this topic and highlight directions for future research.

Two fundamental assumptions of the BMS model are that the market is dynamically complete and frictionless. The theory of stochastic dominance is preference-free and relaxes the assumptions that the market is dynamically complete and frictionless. Constantinides and Perrakis (2002) derived bounds on the prices of options in the absence of stochastic dominance but in the presence of proportional transaction costs that are invariant to the allowed frequency of trading the bond and stock over the life of the options. Potential violation of these bounds implies that traders, irrespective of their preferences, can increase their expected utility by engaging in a zero-net-cost trade.

Constantinides *et al.* (2009) empirically investigated whether the cross-sections of 1-month S&P 500 Index option prices from 1986 to 2006 are consistent with various economic models that explicitly allow for a dynamically incomplete market and also an imperfect market that recognizes trading costs and bid–ask spreads. They documented widespread violations of stochastic dominance even with generous transaction costs in trading the index and the options. They allowed the volatility of the index return to be state dependent and employed the estimated conditional volatility. Even though pre-crash option prices conform to the BMS model reasonably well, once the constant volatility input to the BMS formula is judiciously chosen, this does not speak on the rationality of option prices. The novel finding is that pre-crash options are incorrectly priced if the distribution of the index return is estimated from time-series data even with a variety of statistical adjustments.

The interpretation of these results is that, before the crash, option traders were extensively using the BMS pricing model and the dictates of this model were imposed on the option prices even though these dictates were not necessarily consistent with the time-series behavior of index prices. There are substantial violations by out-of-the-money calls under both the fixed and proportional transaction costs regimes. This observation is novel and contradicts the common inference drawn from the observed implied volatility smile that the problem primarily lies with the left-hand tail of the index return distribution. The decrease in violations from the 1988 to 1995 post-crash period is followed by a substantial increase in violations from 1997 to 2003. This is a novel finding and casts doubts on the hypothesis that the options market is becoming more rational over time, particularly after the crash. Constantinides *et al.* (2011) confirmed these results with out-of-sample tests on options on S&P 500 Index futures.

Constantinides *et al.* (2013) suggested that the three Fama–French factors and related factors used to adjust stock returns for risk may not capture factors appropriate for adjusting option returns. They created three factors from the S&P 500 Index options that capture stochastic volatility, price jumps, and volatility jumps. They did not reject the hypothesis that any one of these factors, combined with the market factor, explains index option returns, except for the returns of short dated out-of-the-money puts. However, the combination of the volatility jump factor and the liquidity factor of Pastor and Stambaugh (2013) explains away the abnormal returns of all option portfolios, including the returns of short dated out-of-the-money puts. Even though the liquidity factor was designed to address stock returns, it is remarkable that it plays a major role in explaining option returns as well. I further address the importance of liquidity and intermediary asset pricing shortly.

Constantinides *et al.* (2020) upset the above resolution of abnormal returns. They reported that portfolios of the S&P 500 Index, bonds, and index options dominate portfolios without index options. The portfolios with options include primarily short calls and are particularly profitable when maturity is short and volatility is high. Similar results obtained for the DAX and CAC index options. Neither priced factors nor a non-monotonic stochastic discount factor explains these results. Bates (2003) emphasized that priced factors cannot fully capture—much less explain—the empirical properties of option prices and concluded that there is a need for a new approach to pricing derivatives that focuses on the financial intermediation of the underlying risks by option market makers.

A substantial amount of economic research focuses on financial intermediation and the capital constraints of intermediaries. A leading example is He and Krishnamurthy (2013) who pointed out that traditional approaches to asset pricing ignore intermediation by assuming that intermediaries' actions reflect the preferences of their client-investors.

A parallel research developed in the option pricing literature that focuses on the supply and demand for options and the role played by market makers, including hedge funds. Bollen and Whaley (2004) examined the relation between the net buying pressure of index options and found that the implied volatility of index options is directly related to the buying pressure of index puts, particularly out-of-the-money puts.

Gârleanu *et al.* (2009) introduced the demand pressure hypothesis where supply shifts in options by market makers are endogenous while demand in options by customers is exogenous. They obtained explicit expressions for the effects of demand on option prices, provided empirical evidence consistent with the demand-pressure hypothesis, and showed that demand-pressure effects can play a role in resolving the main option-pricing puzzles. The equilibrium net buy equals the exogenous customer demand and the paper does not provide testable implications regarding the net buy response to risk and option prices.

Etula (2013) modeled a commodities market with risk-averse producers and consumers and risk-neutral broker-dealers who are subject to a Value-at-Risk constraint and found empirical support for the prediction that the broker-dealers' risk-bearing capacity forecasts energy returns. Muravyev (2016) showed that inventory risk faced by market makers has a first-order effect on option order flow and option prices. Chen *et al.* (2019) proxied the variation in the financial intermediary constraint with the net buy of deep out-of-the-money puts and showed that this measure explains the variance risk premium embedded in puts and predicts the future excess returns of a variety of assets. Fournier and Jacobs (2020) modeled a market maker with limited capital and found that most of the variance risk premium for index options is due to inventory risk.

Constantinides and Lian (2021) modeled the endogenous demand shifts by customers in addition to the endogenous supply shifts by market makers in the market for S&P 500 Index put options. Therefore, they provided testable implications regarding the net buy response to risk and option prices, implications which are empirically verified. They model the supply of at-the-money and out-of-the-money S&P 500 Index put options by risk-averse market makers and their demand by risk-averse customers who hold the index and a risk-free asset and buy puts as downside-risk protection. In equilibrium, market makers are net sellers and customers are net buyers of index puts.

The model implies that, when the risk increases, the supply curve by market makers shifts to the left and the demand curve by customers shift to right. It turns out that the shift to the left of the supply curve dominates the shift to the right of the demand curve and the net buy of puts by customers is decreasing in the risk. This prediction is verified in the time series of the net buy of at-the-money and out-of-the-money puts. This highlights the importance of considering the endogenous response of both the demand by customers and the supply by market makers.

There is much more to be done in exploring the financial intermediation of options. For example, there is no identifiable empirical pattern in the net buy of calls and this needs to be further investigated. I suspect that we need to allow for customer clienteles for calls and puts of different moneyness to provide a richer theory of the demand and supply of options. Overall, I have identified the important role of financial intermediation in understanding the pricing and supply and demand of options. Much exciting research lies ahead.

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