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THE OPTIONS-INFERRED EQUITY PREMIUM AND THE SLIPPERY SLOPE OF THE NEGATIVE CORRELATION CONDITION

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The negative correlation condition *(NCC) of Martin (2017) is that* $\text{cov}_t^{\mathbb{P}}(M_T\, R_T,\, R_T) \leq 0$ *for all* M_T*, where* M_T *is the SDF and* R_T *is the gross market return. He employs this assumption to derive a lower bound of the equity premium. This paper exploits theoretical and empirical constructions to refute the hypothesis of the NCC. Using options on the S&P 500 index and STOXX 50 equity index, our tests favor rejection. Our empirical counterexamples of* M_T *contradict the universality of the NCC, exhibit variance-dependence and incorporate an increasing region to the return upside.*

I still remember the teasing we financial economists, Harry Markowitz, William Sharpe, and I, had to put up with from the physicists and chemists in Stockholm when we conceded that the basic unit of our research, the expected rate of return, was not actually observable—Merton Miller, The History of Finance, Journal of Portfolio Management 1999 (Summer), 95–101.

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1 Introduction

The spread between the expected return on a stock portfolio and the return of a risk-free bond—known as the equity premium—holds enormous significance in many aspects of financial decision-making, including optimal portfolio selection and determining the cost of capital. When estimating the market's expected return, a convenient method is to use the sample average return, often derived from the performance history of the S&P 500 stock index. However, the rather significant gap between the sample average returns on stocks and risk-free bonds has perplexed both financial economists and market observers. Furthermore, the bootstrap confidence intervals on estimated equity premium are sizable. If measuring the equity premium unconditionally is hard, measuring the conditional equity premium is even harder. Key questions are: What is the magnitude of the conditional equity premium? How does it vary with economic conditions?

Research in this field can be categorized into three approaches. The first one is to calculate the equity premium by looking at historical average returns. Second method is to use the dividend discount model, which involves finding the internal rate of return based on the price and expected dividend stream of an index. This approach uses various variables such as projected earnings, market estimates, and long-term growth. Finally, the constant Sharpe ratio method can be applied, which considers the average Sharpe ratio and solves for the conditional equity premium by multiplying it with a measure of market volatility, like the VIX volatility index. This approach is similar to the method proposed by Merton (1980).

Martin (2017) presents an alternative method for estimating the conditional expected excess return of the market. This approach utilizes option prices and economic theory to establish a lower bound on the market portfolio, which can be seen as a prediction of subsequent returns. This lower bound is based on an assumption termed the negative correlation condition, which states that the product of the stochastic discount factor and the gross market return should have a negative correlation with the gross market returns. This approach relies on data from S&P 500 option prices—more precisely, the risk-neutral market variance—to provide a measure of equity premium.

The lower bound on the equity premium touches a theme dating back to Merton (1980), Black (1993), Elton (1999), and Ross (2015) on how elusive it is to estimate the (conditional) equity premium. The topic is cutting-edge, and peer-reviewed academic work on the lower bound of the equity premium remains relevant but controversial. It has stimulated advancements in both theoretical and practical research, as evidenced by recent studies such as Schneider and Trojani (2019), Chabi-Yo and Loudis (2020), Kadan and Tang (2020) and Bakshi *et al.* (2023b, 2024).

The concept of negative correlation condition can be likened to the notion of "dark matter" discussed in previous studies (e.g., Chen *et al.*, 2024; Bakshi *et al.*, 2022). It is crucial for establishing a lower bound on the equity premium at each time point, yet there is no explicit measurement or quantification of this condition. To address this, we develop the underlying asset-pricing restrictions that give rise to the negative correlation condition and suggest potential empirical tests that can be used to validate these restrictions.

The assumption of the negative correlation condition is not transparent because the stochastic discount factor is not directly observable and must be estimated. This assumption is crucial to the lower bound formula for the equity premium, so it is necessary to investigate its generality and applicability. In our analysis, we use the law of total covariance formula to examine the negative correlation condition, introducing new methods to evaluate it. Our approach is motivated by the use of SDF projection, which aligns with existing models. Our findings show that the negative correlation condition does not hold, even on average, and is not a consistent assumption over time.

The remainder of this paper is organized as follows. We begin by describing the negative correlation condition and discussing its relationship with the equity premium in Section 2. In Section 3, we present the S&P 500 index options data. We then provide a framework for testing the asset-pricing restrictions of the negative correlation condition. Our empirical evidence calls into question the universality of the negative correlation condition.[1]

2 Equity Premium and Negative Correlation Condition

We employ the following notations:

S_{t+T} = price of the equity market index (inclusive of dividends) at future date $t+T$;

$R_T \equiv \frac{S_{t+T}}{S_t}$ = gross return of the equity market index over t to $t+T$. Assume $R_T > 0$;

$\mathbb{E}_t^{\mathbb{P}}(.)$ = expectation, at date t, under the real-world probability measure, $\mathbb{P}$;

$\mathbb{E}_t^{\mathbb{Q}}(.)$ = expectation, at date t, under the risk-neutral probability measure, $\mathbb{Q}$;

M_T = stochastic discount factor (SDF) with $\mathbb{E}_t^{\mathbb{P}}(M_T\, R_T) = 1$ holding;

$R_{f,t} = \frac{1}{\mathbb{E}_t^{\mathbb{P}}(M_T)} = \mathbb{E}_t^{\mathbb{Q}}(R_T)$ = gross risk-free return over t to $t+T$ (known at date t);

$\mathbf{1}$ = conformable vector of ones;

$\mathbf{Z}_T[R_T]$ = gross return vector contingent on R_T and satisfying $\mathbb{E}_t^{\mathbb{P}}(M_T\, \mathbf{Z}_T[R_T]) = \mathbf{1}$;

$M_T[R_T]$ = SDF projection (details to come), with $\mathbb{E}_t^{\mathbb{P}}(M_T[R_T]\, \mathbf{Z}_T[R_T]) = \mathbf{1}$ holding;

$\text{cov}_t^{\mathbb{P}}(\tilde{x}, \tilde{y})$ = conditional covariance between two random variables $\tilde{x}$ and $\tilde{y}$ under $\mathbb{P}$; and

$\text{var}_t^{\mathbb{Q}}(R_T)$ = conditional variance of R_T under the risk-neutral measure $\mathbb{Q}$.

Definition 1 (Negative correlation condition, NCC). The NCC is the assumption that $\text{cov}_t^{\mathbb{P}}(M_T\, R_T, R_T) \leq 0$, for all M_T. ■

Martin (2017) considers the relation in Equation (1), imposes $\mathbb{E}_t^{\mathbb{P}}(M_T\, R_T) = 1$, and shows the *lower bound of the equity premium* (expected excess return of the market), as follows:

$$\underbrace{\text{cov}_t^{\mathbb{P}}(M_T\, R_T, R_T)}_{\leq\, 0 \text{ if NCC holds}} = \mathbb{E}_t^{\mathbb{P}}(M_T\, R_T^2) - \underbrace{\mathbb{E}_t^{\mathbb{P}}(M_T\, R_T)}_{=1}\, \mathbb{E}_t^{\mathbb{P}}(R_T) \quad (1)$$

or

$$\underbrace{\mathbb{E}_t^{\mathbb{P}}(R_T) - R_{f,t}}_{\text{conditional equity premium}} \geq \underbrace{\mathbb{E}_t^{\mathbb{P}}(M_T\, R_T^2) - R_{f,t}}_{=\, R_{f,t}^{-1}\, \text{var}_t^{\mathbb{Q}}(R_T)}.$$

$$\text{(provided the NCC holds)} \quad (2)$$

Is $\text{cov}_t^{\mathbb{P}}(M_T\, R_T, R_T) \leq 0$ (that is, the NCC) a generic property that holds *at each date t?* The following two features guide our theoretical and empirical investigation:

- First, the inequality in Equation (2) would need to hold under all change-of-measure densities (that is, for *every* M_T) such that $-\text{cov}_t^{\mathbb{P}}(M_T\, R_T, R_T)$ is nonnegative for each t.
- Second, the lower bound ($R_{f,t}^{-1}\, \text{var}_t^{\mathbb{Q}}(R_T)$) is theoretically unsupportable if one could find economically plausible M_T for which $\text{cov}_t^{\mathbb{P}}(M_T\, R_T, R_T)$ is *positive*.

In analogy to the dark matter property and susceptibility of an asset-pricing model to misspecification, we show that if $\text{cov}_t^{\mathbb{P}}(M_T\, R_T, R_T)$ is nonnegative, then $R_{f,t}^{-1}\, \text{var}_t^{\mathbb{Q}}(R_T)$ is an *upper* bound and not a *lower* bound of the equity premium. Simply put, if the NCC were not a generic property, what, at first sight, appears to be the lower bound of the equity premium, could, in fact, be an upper bound. Crucial to theory and practice, knowledge of the derived *lower bound* of the equity premium is of limited value if the NCC were violated.

We consider SDF projections (following Rosenberg and Engle, 2002) that are equipped with sensitivity to market returns and returns of market variance. Taking cues from the evidence in Aït-Sahalia and Lo (2000), Jackwerth (2000), Rosenberg and Engle (2002), Bakshi *et al.* (2010), and Christoffersen *et al.* (2013), our treatment exploits SDFs that incorporate exposure to market variance and maintain pricing consistency with the returns of index options. While the considered SDFs can be distinct from some other models in their dependency structure of uncertainty, these SDFs share their economic realism along several data dimensions.

Our empirical validation exercises rely on options on both the S&P 500 equity index and STOXX 50 equity index. All in all, we construct SDF specifications to show that $\text{cov}_t^{\mathbb{P}}(M_T\, R_T, R_T)$ is *positive*. The bootstrap-based tests reject the premise of the NCC with the highest p-value of 0.012. Our theory formalizes asset-pricing restrictions of the NCC, and our empirical work falsifies the universality of the NCC. Our exceptions to the NCC are grounded in established asset-pricing theory. Specifically, the featured SDF specifications admit an increasing region to the upside of market returns and are aligned with the salient properties of index option returns.

3 Testing the Asset-Pricing Restrictions of the NCC

This section develops the theoretical and empirical restrictions on M_T to examine the generality of the NCC.

Consider a discrete-time environment. We assume that there exists an *SDF projection*, denoted as $M_T[R_T]$, in the sense of Rosenberg and Engle (2002, Section 2.2) and Bakshi *et al.* (2010), satisfying (i) $M_T[R_T] > 0$, (ii) $\mathbb{E}_t^{\mathbb{P}}(M_T[R_T]) < \infty$, (iii) $\mathbb{E}_t^{\mathbb{P}}(M_T[R_T]\, R_T) = 1$, and (iv) $\mathbb{E}_t^{\mathbb{P}}(\{M_T[R_T]\}^2) < \infty$. The length of the time period T for $M_T[R_T]$ can be arbitrary.

Why focus on M_T that are a function of R_T? For one reason, Martin (2017) posits that if $M_T = \frac{1}{R_T}$, then $\text{cov}_t^{\mathbb{P}}(M_T R_T, R_T) = 0$, and $R_{f,t}^{-1}\, \text{var}_t^{\mathbb{Q}}(R_T)$ is the *tightest* lower bound for $\mathbb{E}_t^{\mathbb{P}}(R_T) - R_{f,t}$. For another reason, when it comes to considering claims written on R_T for which data is there, it suffices to work with the SDF projection. Thus, we only need to model $M_T[R_T] \equiv \mathbb{E}^{\mathbb{P}}(\mathcal{M} \mid R_T)$, for any $\mathcal{M}$ that represents the change of probability measure with the bond price as numeraire.

We first present a theoretical result that gives a sufficient condition on the sign of the conditional covariance between a random variable X and a function $g[X]$ (suppressing subscript T on X_T).

Lemma 1 (Sign of conditional covariance). *Let X be a random variable with a finite second moment, $\mathcal{D}$ be a subset of the real line $\mathbb{R}$ with $\mathbb{E}_t^{\mathbb{P}}(X) \in \mathcal{D}$, and $g : \mathcal{D} \to \mathbb{R}$ be a function for which $g[X]$ has a finite second moment. The following statements are true*:

$$\text{If } g[X] \text{ is a decreasing function on } \mathcal{D}, \text{ then } \text{cov}_t^{\mathbb{P}}(g[X], X) \leq 0. \tag{3}$$

$$\text{If } g[X] \text{ is an increasing function on } \mathcal{D}, \text{ then } \text{cov}_t^{\mathbb{P}}(g[X], X) \geq 0. \tag{4}$$

Proof. Such a result may have appeared elsewhere, but see Appendix (Section A). ■

The essence of Lemma 1 is that if $\frac{dg[X]}{dX} \equiv g'[X] \leq (\geq) 0$, then $g[X]$ is decreasing (increasing), so Lemma 1 tells us that the conditional covariance $\text{cov}_t^{\mathbb{P}}(g[X], X)$ is nonpositive (nonnegative).

In view of Lemma 1, what are the theoretical restrictions on plausible M_T under which the NCC does not hold? We answer this question by developing a result on the sign of $\text{cov}_t^{\mathbb{P}}(M_T[R_T]R_T^n, R_T^n)$ for any positive integer n and then specializing the result to $n = 1$. Let $M_T'[R_T] \equiv \frac{dM_T[R_T]}{dR_T}$.

Result 1 (Restrictions of the NCC). *The following hold for any positive integer n:*

$$\text{If } \frac{M'_T[R_T]}{M_T[R_T]} + \frac{n}{R_T} \leq 0 \text{ almost surely, then } \text{cov}_t^{\mathbb{P}}(M_T[R_T]R_T^n, R_T^n) \leq 0. \tag{5}$$

$$\text{If } \frac{M'_T[R_T]}{M_T[R_T]} + \frac{n}{R_T} \geq 0 \text{ almost surely, then } \text{cov}_t^{\mathbb{P}}(M_T[R_T]R_T^n, R_T^n) \geq 0. \tag{6}$$

Proof. This result follows from Lemma 1 and the steps in the Appendix (Section B). ■

The conditions in Result 1 are *sufficient* but not necessary. This is because derivatives are local properties. Result 1 has implications for lower and upper bounding the equity premium. In this regard, we note that (as described in Internet Appendix (Section B)) for any positive integer n

$$\text{If } \frac{M'_T[R_T]}{M_T[R_T]} + \frac{n}{R_T} \leq 0, \quad \text{then} \quad \mathbb{E}_t^{\mathbb{P}}(R_T^n) \geq \overbrace{\frac{\mathbb{E}_t^{\mathbb{Q}}(R_T^{2n})}{\mathbb{E}_t^{\mathbb{Q}}(R_T^n)}}^{\text{lower bound}}, \quad \text{and} \tag{7}$$

$$\text{if } \frac{M'_T[R_T]}{M_T[R_T]} + \frac{n}{R_T} \geq 0, \quad \text{then} \quad \mathbb{E}_t^{\mathbb{P}}(R_T^n) \leq \underbrace{\frac{\mathbb{E}_t^{\mathbb{Q}}(R_T^{2n})}{\mathbb{E}_t^{\mathbb{Q}}(R_T^n)}}_{\text{upper bound}}. \tag{8}$$

Consider Equations (7) and (8) and set $n = 1$:

$$\text{If } \frac{M'_T[R_T]}{M_T[R_T]} + \frac{1}{R_T} \leq 0, \quad \text{then} \quad \mathbb{E}_t^{\mathbb{P}}(R_T) \geq \frac{\mathbb{E}_t^{\mathbb{Q}}(R_T^2)}{\mathbb{E}_t^{\mathbb{Q}}(R_T)}. \tag{9}$$

$$\text{If } \frac{M'_T[R_T]}{M_T[R_T]} + \frac{1}{R_T} \geq 0, \quad \text{then} \quad \mathbb{E}_t^{\mathbb{P}}(R_T) \leq \frac{\mathbb{E}_t^{\mathbb{Q}}(R_T^2)}{\mathbb{E}_t^{\mathbb{Q}}(R_T)}. \tag{10}$$

If the restriction $\frac{M'_T[R_T]}{M_T[R_T]} + \frac{1}{R_T} \leq 0$ holds, then rearranging Equation (9) implies a conditional lower bound of the equity premium; that is, $\mathbb{E}_t^{\mathbb{P}}(R_T) - R_{f,t} \geq R_{f,t}^{-1} \text{var}_t^{\mathbb{Q}}(R_T)$, in view of $\mathbb{E}_t^{\mathbb{Q}}(R_T) = R_{f,t}$.

What do we convey that is different from Martin (2017)? Equation (10) uncovers that $\mathbb{E}_t^{\mathbb{P}}(R_T) - R_{f,t} \leq R_{f,t}^{-1} \text{var}_t^{\mathbb{Q}}(R_T)$, so $R_{f,t}^{-1} \text{var}_t^{\mathbb{Q}}(R_T)$ represents an *upper* bound, when $\frac{M'_T[R_T]}{M_T[R_T]} + \frac{1}{R_T} \geq 0$ (that is, when $\text{cov}_t^{\mathbb{P}}\big(M_T[R_T]R_T, R_T\big) \geq 0$).

Since the sign of $\frac{M'_T[R_T]}{M_T[R_T]} + \frac{1}{R_T}$ could potentially alternate through (calendar) time, our characterizations do not preclude the possibility that $R_{f,t}^{-1} \text{var}_t^{\mathbb{Q}}(R_T)$ represents an *upper* bound of the equity premium at some time t, while it could represent a *lower* bound at some other time $t^{\bullet}$. Such a possibility is at the crux of our theoretical treatment and empirical approach.

3.1 *Empirical approach and evidence on the NCC*

Making empirical connections, the hypothesis of the NCC is that the conditional covariance $\text{cov}_t^{\mathbb{P}}(M_T\, R_T, R_T) \leq 0$, for *all* M_T. Therefore,

$$\text{we can reject the hypothesis of the NCC if } \text{cov}_t^{\mathbb{P}}(M_T[R_T]\, R_T, R_T) > 0, \text{ for some } M_T[R_T]. \tag{11}$$

Denoting the conditional covariance by $\mathbb{E}_t^{\mathbb{P}}(\tilde{\mathbf{c}}_T)$, where the demeaned cross-product $\tilde{\mathbf{c}}_T$ is given by $\tilde{\mathbf{c}}_T \equiv \{M_T[R_T]\, R_T - \mathbb{E}_t^{\mathbb{P}}(M_T[R_T]\, R_T)\}\{R_T - \mathbb{E}_t(R_T)\}$, the following comments are in order:

- First, suppose we wish to assess the hypothesis $\mathbb{H}: \mathbb{E}_t^{\mathbb{P}}(\tilde{\mathbf{c}}_T) > 0$. Having infinitely many, but countable, observations of positive $\tilde{\mathbf{c}}_T$ does not violate the possibility that $\mathbb{E}_t^{\mathbb{P}}(\tilde{\mathbf{c}}_T) \leq 0$.
- Second, suppose we wish to assess the same hypothesis $\mathbb{H}: \mathbb{E}_t^{\mathbb{P}}(\tilde{\mathbf{c}}_T) > 0$ by utilizing the sufficient condition in Result 1; then it seems that we need to test $\mathbb{H}^\bullet : \frac{M_T'[R_T]}{M_T[R_T]} + \frac{1}{R_T} > 0$ almost surely. While straightforward, this condition is not easily testable because, even if we have infinitely many, but countable, observations of positive $\frac{M_T'[R_T]}{M_T[R_T]} + \frac{1}{R_T}$, it does not exclude the possibility that $\frac{M_T'[R_T]}{M_T[R_T]} + \frac{1}{R_T} < 0$ almost surely.

Motivated by the discussions above, the next result is at the center of our empirical exercises.

Result 2. *Let* $\mathbb{E}^{\mathbb{P}}(.)$ *and* $\mathrm{cov}^{\mathbb{P}}(.,.)$ *denote the unconditional expectation and unconditional covariance, respectively. The following statement is true*:

$$\underbrace{\mathrm{cov}^{\mathbb{P}}(M_T[R_T]\,R_T,\,R_T)}_{\text{unconditional covariance}} = \underbrace{\mathbb{E}^{\mathbb{P}}(\mathrm{cov}_t^{\mathbb{P}}(M_T[R_T]\,R_T,\,R_T)).}_{\text{unconditional expectation of conditional covariance}} \quad (12)$$

Proof. By the law of total covariance formula,

$$\begin{aligned}\mathrm{cov}^{\mathbb{P}}&(M_T[R_T]\,R_T,\,R_T)\\ &= \mathbb{E}^{\mathbb{P}}(\mathrm{cov}_t^{\mathbb{P}}(M_T[R_T]\,R_T,\,R_T))\\ &\quad + \mathrm{cov}^{\mathbb{P}}(\mathbb{E}_t^{\mathbb{P}}(M_T[R_T]\,R_T), \mathbb{E}_t^{\mathbb{P}}(R_T)). \quad (13)\end{aligned}$$

Since $\mathbb{E}_t^{\mathbb{P}}(M_T[R_T]\,R_T) = 1$, the result in Equation (12) follows. ■

If the NCC holds almost surely, that is, if, $\mathrm{cov}_t^{\mathbb{P}}(M_T[R_T]\,R_T,\,R_T) \leq 0$ almost surely, then the unconditional covariance $\mathrm{cov}^{\mathbb{P}}(M_T[R_T]\,R_T,\,R_T) \leq 0$.

For comparability of magnitudes across models of $M_T[R_T]$, we test the hypothesis that

$$\mathbb{H}_0 : \ \mathrm{NCC}_T \leq 0, \ \text{where } \mathrm{NCC}_T \text{ is the unconditional } \textit{correlation} \text{ between } M_T[R_T]R_T \text{ and } R_T. \quad (14)$$

Specifically, if we reject $\mathbb{H}_0$, then we reject the NCC, meaning that $\mathrm{cov}_t^{\mathbb{P}}(M_T[R_T]\,R_T,\,R_T) > 0$ holds with a strictly positive probability. Such a restriction has not been evaluated to our knowledge.

We next present empirical evidence on the negative *correlation* condition from three models. While one could feature more models of M_T (the Internet Appendix contains additional models and supportive evidence), we highlight settings that are theoretically and empirically revealing as well as amenable to straightforward implementation and validation.

In what follows, we first examine data from the S&P 500 equity index options market from January 1990 to December 2020 (31 years, 372 option expiration cycles). The returns of the equity market index and of options correspond to 28 day expiration cycles. Thus, we set $T = 28$ days.

Model A: $M_T[R_T]$ depends on the gross return of the equity market (R_T) and the gross return of an at-the-money straddle (R_T^{straddle}), as follows:

$$\underbrace{M_T[R_T]}_{\substack{\text{volatility}\\ \text{dependent}}} = \mathbf{Z}_T^\top[R_T]\,\boldsymbol{\alpha} \quad \text{with}$$

$$\mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ R_T^{\text{straddle}} \end{pmatrix} \quad \text{and}$$

$$\boldsymbol{\alpha} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^\top[R_T])\}^{-1}\,\mathbf{1}, \quad (15)$$

where $R_T^{\text{straddle}} \equiv \frac{S_t \max(1-R_T,0)+S_t \max(R_T-1,0)}{\text{put}_{t,T}[S_t]+\text{call}_{t,T}[S_t]}$. We enforce $\mathbb{E}^{\mathbb{P}}(M_T[R_T]\,\mathbf{Z}_T[R_T]) = \mathbf{1}$.

We follow a standard procedure to obtain estimates of $\boldsymbol{\alpha}$ that is consistent with a minimum variance discrepancy problem for SDF projection (e.g., Hansen and Jagannathan, 1997; Bakshi *et al.*, 2023b). Intuitively, $M_T[R_T]$ correctly prices the market return and the straddle return (proxying for the return of variance) and has an increasing region to the return upside. Empirically, the sensitivity of $M_T[R_T]$ to R_T^{straddle} is positive (Table 1). Additionally, $M_T[R_T]$ is convex in R_T.

Model B: $M_T[R_T]$ depends on the gross return of the equity market and the gross return of a 2% out-of-the-money strangle, defined as $R_T^{\text{strangle}} \equiv \frac{S_t \max(e^{-0.02}-R_T,0)+S_t \max(R_T-e^{0.02},0)}{\text{put}_{t,T}[S_t e^{-0.02}]+\text{call}_{t,T}[S_t e^{0.02}]}$), as follows:

$$\underbrace{M_T[R_T]}_{\substack{\text{volatility}\\ \text{dependent}}} = \mathbf{Z}_T^{\top}[R_T]\,\boldsymbol{\alpha} \quad \text{with}$$

$$\mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ R_T^{\text{strangle}} \end{pmatrix} \quad \text{and}$$

$$\boldsymbol{\alpha} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\mathbf{Z}_T^{\top}[R_T])\}^{-1}\,\mathbf{1}. \qquad (16)$$

$M_T[R_T]$ loads on tail-sensitive variance and has an increasing region to the upside. Models A and B are distinct since R_T^{straddle} and R_T^{strangle} deviate across states and are imperfectly correlated.

Model C: $M_T[R_T]$ is linear in R_T and $(1 + \{R_T^{\text{straddle}} - R_T^{\text{strangle}}\})$, as follows:

$$\underbrace{M_T[R_T]}_{\substack{\text{volatility}\\ \text{dependent}}} = \mathbf{Z}_T^{\top}[R_T]\,\boldsymbol{\alpha} \quad \text{with}$$

$$\mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ 1 + \underbrace{\{R_T^{\text{straddle}} - R_T^{\text{strangle}}\}}_{\text{return spread}} \end{pmatrix}, \qquad (17)$$

where, again, $\boldsymbol{\alpha} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^{\top}[R_T])\}^{-1}\,\mathbf{1}$. The estimates of $\boldsymbol{\alpha}$ reflect the correct pricing of R_T and $1 + \{R_T^{\text{straddle}} - R_T^{\text{strangle}}\}$ and the pricing restrictions are imposed unconditionally.

Figure 1 plots $\{R_T^{\text{straddle}} - R_T^{\text{strangle}}\}$ over our sample period, which indicates that the return spread between the straddle and strangle tends to be *negative* for large movements in R_T (more so for the downside than for the upside). Notably, the correlation between $\{R_T^{\text{straddle}} - R_T^{\text{strangle}}\}$ and changes in return quadratic variation (respectively, $\{R_T - 1\}$) over expiration cycles is -0.48 (respectively, 0.30). The correlation matrix among considered variables is presented in the Internet Appendix (Table I.1). Additionally, $M_T[R_T]$ loads negatively (and significantly) on $\{R_T^{\text{straddle}} - R_T^{\text{strangle}}\}$. See the bootstrap intervals—which do not bracket zero—in Table 3.

I. *Big picture from S&P 500 equity index and options.* Tables 1 through 3 present our findings from implementing Models A, B, and C, respectively (over the 31-year sample period). The takeaway is that the unconditional correlation between $M_T[R_T]R_T$ and R_T, that is, NCC_T, is *positive*. The value of NCC_T ranges from 0.40 (Model B) to 0.50 (Model C).

For each model, we additionally verified that $M_T[R_T]$ does not attain negative values (that is, the minimum value of $M_T[R_T]$ is positive). For economic realism, it is important that $M_T[R_T]$ be positive in every state and that $M_T[R_T]$ prices the gross equity market return R_T.

The methodological appeal of Result 2 is that the sign of the expected conditional covariance between $M_T[R_T]R_T$ and R_T can be ascertained by the sign of the unconditional covariance between $M_T[R_T]R_T$ and R_T. It would suffice to construct *one* counterexample in which the negative correlation condition does not hold.

Table 1 Testing the NCC when $\mathbf{Z}_T[R_T]$ contains the gross return of the equity market and the gross return of an at-the-money straddle (Model A).

		Bootstrap percentiles						
	Estimate	Mean	SD	2.5^{th}	5^{th}	50^{th}	95^{th}	97.5^{th}
Panel A: Estimates of NCC_T								
NCC_T	**0.45**	0.48	0.17	0.23	0.26	0.46	0.84	0.91
p-value, $\mathbb{H}_0 : \text{NCC}_T \leq 0$	{0.000}							
Panel B: Properties of the SDF projection								
α_{market}	**0.83**	0.81	0.08	0.65	0.68	0.82	0.92	0.94
α_{straddle}	**0.17**	0.20	0.09	0.04	0.06	0.18	0.37	0.40
Standard deviation of M_T (annualized)	**0.50**	0.54	0.21	0.19	0.22	0.51	0.91	0.97
Mean of of M_T	**0.991**	0.990	0.003	0.985	0.986	0.990	0.995	0.996
Minimum of M_T	**0.82**	0.80	0.06	0.65	0.67	0.81	0.88	0.89

All reported results rely on data from the S&P 500 equity index options market from January 1990 to December 2020 (31 years, 372 option expiration cycles, $T = 28$ days). The first expiration cycle starts 01/19/1990, and the last expiration cycle starts 12/18/2020. In Model A, the gross returns in $\mathbf{Z}_T[R_T]$ are

$$\mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ R_T^{\text{straddle}} \end{pmatrix}, \quad \text{where } R_T^{\text{straddle}} = \frac{S_t \max(1 - R_T, 0) + S_t \max(R_T - 1, 0)}{\text{put}_{t,T}[S_t] + \text{call}_{t,T}[S_t]}.$$

The option return calculations are based on the option ask prices. The form of $M_T[R_T]$ is

$$M_T[R_T] = \mathbf{Z}_T^\top[R_T]\boldsymbol{\alpha}, \quad \text{and we infer } \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{straddle}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\mathbf{Z}_T^\top[R_T])\}^{-1}\mathbf{1}.$$

We compute $\boldsymbol{\alpha}$ following Cochrane (2005, pages 65–66). Reported is the unconditional correlation between $M_T[R_T]R_T$ and R_T, denoted as NCC_T. We adopt a bootstrap procedure and draw $\mathbf{Z}_T[R_T]$ with replacement. Then, we reestimate $\boldsymbol{\alpha}$. The reported bootstrap percentiles and standard deviation (SD) are based on 10,000 bootstrap samples. Reported also are the p-values, in curly brackets, for the hypothesis $\text{NCC}_T \leq 0$, which represents the proportion of bootstrap replications for which the estimates of $\text{NCC}_T \leq 0$ (that is, the negative correlation condition holds).

The conclusion that we draw is that, in *each* of the three models, the NCC fails to hold even *on average.*

Elaborating on these findings, we pose $\text{cov}^{\mathbb{P}}(M_T[R_T]\,R_T, R_T)$ being negative as an explicit hypothesis and consider a bootstrap procedure. We bootstrap (with replacement) the gross returns in $\mathbf{Z}_T[R_T]$ and reestimate $\boldsymbol{\alpha}$ in the context of Models A, B, and C. Then we reconstruct $M_T[R_T] = \mathbf{Z}_T^\top[R_T]\,\boldsymbol{\alpha}$ and R_T. The key observation is that the 2.5^{th} and 97.5^{th} bootstrap values for NCC_T are positive (do not bracket zero), and we can reject the hypothesis of the NCC.

Complementing our evidence, we report the p-values for the hypothesis $\text{NCC}_T \leq 0$, which represents the proportion of bootstrap replications for which the estimates of $\text{NCC}_T \leq 0$. The p-value is 0.000, 0.001, and 0.000 for Models A, B, and C, respectively, establishing that our exercises do not support the negative correlation condition.

What is the impact of estimation noise in $\boldsymbol{\alpha}$ on the constructed $M_T[R_T]$? Our bootstrap-based evidence—that recomputes $\boldsymbol{\alpha}$ in each bootstrap draw—shows that our findings about NCC not holding appear robust to possible estimation noise.

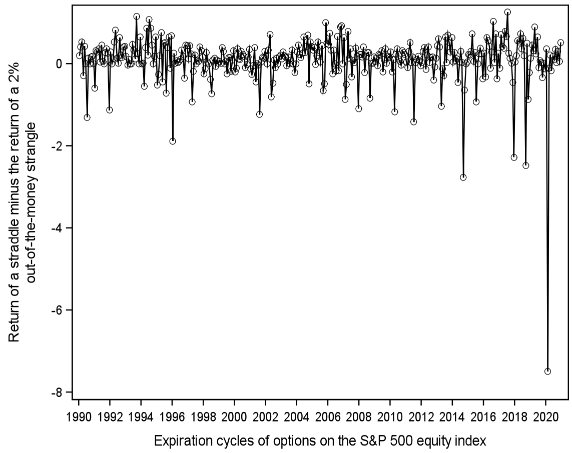

Figure 1 Return of a straddle minus the return of a 2% out-of-the-money strangle.

Plotted is the return of a straddle minus the return of a 2% out-of-the-money (OTM) strangle on the S&P 500 equity index. We compute $R_T^{\text{straddle}} = \frac{S_t \max(1-R_T,0)+S_t \max(R_T-1,0)}{\text{put}_{t,T}[S_t]+\text{call}_{t,T}[S_t]}$ and $R_T^{\text{strangle}} = \frac{S_t \max(e^{-0.02}-R_T,0)+S_t \max(R_T-e^{0.02},0)}{\text{put}_{t,T}[S_t e^{-0.02}]+\text{call}_{t,T}[S_t e^{0.02}]}$. The study period is January 1990 to December 2020 (372 option expiration cycles). We tabulate the summary statistics of the (28-day) $R_T^{\text{straddle}} - R_T^{\text{strangle}}$ series as follows:

Mean	SD	$1_{\{>0\}}$	Min.	Max.	Skewness	Kurtosis
0.09	0.61	77%	−7.5	1.26	−6.1	67.3

where SD is the standard deviation and $1_{\{>0\}}$ is the proportion of the return observations that are positive.

Table 4 (respectively, Table 5) shows evidence based on 13 expanding (respectively, 12 rolling) samples. Our findings about NCC being negated is valid under a variety of market conditions. The low p-values for $\text{NCC}_T \leq 0$ indicate rejection.

Our evidence refutes the notion that the NCC holds *point by point.* In essence, there is reliable evidence (from Tables 1, 2, 3, and 4) that the NCC is *not* a generic property, and, consequently, $R_{f,t}^{-1} \text{var}_t^{\mathbb{Q}}(R_T)$ may sometimes be an *upper* bound and not a *lower* bound of the equity premium.

What is special about Models A, B, and C? The distinguishing attribute is the dependence of $M_T[R_T]$ on a convex function (that is, $M_T''[R_T] \equiv \frac{d^2 M_T[R_T]}{d R_T^2} > 0$) of R_T that manifests decreasing and increasing regions. This is visually affirmed through the U-shape of the extracted M_T in Figure 2. The economic driver is equity volatility. Essentially, $M_T[R_T]$ tends to be high when R_T^{straddle} or R_T^{strangle} is high, which, intuitively, reflects sensitivity to equity volatility.

$M_T[R_T]$ with the said properties has been studied and is consistent with the empirical observation of *negative* average returns of deep OTM call options on the equity market index.[2] The model class considered can be differentiated from the ones in Martin (2017), which he employs to convey the empirical relevance of the NCC. We establish the reliability of our conclusions about the NCC by implementing three additional models of M_T in the Internet Appendix. See Table I.2.

II. *Evidence from options on the STOXX 50 equity index is consistent in* not *supporting the premise of the NCC.* From OptionMetrics, we construct options data on the STOXX 50 equity index and present our findings from implementing Models A, B, and C in Table 6.

Table 2 Testing the NCC when $\mathbf{Z}_T[R_T]$ contains the gross return of the equity market and the gross return of a 2% out-of-the-money strangle (Model B).

	Estimate	Bootstrap percentiles						
		Mean	SD	2.5^{th}	5^{th}	50^{th}	95^{th}	97.5^{th}
Panel A: Estimates of NCC_T								
NCC_T	**0.40**	0.43	0.18	0.17	0.20	0.39	0.79	0.88
p-value, $\mathbb{H}_0 : \text{NCC}_T \leq 0$	{0.001}							
Panel B: Properties of the SDF projection								
α_{market}	**0.89**	0.87	0.05	0.75	0.77	0.88	0.95	0.96
α_{strangle}	**0.12**	0.15	0.08	0.03	0.04	0.13	0.30	0.33
Standard deviation of M_T (annualized)	**0.54**	0.60	0.25	0.19	0.24	0.57	1.06	1.16
Mean of M_T	**0.991**	0.991	0.003	0.985	0.986	0.991	0.995	0.996
Minimum of M_T	**0.87**	0.83	0.04	0.73	0.75	0.84	0.89	0.90

All reported results rely on data from the S&P 500 equity index options market from January 1990 to December 2020 (31 years, 372 option expiration cycles, $T = 28$ days). The first expiration cycle starts 01/19/1990, and the last expiration cycle starts 12/18/2020. In Model B, the gross returns in $\mathbf{Z}_T[R_T]$ are

$$\mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ R_T^{\text{strangle}} \end{pmatrix}, \quad \text{where } R_T^{\text{strangle}} = \frac{S_t \max(e^{-0.02} - R_T, 0) + S_t \max(R_T - e^{0.02}, 0)}{\text{put}_{t,T}[S_t e^{-0.02}] + \text{call}_{t,T}[S_t e^{0.02}]}.$$

The option return calculations are based on the option ask prices. The form of $M_T[R_T]$ is

$$M_T[R_T] = \mathbf{Z}_T^\top[R_T]\boldsymbol{\alpha}, \quad \text{and we infer } \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{strangle}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\mathbf{Z}_T^\top[R_T])\}^{-1}\mathbf{1}.$$

We compute $\boldsymbol{\alpha}$ following Cochrane (2005, pages 65–66). Reported is the unconditional correlation between $M_T[R_T]R_T$ and R_T, denoted as NCC_T. We adopt a bootstrap procedure and draw $\mathbf{Z}_T[R_T]$ with replacement. Then, we reestimate $\boldsymbol{\alpha}$. The reported bootstrap percentiles and standard deviation (SD) are based on 10,000 bootstrap samples. Reported also are the p-values, in curly brackets, for the hypothesis $\text{NCC}_T \leq 0$, which represents the proportion of bootstrap replications for which the estimates of $\text{NCC}_T \leq 0$ (that is, the negative correlation condition holds).

The sample period is January 2002 to December 2019 (216 option expiration cycles) and these options expire on the third Friday of each month ($T = 28$ days). The STOXX 50 equity index covers an important equity market segment in the Eurozone and the options are actively traded.

Our results based on the STOXX 50 equity index options share a number of features with their S&P 500 equity index counterparts. First and foremost, the estimates of NCC_T are positive (range between 0.46 and 0.62), and the hypothesis of $\text{NCC}_T \leq 0$ is rejected with the highest p-value of 0.012. In line with our evidence from S&P 500 equity index options, the 2.5^{th} and 97.5^{th} bootstrap values for NCC_T are positive for each of Models A, B, and C.[3]

The implication is that our findings about the sign of NCC_T are not specific to the United States and also hold for M_T that are constructed from (index and options) data on the European equity market. Our conclusions about the NCC are reinforced because the sample periods of our empirical evaluation do not fully intersect across the two equity markets.

Table 3 Testing the NCC when $\mathbf{Z}_T[R_T]$ contains the gross return of the equity market and the gross return of a position that reflects the return spread between a straddle and a 2% out-of-the-money strangle (Model C).

		Bootstrap percentiles							
	Estimate	Mean	SD	2.5^{th}	5^{th}	50^{th}	95^{th}	97.5^{th}
Panel A: Estimates of NCC_T								
NCC_T	**0.50**	0.52	0.18	0.26	0.28	0.49	0.88	0.95
p-value, $\mathbb{H}_0 : \text{NCC}_T \leq 0$	{0.000}							
Panel B: Properties of the SDF projection								
α_{market}	**1.24**	1.32	0.23	1.03	1.05	1.26	1.76	1.88
$\alpha_{\text{straddle minus strangle}}$	**−0.24**	−0.31	0.20	−0.80	−0.70	−0.26	−0.07	−0.04
Standard deviation of M_T (annualized)	0.49	0.56	0.25	0.18	0.20	0.51	1.02	1.14
Mean of M_T	0.990	0.990	0.003	0.984	0.985	0.990	0.994	0.995
Minimum of M_T	0.68	0.61	0.20	0.08	0.19	0.68	0.79	0.81

All reported results rely on data from the S&P 500 equity index options market from January 1990 to December 2020 (31 years, 372 option expiration cycles, $T = 28$ days). The first expiration cycle starts 01/19/1990, and the last expiration cycle starts 12/18/2020. In Model C, the gross returns in $\mathbf{Z}_T[R_T]$ are

$$\mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ 1 + \underbrace{\{R_T^{\text{straddle}} - R_T^{\text{strangle}}\}}_{\text{return spread}} \end{pmatrix},$$

where $R_T^{\text{straddle}} = \frac{S_t \max(1-R_T,0)+S_t \max(R_T-1,0)}{\text{put}_{t,T}[S_t]+\text{call}_{t,T}[S_t]}$ and $R_T^{\text{strangle}} = \frac{S_t \max(e^{-0.02}-R_T,0)+S_t \max(R_T-e^{0.02},0)}{\text{put}_{t,T}[S_t e^{-0.02}]+\text{call}_{t,T}[S_t e^{0.02}]}$. The option return calculations are based on the option ask prices. The form of $M_T[R_T]$ is

$$M_T[R_T] = \mathbf{Z}_T^{\top}[R_T]\,\boldsymbol{\alpha}, \quad \text{and we infer } \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{straddle minus strangle}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^{\top}[R_T])\}^{-1}\,\mathbf{1}.$$

We compute $\boldsymbol{\alpha}$ following Cochrane (2005, pages 65–66). Reported is the unconditional correlation between $M_T[R_T]R_T$ and R_T, denoted as NCC_T. We adopt a bootstrap procedure and draw $\mathbf{Z}_T[R_T]$ with replacement. Then, we reestimate $\boldsymbol{\alpha}$. The reported bootstrap percentiles and standard deviation (SD) are based on 10,000 bootstrap samples. Reported also are the p-values, in curly brackets, for the hypothesis $\text{NCC}_T \leq 0$, which represents the proportion of bootstrap replications for which the estimates of $\text{NCC}_T \leq 0$ (that is, the negative correlation condition holds).

3.2 *Identifying economic states when the NCC is not supported*

In light of Equations (7)–(8) evaluated at $n = 1$, we consider $\frac{M_T'[R_T]}{M_T[R_T]} + \frac{1}{R_T}$ and develop example economies in which $\frac{1}{R_T}$ is sufficiently positive to counteract the possible negative value of $\frac{M_T'[R_T]}{M_T[R_T]}$.

Complementing our empirical approach to assess the sign of NCC_T unconditionally, the essential point of the theoretical approach is to present economies that disprove the premise of the NCC in certain economic states. If the NCC is provably nullified in certain economic states, the generality and relevance of the lower bound expression for the equity premium is in doubt.

We focus here on $M_T[R_T]$ that are positive by construction, are differentiable in R_T, and enable the analytical tractability of $\frac{M_T'[R_T]}{M_T[R_T]}$.

Table 4 Expanding sample evidence on testing the NCC (Models A, B, and C).

				Model A Minimum of M_T is 0.70				**Model B** Minimum of M_T is 0.77				**Model C** Minimum of M_T is 0.29			
	Start	End	N	NCC_T	p-val. $\text{NCC}_T \leq 0$	α_{market}	α_{straddle}	NCC_T	p-val. $\text{NCC}_T \leq 0$	α_{market}	α_{strangle}	NCC_T	p-val. $\text{NCC}_T \leq 0$	α_{market}	$\alpha_{\text{straddle minus strangle}}$
1	1990:01	2007:12	216	**0.33**	0.00	0.70	0.35	**0.27**	0.01	0.79	0.28	**0.34**	0.00	1.71	−0.66
2	1990:01	2008:12	228	**0.29**	0.00	0.73	0.32	**0.23**	0.02	0.80	0.27	**0.34**	0.00	1.75	−0.68
3	1990:01	2009:12	240	**0.28**	0.00	0.71	0.34	**0.23**	0.02	0.79	0.29	**0.35**	0.00	1.76	−0.70
4	1990:01	2010:12	252	**0.28**	0.00	0.72	0.33	**0.23**	0.01	0.79	0.27	**0.34**	0.00	1.74	−0.69
5	1990:01	2011:12	264	**0.28**	0.00	0.73	0.31	**0.22**	0.02	0.80	0.25	**0.33**	0.00	1.70	−0.65
6	1990:01	2012:12	276	**0.28**	0.00	0.73	0.30	**0.22**	0.02	0.80	0.25	**0.33**	0.00	1.71	−0.66
7	1990:01	2013:12	288	**0.31**	0.00	0.74	0.29	**0.23**	0.01	0.80	0.25	**0.33**	0.00	1.72	−0.67
8	1990:01	2014:12	300	**0.33**	0.00	0.75	0.27	**0.27**	0.01	0.82	0.22	**0.38**	0.00	1.53	−0.50
9	1990:01	2015:12	312	**0.31**	0.00	0.75	0.28	**0.25**	0.00	0.82	0.22	**0.36**	0.00	1.55	−0.52
10	1990:01	2016:12	324	**0.30**	0.00	0.74	0.29	**0.24**	0.01	0.81	0.23	**0.35**	0.00	1.58	−0.55
11	1990:01	2017:12	336	**0.33**	0.00	0.75	0.27	**0.27**	0.00	0.82	0.22	**0.38**	0.00	1.52	−0.49
12	1990:01	2018:12	348	**0.35**	0.00	0.77	0.25	**0.28**	0.00	0.83	0.20	**0.38**	0.00	1.47	−0.44
13	1990:01	2019:12	360	**0.36**	0.00	0.77	0.25	**0.28**	0.00	0.83	0.20	**0.37**	0.00	1.49	−0.46

All reported results rely on data (13 expanding samples) from the S&P 500 equity index options market. In each model, $M_T[R_T] = \mathbf{Z}_T^{\top}[R_T]\,\boldsymbol{\alpha}$. We consider the following:

$$\underline{\text{Model A}}: \ \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ R_T^{\text{straddle}} \end{pmatrix}, \quad \text{and we infer} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{straddle}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^{\top}[R_T])\}^{-1}\,\mathbf{1},$$

$$\underline{\text{Model B}}: \ \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ R_T^{\text{strangle}} \end{pmatrix}, \quad \text{and we infer} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{strangle}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^{\top}[R_T])\}^{-1}\,\mathbf{1},$$

$$\underline{\text{Model C}}: \ \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ 1 + \{R_T^{\text{straddle}} - R_T^{\text{strangle}}\} \end{pmatrix}, \quad \text{and we infer} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{straddle minus strangle}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^{\top}[R_T])\}^{-1}\,\mathbf{1}.$$

Reported is the unconditional correlation between $M_T[R_T]R_T$ and R_T, denoted as NCC_T. We adopt a bootstrap procedure and draw $\mathbf{Z}_T[R_T]$ with replacement. Then, we reestimate $\boldsymbol{\alpha}$. The reported bootstrap percentiles and standard deviation (SD) are based on 10,000 bootstrap samples. Reported also are the p-values for the hypothesis $\text{NCC}_T \leq 0$, which represents the proportion of bootstrap replications for which the estimates of $\text{NCC}_T \leq 0$ (that is, the negative correlation condition holds). N is the number of time-series observations in the expanding sample. *The extracted M_T for each model is positive in every state.*

Table 5 Rolling sample evidence on testing the NCC (Models A, B, and C).

				Model A Minimum of M_T is 0.70				**Model B** Minimum of M_T is 0.77				**Model C** Minimum of M_T is 0.27			
	Start	End	N	NCC_T	p-val. $\text{NCC}_T \leq 0$	α_{market}	α_{straddle}	NCC_T	p-val. $\text{NCC}_T \leq 0$	α_{market}	α_{strangle}	NCC_T	p-val. $\text{NCC}_T \leq 0$	α_{market}	$\alpha_{\text{straddle minus strangle}}$
1	1990:01	2009:12	240	**0.28**	0.00	0.71	0.34	**0.23**	0.02	0.79	0.29	**0.35**	0.00	1.76	−0.70
2	1991:01	2010:12	240	**0.30**	0.00	0.71	0.33	**0.24**	0.01	0.79	0.28	**0.35**	0.00	1.77	−0.71
3	1992:01	2011:12	240	0.26	0.00	0.72	0.32	0.18	0.04	0.79	0.27	**0.29**	0.00	1.78	−0.72
4	1993:01	2012:12	240	**0.29**	0.00	0.74	0.29	**0.21**	0.03	0.80	0.24	**0.31**	0.00	1.71	−0.66
5	1994:01	2013:12	240	**0.37**	0.00	0.77	0.24	**0.26**	0.01	0.82	0.21	**0.35**	0.00	1.66	−0.62
6	1995:01	2014:12	240	**0.46**	0.00	0.80	0.20	**0.39**	0.00	0.85	0.16	**0.51**	0.00	1.37	−0.37
7	1996:01	2015:12	240	**0.37**	0.00	0.78	0.24	**0.32**	0.01	0.84	0.18	**0.48**	0.00	1.39	−0.39
8	1997:01	2016:12	240	**0.32**	0.00	0.76	0.26	**0.25**	0.01	0.83	0.21	**0.38**	0.00	1.53	−0.50
9	1998:01	2017:12	240	**0.38**	0.00	0.79	0.22	**0.28**	0.02	0.84	0.18	**0.39**	0.00	1.46	−0.44
10	1999:01	2018:12	240	**0.40**	0.00	0.81	0.20	**0.28**	0.01	0.86	0.17	**0.36**	0.00	1.44	−0.41
11	2000:01	2019:12	240	**0.43**	0.00	0.82	0.18	**0.27**	0.01	0.86	0.16	**0.34**	0.00	1.45	−0.43
12	2001:01	2020:12	240	**0.78**	0.00	0.91	0.08	**0.65**	0.00	0.93	0.07	**0.63**	0.00	1.16	−0.17

All reported results rely on data (12 rolling samples) from the S&P 500 equity index options market. In each model, $M_T[R_T] = \mathbf{Z}_T^\top[R_T]\boldsymbol{\alpha}$. We consider the following:

$$\underline{\text{Model A}}: \ \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ R_T^{\text{straddle}} \end{pmatrix}, \quad \text{and we infer} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{straddle}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^\top[R_T])\}^{-1}\,\mathbf{1},$$

$$\underline{\text{Model B}}: \ \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ R_T^{\text{strangle}} \end{pmatrix}, \quad \text{and we infer} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{strangle}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^\top[R_T])\}^{-1}\,\mathbf{1},$$

$$\underline{\text{Model C}}: \ \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ 1 + \{R_T^{\text{straddle}} - R_T^{\text{strangle}}\} \end{pmatrix}, \quad \text{and we infer} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{straddle minus strangle}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^\top[R_T])\}^{-1}\,\mathbf{1}.$$

Reported is the unconditional correlation between $M_T[R_T]R_T$ and R_T, denoted as NCC_T. We adopt a bootstrap procedure and draw $\mathbf{Z}_T[R_T]$ with replacement. Then, we reestimate $\boldsymbol{\alpha}$. The reported bootstrap percentiles and standard deviation (SD) are based on 10,000 bootstrap samples. Reported also are the p-values for the hypothesis $\text{NCC}_T \leq 0$, which represents the proportion of bootstrap replications for which the estimates of $\text{NCC}_T \leq 0$ (that is, the negative correlation condition holds). N is the number of time-series observations in the expanding sample. *The extracted M_T for each model is positive in every state.*

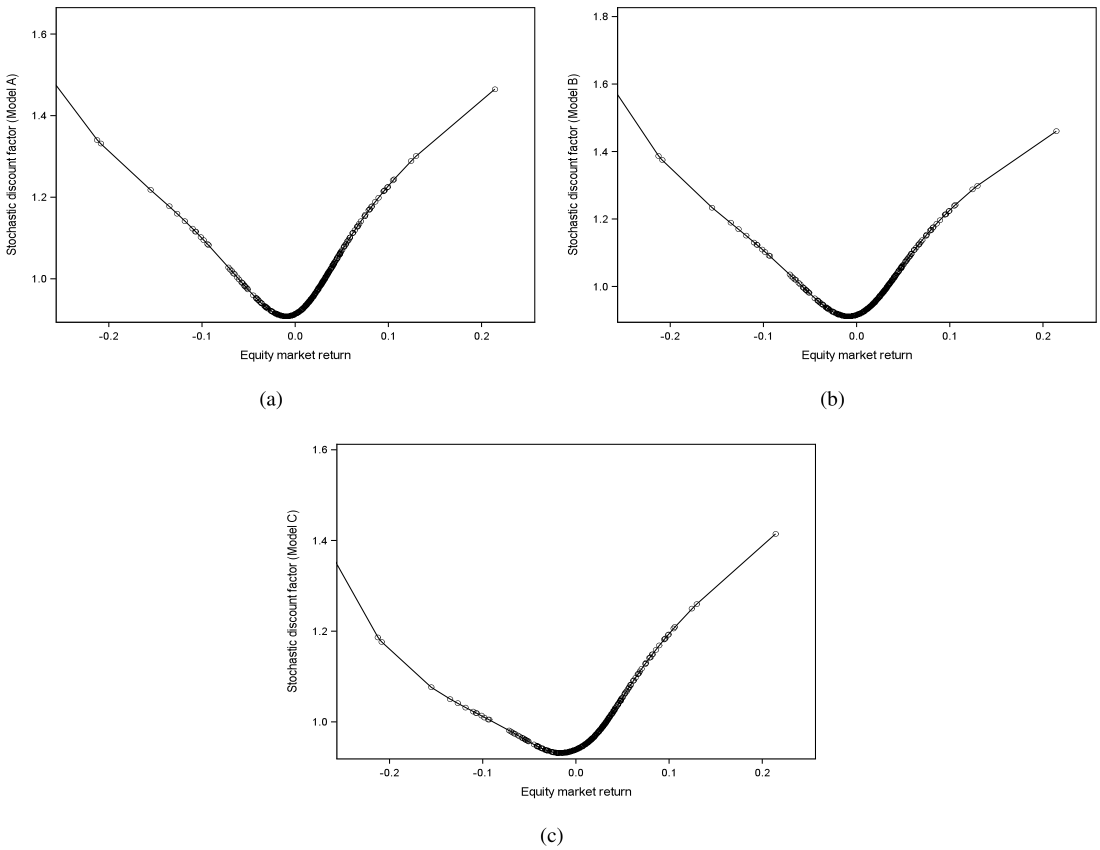

Figure 2 Shape of the extracted SDF's (M_T) for Models A, B, and C.

Plotted is M_T as a function of market return for the estimated sensitivity coefficients reported in Tables 1 through 3. Each M_T prices the market in combination with option returns.

Economy 1. Suppose

$$M_T[R_T] = \exp\left(m_0 + \Lambda_{\text{straddle}} \underbrace{R_T^{\text{straddle}}}_{\substack{\text{gross return}\\ \text{of straddle}}} \right), \quad \text{with } \Lambda_{\text{straddle}} > 0. \tag{18}$$

Then,

$$\frac{M_T'[R_T]}{M_T[R_T]} + \frac{1}{R_T} = \begin{cases} \dfrac{\Lambda_{\text{straddle}}\, S_t}{\text{call}_{t,T}[S_t] + \text{put}_{t,T}[S_t]} + \dfrac{1}{R_T} \;\boxed{\;>\;0\;} & \text{if } R_T > 1 \\ -\frac{\Lambda_{\text{straddle}}\, S_t}{\text{call}_{t,T}[S_t] + \text{put}_{t,T}[S_t]} + \frac{1}{R_T} & \text{if } R_T < 1. \end{cases} \tag{19}$$

The SDF in Equation (18) depends on the return of a variable that is sensitive to volatility (e.g., Fung and Hsieh, 2001), has a decreasing region, has an increasing region, and is convex in R_T. ♣

Economy 2. Suppose, for constants m_0, $\upsilon > 0$, and $\delta > 0$,

$$M_T[R_T] = \exp\left(m_0 + \frac{\upsilon}{2} \underbrace{\frac{R_T^2}{\mathbf{q}_{t,\{R_T^2\}}}}_{\substack{\text{gross return}\\ \text{of squared contract}}} + \delta \underbrace{\frac{\frac{1}{R_T}}{\mathbf{q}_{t,\{\frac{1}{R_T}\}}}}_{\substack{\text{gross return}\\ \text{of inverse contract}}} \right), \quad \text{for } R_T \in [\underline{R}, \overline{R}], \tag{20}$$

Table 6 Evidence on the NCC based on the STOXX 50 equity index and options.

	Estimate	Mean	SD	Bootstrap percentiles 2.5th	5th	50th	95th	97.5th
Model A: $\mathbf{Z}_T[R_T]$ contains R_T and R_T^{straddle}								
NCC_T	**0.54**	0.55	0.30	0.05	0.10	0.53	0.99	1.00
p-value, $\mathbb{H}_0$: $\text{NCC}_T \leq 0$	{0.011}							
α_{market}	**0.90**	0.89	0.07	0.74	0.76	0.89	1.01	1.03
α_{straddle}	**0.11**	0.12	0.09	−0.03	−0.01	0.11	0.29	0.32
Standard deviation of M_T (annualized)	**0.33**	0.40	0.21	0.16	0.16	0.35	0.82	0.88
Mean of M_T	**0.998**	0.998	0.003	0.992	0.993	0.998	1.004	1.005
Minimum of M_T	**0.86**	0.84	0.05	0.70	0.73	0.85	0.90	0.91
Model B: $\mathbf{Z}_T[R_T]$ contains R_T and R_T^{strangle}								
NCC_T	**0.46**	0.49	0.29	0.03	0.08	0.45	0.99	1.00
p-value, $\mathbb{H}_0$: $\text{NCC}_T \leq 0$	{0.012}							
α_{market}	**0.92**	0.91	0.06	0.79	0.81	0.91	0.99	1.01
α_{strangle}	**0.09**	0.11	0.08	0.00	0.00	0.10	0.25	0.30
Standard deviation of M_T (annualized)	**0.40**	0.46	0.24	0.16	0.17	0.42	0.92	1.04
Mean of M_T	**0.999**	0.998	0.003	0.992	0.993	0.998	1.004	1.005
Minimum of M_T	**0.86**	0.85	0.04	0.75	0.78	0.85	0.90	0.90
Model C: $\mathbf{Z}_T[R_T]$ contains R_T and $(1 + \{R_T^{\text{straddle}} - R_T^{\text{strangle}}\})$								
NCC_T	**0.62**	0.64	0.21	0.30	0.34	0.60	0.99	1.00
p-value, $\mathbb{H}_0$: $\text{NCC}_T \leq 0$	{0.000}							
α_{market}	**1.23**	1.29	0.24	0.96	0.99	1.24	1.74	1.89
$\alpha_{\text{straddle minus strangle}}$	**−0.22**	−0.27	0.21	−0.79	−0.67	−0.23	0.01	0.03
Standard deviation of M_T (annualized)	**0.42**	0.48	0.25	0.17	0.18	0.44	0.94	1.08
Mean of M_T	**0.997**	0.997	0.003	0.990	0.991	0.997	1.002	1.003
Minimum of M_T	**0.78**	0.70	0.14	0.31	0.40	0.76	0.83	0.84

Reported are the findings based on the STOXX 50 equity index and its (European-style) options. The sample period of STOXX 50 equity index options is from January 2002 to December 2019 (18 years, 216 option expiration cycles). The options data is from OptionMetrics. The first expiration cycle starts 01/18/2002, and the last expiration cycle starts 12/20/2019. The form of $M_T[R_T]$ is

$$M_T[R_T] = \mathbf{Z}_T^\top[R_T]\boldsymbol{\alpha}, \quad \text{and we infer } \boldsymbol{\alpha} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\mathbf{Z}_T^\top[R_T])\}^{-1}\mathbf{1}.$$

We consider the following models:

$$\underline{\text{Model A}}: \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ R_T^{\text{straddle}} \end{pmatrix}, \quad \underline{\text{Model B}}: \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ R_T^{\text{strangle}} \end{pmatrix}, \quad \underline{\text{Model C}}: \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ 1 + \{R_T^{\text{straddle}} - R_T^{\text{strangle}}\} \end{pmatrix}.$$

We compute $R_T^{\text{straddle}} = \frac{S_t \max(1-R_T,0)+S_t \max(R_T-1,0)}{\text{put}_{t,T}[S_t]+\text{call}_{t,T}[S_t]}$, $R_T^{\text{strangle}} = \frac{S_t \max(e^{-0.02}-R_T,0)+S_t \max(R_T-e^{0.02},0)}{\text{put}_{t,T}[S_t e^{-0.02}]+\text{call}_{t,T}[S_t e^{0.02}]}$, and $\boldsymbol{\alpha}$ following Cochrane (2005, pages 65–66).

Reported is the unconditional correlation between $M_T[R_T]R_T$ and R_T, denoted as NCC_T. We adopt a bootstrap procedure and draw $\mathbf{Z}_T[R_T]$ with replacement. Then, we reestimate $\boldsymbol{\alpha}$. The reported bootstrap percentiles and standard deviation (SD) are based on 10,000 bootstrap samples. Reported also are the p-values, in curly brackets, for the hypothesis $\text{NCC}_T \leq 0$, which represents the proportion of bootstrap replications for which the estimates of $\text{NCC}_T \leq 0$ (that is, the negative correlation condition holds).

where $\underline{R} > 0$ and $\overline{R} < \infty$. Here $\mathbf{q}_{t,\{R_T^2\}} = R_{f,t}^{-1}\,\mathbb{E}_t^{\mathbb{Q}}(R_T^2)$ (respectively, $\mathbf{q}_{t,\{\frac{1}{R_T}\}} = R_{f,t}^{-1}\,\mathbb{E}_t^{\mathbb{Q}}(R_T^{-1})$) is the price of the payoff R_T^2 (respectively, $\frac{1}{R_T}$) and can be synthesized via a positioning in options.

Then,

$$\frac{M_T'[R_T]}{M_T[R_T]} + \frac{1}{R_T} = \frac{\frac{\upsilon}{\mathbf{q}_{t,\left\{R_T^2\right\}}} R_T^3 + R_T - \frac{\delta}{\mathbf{q}_{t,\left\{\frac{1}{R_T}\right\}}}}{R_T^2}. \quad (21)$$

With $\upsilon > 0$, $\frac{\upsilon}{\mathbf{q}_{t,\{R_T^2\}}} R_T^3 + R_T$ is strictly increasing in R_T, which implies that the equation $\frac{\upsilon}{\mathbf{q}_{t,\{R_T^2\}}} R_T^3 + R_T - \frac{\delta}{\mathbf{q}_{t,\{\frac{1}{R_T}\}}} = 0$ has a unique real solution given by Weisstein (2010, equation (80)), as follows:

$$R_T^{\text{critical}} = -2\sqrt{\frac{\mathbf{q}_{t,\{R_T^2\}}}{3\upsilon}}\sinh \times \left(\frac{1}{3}\text{arcsinh}\left(-\frac{3\delta}{2\mathbf{q}_{t,\left\{\frac{1}{R_T}\right\}}}\sqrt{\frac{3\upsilon}{\mathbf{q}_{t,\left\{R_T^2\right\}}}}\right)\right). \quad (22)$$

R_T^{critical} is crucial for determining the sign of $\frac{M_T'[R_T]}{M_T[R_T]} + \frac{1}{R_T}$ and, thus, of $\text{cov}_t^{\mathbb{P}}(M_T[R_T]R_T, R_T)$.

Suppose $R_T^{\text{critical}} < \underline{R}$. Then, since $R_T \in [\underline{R}, \overline{R}]$, we have $R_T > R_T^{\text{critical}}$, and the numerator of Equation (21) is positive. Therefore, $\text{cov}_t^{\mathbb{P}}(M_T[R_T]\,R_T, R_T) > 0$, and the NCC is violated. ♣

Economy 3 Suppose $M_T[R_T]$ is the *weighted* sum of a completely monotone function and an absolutely monotone function of the type (motivated by Bakshi *et al.*, 2010)

$$M_T[R_T] = w\,R_T^{-\theta} + (1-w)\left(\frac{1}{R_T}\right)^{-\theta},$$
$$\text{for some constants } \theta > 1 \text{ and } 0 < w < 1. \quad (23)$$

This $M_T[R_T]$ is convex in R_T, and is negatively (positively) sloped at low (large) R_T, with $(\frac{1}{R_T})^{-\theta}$ reflecting the marginal utility of agents long the *inverse of the equity market* (that is, their wealth declines in the region $R_T > 1$).

Consequently, we have the following result:

$$\frac{M_T'[R_T]}{M_T[R_T]} + \frac{1}{R_T} = \frac{\frac{\theta}{R}\{-w\,R_T^{-\theta} + (1-w)\,R_T^{\theta}\}}{w\,R_T^{-\theta} + (1-w)\,R_T^{\theta}} + \frac{1}{R_T} > 0,$$
$$\text{when } R_T > \left(\frac{w\,(\theta-1)}{(1-w)(\theta+1)}\right)^{\frac{1}{2\theta}}. \quad (24)$$

Thus, $\text{cov}_t^{\mathbb{P}}(M_T\,R_T, R_T)$ can be positive in certain economic states, which poses a challenge to the notion of the NCC.[4] ♣

4 Conclusion

The statement of the negative correlation condition is that $\text{cov}_t^{\mathbb{P}}(M_T\,R_T, R_T) \leq 0$, for all t and for all M_T, where M_T is the stochastic discount factor and R_T is the gross equity market return. The assumption of the negative correlation condition is crucial to Martin (2017), who shows that the discounted risk-neutral variance (that is, $R_{f,t}^{-1}\,\text{var}_t^{\mathbb{Q}}(R_T)$) represents the lower bound of the conditional equity premium.

But does the negative correlation condition hold for all M_T and in every economic state? In this paper, we address this concern both empirically and theoretically.

Our empirical approach rejects the hypothesis that the negative correlation condition is a representation of the asset market data in every economic state. Essentially, we feature empirical counterexamples in which the estimate of $\text{cov}_t^{\mathbb{P}}(M_T\, R_T, R_T)$ is positive. We find that for each of the constructed M_T, the negative correlation condition fails to hold even on average—let alone for all t. In particular, our bootstrap procedure and associated p-values convey the statistical rejection of the negative correlation condition. Our empirical findings are robust and rely on data drawn from the S&P 500 equity index, as well as on the STOXX 50 equity index, and their options.

In our investigation, we consider empirical counterexamples of M_T that are market variance-dependent and support correct pricing of equity index option returns (and also the equity market return). The consistency of the SDF M_T with equity index option returns is pertinent, as the lower bound of the equity premium is itself extracted from options data.

We emphasize that the applicability of the lower bound on the equity premium rests on the premise of the negative correlation condition holding in every state. Taken all together, we show that the negative correlation condition is not supported in empirical and theoretical models of M_T that admit an increasing region to the upside of market returns. Our empirical evidence undermines the universality of the negative correlation condition.

The equity premium is one of the most important numbers in financial economics. Crucial to theory and practice, knowledge of the lower bound of the equity premium is of limited value if the NCC were violated. The problem of determining the conditional equity premium—or even computing a universal lower bound—as rooted in the tradition of Merton (1980) and Black (1993), still needs consensus and resolution.

Appendix

A Proof of Lemma 1 (sign of conditional covariance)

In what follows, we suppress the subscript T on the random variable X_T.

Since $\mathbb{E}_t^{\mathbb{P}}(X)$ is in the domain of g, we have

$$\begin{aligned}
&\text{cov}_t^{\mathbb{P}}(g[X], X) \\
&\quad = \mathbb{E}_t^{\mathbb{P}}(\{X - \mathbb{E}_t^{\mathbb{P}}(X)\}\{g[X] - \mathbb{E}_t^{\mathbb{P}}(g[X])\}) \qquad \text{(A.1)}
\end{aligned}$$

$$\begin{aligned}
&\quad = \mathbb{E}_t^{\mathbb{P}}(\{X - \mathbb{E}_t^{\mathbb{P}}(X)\}\{g[X] - g[\mathbb{E}_t^{\mathbb{P}}(X)]\}) \\
&\qquad + \mathbb{E}_t^{\mathbb{P}}(\{X - \mathbb{E}_t^{\mathbb{P}}(X)\}\{g[\mathbb{E}_t^{\mathbb{P}}(X)] \\
&\qquad - \mathbb{E}_t^{\mathbb{P}}(g[X])\}) \\
&\quad = \mathbb{E}_t^{\mathbb{P}}(\{X - \mathbb{E}_t^{\mathbb{P}}(X)\}\{g[X] - g[\mathbb{E}_t^{\mathbb{P}}(X)]\}) \\
&\qquad + \{g[\mathbb{E}_t^{\mathbb{P}}(X)] - \mathbb{E}_t^{\mathbb{P}}(g[X])\} \\
&\qquad \times \underbrace{\mathbb{E}_t^{\mathbb{P}}(X - \mathbb{E}_t^{\mathbb{P}}(X))}_{=0} \\
&\quad = \mathbb{E}_t^{\mathbb{P}}(\{X - \mathbb{E}_t^{\mathbb{P}}(X)\}\{g[X] - g[\mathbb{E}_t^{\mathbb{P}}(X)]\}). \qquad \text{(A.2)}
\end{aligned}$$

The lemma follows by noticing that if $g[X]$ is decreasing on $\mathcal{D}$, when $X \leq (\geq)\, \mathbb{E}_t^{\mathbb{P}}(X)$, we have $g[X] \geq (\leq)\, g[\mathbb{E}_t^{\mathbb{P}}(X)]$, which implies that $(X - \mathbb{E}_t^{\mathbb{P}}(X))(g[X] - g[\mathbb{E}_t^{\mathbb{P}}(X)]) \leq 0$ everywhere.

Furthermore, if $g[X]$ is increasing on $\mathcal{D}$, when $X \leq (\geq)\, \mathbb{E}_t^{\mathbb{P}}(X)$, we have $g[X] \leq (\geq)\, g[\mathbb{E}_t^{\mathbb{P}}(X)]$, which implies that $(X - \mathbb{E}_t^{\mathbb{P}}(X))(g[X] - g[\mathbb{E}_t^{\mathbb{P}}(X)]) \geq 0$ everywhere. ■

B Proof of Result 1 (restrictions of the NCC)

With the understanding that the length of the period T for M_T can be arbitrary, we again suppress the subscript T on the random variable X_T. It suffices to notice that if $g'[X] \leq (\geq) 0$, then $g[X]$ is decreasing (increasing). Applying Lemma 1 by setting, for any positive integer n,

$$X = R_T^n \quad \text{and} \quad g[X] = M_T[X^{\frac{1}{n}}]X. \tag{B.1}$$

Then,

$$g'[X] = M_T'[X^{\frac{1}{n}}]\frac{1}{n}X^{\frac{1}{n}-1}X + M_T[X^{\frac{1}{n}}] \tag{B.2}$$

$$= \frac{M_T'[R_T]R_T}{n} + M_T[R_T] \tag{B.3}$$

$$= \underbrace{\frac{M_T[R_T]\,R_T}{n}}_{>0}\left(\frac{M_T'[R_T]}{M_T[R_T]} + \frac{n}{R_T}\right). \tag{B.4}$$

Thus, combining Equations (A.2) and (B.4), we have that, if $\frac{M_T'[R_T]}{M_T[R_T]} + \frac{n}{R_T} \leq 0$ almost surely, then $\text{cov}_t^{\mathbb{P}}(M_T[R_T]R_T^n, R_T^n) \leq 0$.

Next, to obtain $\mathbb{E}_t^{\mathbb{P}}(R_T^n)$, notice that

$$\text{cov}_t^{\mathbb{P}}(M_T[R_T]R_T^n, R_T^n) = \mathbb{E}_t^{\mathbb{P}}(M_T[R_T]R_T^{2n}) - \mathbb{E}_t^{\mathbb{P}}(M_T[R_T]\,R_T^n)\mathbb{E}_t^{\mathbb{P}}(R_T^n) \tag{B.5}$$

$$= \mathbb{E}_t^{\mathbb{P}}(M_T[R_T])\mathbb{E}_t^{\mathbb{Q}}(R_T^{2n}) - \mathbb{E}_t^{\mathbb{P}}(M_T[R_T])\mathbb{E}_t^{\mathbb{Q}}(R_T^n)\mathbb{E}_t^{\mathbb{P}}(R_T^n) \tag{B.6}$$

$$= \frac{\mathbb{E}_t^{\mathbb{Q}}(R_T^{2n})}{R_{f,t}} - \frac{\mathbb{E}_t^{\mathbb{Q}}(R_T^n)}{R_{f,t}}\mathbb{E}_t^{\mathbb{P}}(R_T^n). \tag{B.7}$$

Rearranging,

$$\mathbb{E}_t^{\mathbb{P}}(R_T^n) = \frac{\mathbb{E}_t^{\mathbb{Q}}(R_T^{2n}) - R_{f,t}\,\text{cov}_t^{\mathbb{P}}(M_T[R_T]R_T^n, R_T^n)}{\mathbb{E}_t^{\mathbb{Q}}(R_T^n)}. \tag{B.8}$$

In particular, for $n = 1$, and substituting $\mathbb{E}_t^{\mathbb{Q}}(R_T) = R_{f,t}$, we obtain the following:

$$\mathbb{E}_t^{\mathbb{P}}(R_T) = \frac{\mathbb{E}_t^{\mathbb{Q}}(R_T^2) - R_{f,t}\,\text{cov}_t^{\mathbb{P}}(M_T[R_T]R_T, R_T)}{\mathbb{E}_t^{\mathbb{Q}}(R_T)} \tag{B.9}$$

$$= R_{f,t} + \frac{\text{var}_t^{\mathbb{Q}}(R_T)}{R_{f,t}} - \text{cov}_t^{\mathbb{P}}(M_T[R_T]\,R_T, R_T). \tag{B.10}$$

We have provided the intermediate steps of the proof. ■

Complementing theoretical implications, we consider $n = 2$ and derive the following:

$$\mathbb{E}_t^{\mathbb{P}}(R_T^2) = \frac{\mathbb{E}_t^{\mathbb{Q}}(R_T^4) - R_{f,t}\,\text{cov}_t^{\mathbb{P}}(M_T[R_T]R_T^2, R_T^2)}{\mathbb{E}_t^{\mathbb{Q}}(R_T^2)} \tag{B.11}$$

$$= \frac{\mathbb{E}_t^{\mathbb{Q}}(R_T^4)}{\mathbb{E}_t^{\mathbb{Q}}(R_T^2)} - \underbrace{\frac{R_{f,t}}{\mathbb{E}_t^{\mathbb{Q}}(R_T^2)}}_{>0} \times \text{cov}_t^{\mathbb{P}}(M_T[R_T]R_T^2, R_T^2). \tag{B.12}$$

The lower bound on $\mathbb{E}_t^{\mathbb{P}}(R_T^2)$ in Equation (7) is a consequence of Equation (B.12). ■

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Endnotes

[1] Related to our investigation, Back *et al.* (2022) study the accuracy of the equity premium bounds. They use conditional testing and find that the lower bound on equity premium is not tight. This evidence also goes against the usefulness of the negative correlation condition.

[2] We refer the reader to theoretical and empirical defence in, for example, Aït-Sahalia and Lo (2000), Jackwerth (2000), Rosenberg and Engle (2002), Bakshi *et al.* (2010), Christoffersen *et al.* (2013), Hens and Reichlin (2013), Chaudhuri and Schroder (2015), Beare and Schmidt (2016), Wolfgang *et al.* (2017), Schneider and Trojani (2019), and Bakshi *et al.* (2023a). The work of Guo *et al.* (2021) provides a perspective in a consumption-based model.

[3] Equally notable is that Table 6 affirms that the sensitivity coefficients of M_T with respect to R_T, and with respect to R_T^{straddle} and R_T^{strangle}, are positive, while that of $\{R_T^{\text{straddle}} - R_T^{\text{strangle}}\}$ is negative.

[4] The SDF projection in Schneider and Trojani (2019, Section II) is linear in powers of gross returns (they focus on $J = 3$ in implementations; see page 332): $M_T[R_T] = a_0 + a_1\left(\frac{S_{t+T}}{S_t}\right) + a_2\left(\frac{S_{t+T}}{S_t}\right)^2 + a_3\left(\frac{S_{t+T}}{S_t}\right)^3 + \cdots + a_J\left(\frac{S_{t+T}}{S_t}\right)^J$. The essence of Schneider and Trojani is to recover the minimum variance projection of the SDF from option prices and study the recovered equity premium (following Ross, 2015).

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Keywords: Hypothesis and generality of the negative correlation condition; dark matter property; conditional equity premium; options on S&P 500 equity index and STOXX 50 equity index

THE OPTIONS-INFERRED EQUITY PREMIUM AND THE SLIPPERY SLOPE OF THE NEGATIVE CORRELATION CONDITION

Internet Appendix: Not Intended for Publication

We study the generality of the NCC and consider three additional models of M_T in Section A.1. For example, the differentiating aspect of our Model D is that it accommodates an SDF M_T that varies with the returns of 5% out-of-the-money (OTM) puts and 5% OTM calls on the S&P 500 equity index.

A Internet Appendix

A.1 Additional empirical models of M_T that do not support the NCC

The purpose of this section is to consider three additional empirical models of $M_T[R_T]$ that employ out-of-the-money options (or portfolio combinations)—that reflect market variance—to show that the general statement of the NCC is not supported.

Model D: $M_T[R_T] = \mathbf{Z}_T^\top[R_T]\boldsymbol{\alpha}$ and $\mathbf{Z}_T[R_T]$ contains R_T and $(1 + r_T^{\text{tail}})$. Specifically,

$$r_T^{\text{tail}} \equiv \frac{1}{2}\left(\underbrace{\{R_T^{5\%\text{ otm put}} - R_T^{\text{atm put}}\}}_{\text{put return spread}} + \underbrace{\{R_T^{5\%\text{ otm call}} - R_T^{\text{atm call}}\}}_{\text{call return spread}}\right).$$

We estimate

$$\boldsymbol{\alpha} = \begin{pmatrix}\alpha_{\text{market}}\\ \alpha_{\text{tail}}\end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\mathbf{Z}_T^\top[R_T])\}^{-1}\mathbf{1}.$$

As noted in Table I.1, the return r_T^{tail} has a correlation of 0.19 with changes in the return quadratic variation. The return r_T^{tail} has components related to large downside and upside return movements, as follows:

$$R_T^{5\%\text{ otm put}} = \frac{S_t \max(e^{-0.05} - R_T, 0)}{\text{put}_{t,T}[S_t e^{-0.05}]} \quad \text{and}$$

$$R_T^{\text{atm put}} = \frac{S_t \max(1 - R_T, 0)}{\text{put}_{t,T}[S_t]} \qquad \text{(IA.1)}$$

$$R_T^{5\%\text{ otm call}} = \frac{S_t \max(R_T - e^{0.05}, 0)}{\text{call}_{t,T}[S_t e^{0.05}]}, \quad \text{and}$$

$$R_T^{\text{atm call}} = \frac{S_t \max(R_T - 1, 0)}{\text{call}_{t,T}[S_t]}. \qquad \text{(IA.2)}$$

Figure I.1 plots the time-series of r_T^{tail} and shows large returns to realization of tail events.

Model E: $M_T[R_T] = \mathbf{Z}_T^\top[R_T]\boldsymbol{\alpha}$ and $\mathbf{Z}_T[R_T]$ contains R_T and $(1 + \frac{1}{2}\{(R_{f,t} - 1) + r_T^{\text{tail}}\})$. We estimate

$$\boldsymbol{\alpha} = \begin{pmatrix}\alpha_{\text{market}}\\ \alpha_{\text{tail, portfolio}}\end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\mathbf{Z}_T^\top[R_T])\}^{-1}\mathbf{1}.$$

Model F: $M_T[R_T] = \mathbf{Z}_T^\top[R_T]\boldsymbol{\alpha}$ and $\mathbf{Z}_T[R_T]$ contains R_T and $(1 + \frac{1}{2}\{(R_{f,t} - 1) +$

Table I.1 Correlations of variables with changes in the return quadratic variation over option expiration cycles.

	$R_T^{\text{straddle}} - 1$	$R_T^{\text{strangle}} - 1$	$R_T^{\text{straddle}} - R_T^{\text{strangle}}$	r_T^{tail}	$\text{QV}_T - \text{QV}_{T-1}$
$R_T - 1$	−0.21	−0.26	0.30	0.03	−0.53
$R_T^{\text{straddle}} - 1$		0.95	−0.74	0.39	0.44
$R_T^{\text{strangle}} - 1$			−0.91	0.46	0.49
$R_T^{\text{straddle}} - R_T^{\text{strangle}}$				−0.48	−0.48
r_T^{tail}					0.19

All reported results rely on data from the S&P 500 equity index options market from January 1990 to December 2020 (31 years, 372 option expiration cycles, $T = 28$ days). The first expiration cycle starts 01/19/1990, and the last expiration cycle starts 12/18/2020. We define the variables as follows:

$$R_T = \frac{S_{t+T}}{S_t},$$

$$R_T^{\text{straddle}} = \frac{S_t \max(1 - R_T, 0) + S_t \max(R_T - 1, 0)}{\text{put}_{t,T}[S_t] + \text{call}_{t,T}[S_t]},$$

$$R_T^{\text{strangle}} = \frac{S_t \max(e^{-0.02} - R_T, 0) + S_t \max(R_T - e^{0.02}, 0)}{\text{put}_{t,T}[S_t e^{-0.02}] + \text{call}_{t,T}[S_t e^{0.02}]},$$

$$r_T^{\text{tail}} = \frac{1}{2}\left(\underbrace{\{R_T^{5\%\text{ otm put}} - R_T^{\text{atm put}}\}}_{\text{put return spread}} + \underbrace{\{R_T^{5\%\text{ otm call}} - R_T^{\text{atm call}}\}}_{\text{call return spread}} \right),$$

$$\text{QV}_T = \text{return quadratic variation over expiration cycles, and}$$

$$\text{QV}_T - \text{QV}_{T-1} = \text{change in return quadratic variation over adjacent expiration cycles.}$$

Reported is the correlation matrix over the option expiration cycles.

$r_T^{\text{straddle minus strangle}}$}). We estimate

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{straddle minus strangle, portfolio}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^{\top}[R_T])\}^{-1}\,\mathbf{1}.$$

Summary. Table I.2 presents estimates of (i) $\boldsymbol{\alpha}$, (ii) NCC_T, and (iii) p-values for the hypothesis of $\text{NCC}_T \leq 0$. The utilized option combinations allow for model assessments under altered leverage.

Shared with Tables 1 through 3 is the feature that the statement of NCC is not empirically true when M_T depends on the return of deeper OTM puts and calls. We observe that α_{tail} and $\alpha_{\text{tail portfolio}}$ are *positive*. The implication being that a high M_T coincides with tail events, including large market declines. Such an M_T is economically relevant.

Aligning with the objectives at hand, the estimates of NCC_T are positive for each of the models. The p-values for the hypothesis of $\text{NCC}_T \leq 0$ are 0.000, 0.003, and 0.000, for Models D, E, and F, respectively. Our implementations affirm the robustness of our findings through the expanding sample evidence (there are 13 estimates of NCC_T) reported in Table I.3. ■

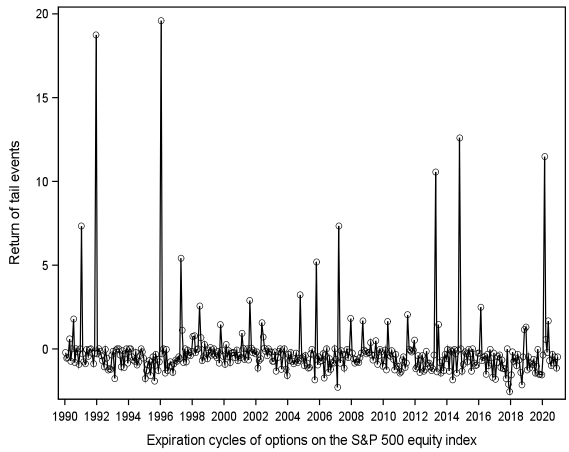

Figure I.1 Return of tail events (r_T^{tail}).

Plotted is the return of tail events on the S&P 500 equity index. We define as follows:

$$r_T^{\text{tail}} \equiv \frac{1}{2}\left(\underbrace{\{R_T^{5\%\text{ otm put}} - R_T^{\text{atm put}}\}}_{\text{put return spread}} + \underbrace{\{R_T^{5\%\text{ otm call}} - R_T^{\text{atm call}}\}}_{\text{call return spread}}\right)$$

The study period is January 1990 to December 2020 (372 option expiration cycles). We tabulate the summary statistics of the (28-day) r_T^{tail} series as follows:

Mean	SD	$1_{\{>0\}}$	Min.	Max.	Skewness	Kurtosis
−0.20	2.06	17%	−2.55	19.6	6.5	50.7

where SD is the standard deviation and $1_{\{>0\}}$ is the proportion of the return observations that are positive.

Table I.2 Testing the negative correlation condition with additional models of M_T.

		Bootstrap Percentiles						
	Estimate	Mean	SD	2.5^{th}	5^{th}	50^{th}	95^{th}	97.5^{th}
Model D: $\mathbf{Z}_T[R_T]$ contains R_T and $(1 + r_T^{\text{tail}})$								
NCC_T	**0.68**	0.64	0.21	0.24	0.28	0.64	0.98	0.99
p-value, $\mathbb{H}_0 : \text{NCC}_T \leq 0$	{0.000}							
α_{market}	0.94	0.93	0.04	0.82	0.85	0.94	0.98	0.98
α_{tail}	0.05	0.07	0.07	−0.01	0.00	0.06	0.21	0.26
Standard deviation of M_T (annualized)	0.39	0.48	0.26	0.17	0.19	0.43	0.98	1.15
Minimum of M_T	0.81	0.75	0.11	0.45	0.53	0.78	0.84	0.86
Model E: $\mathbf{Z}_T[R_T]$ contains R_T and $1 + \frac{1}{2}\{(R_{f,t} - 1) + r_T^{\text{tail}}\}$								
NCC_T	**0.66**	0.62	0.22	0.21	0.25	0.62	0.97	0.99
p-value, $\mathbb{H}_0 : \text{NCC}_T \leq 0$	{0.003}							
α_{market}	0.89	0.85	0.11	0.56	0.64	0.88	0.97	0.99
$\alpha_{\text{tail, portfolio}}$	0.10	0.15	0.15	−0.01	0.01	0.12	0.42	0.53
Standard deviation of M_T (annualized)	0.40	0.49	0.26	0.17	0.19	0.43	0.98	1.18
Minimum of M_T	0.82	0.75	0.12	0.44	0.54	0.79	0.84	0.86

Table I.2 (*Continued*)

	Estimate	Bootstrap Percentiles Mean	SD	2.5^{th}	5^{th}	50^{th}	95^{th}	97.5^{th}
Model F: $\mathbf{Z}_T[R_T]$ contains R_T and $1+\frac{1}{2}\{(R_{f,t}-1)+(R_T^{\text{straddle}}-R_T^{\text{strangle}})\}$								
NCC_T	**0.58**	0.60	0.16	0.35	0.38	0.58	0.92	0.97
p-value, $\mathbb{H}_0 : \text{NCC}_T \leq 0$	{0.000}							
α_{market}	1.44	1.57	0.39	1.04	1.09	1.47	2.33	2.52
$\alpha_{\text{straddle minus strangle, portfolio}}$	−0.44	−0.57	0.37	−1.45	−1.28	−0.47	−0.11	−0.06
Standard deviation of M_T (annualized)	0.46	0.52	0.24	0.18	0.20	0.48	0.95	1.07
Minimum of M_T	0.69	0.62	0.18	0.13	0.24	0.69	0.78	0.80

All reported results rely on data from the S&P 500 equity index options market from January 1990 to December 2020 (31 years, 372 option expiration cycles, $T = 28$ days). The first expiration cycle starts 01/19/1990, and the last expiration cycle starts 12/18/2020. We consider the following models:

$$\underline{\text{Model D}} : M_T[R_T] = \mathbf{Z}_T^\top[R_T]\boldsymbol{\alpha} \quad \text{with } \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ 1 + r_T^{\text{tail}} \end{pmatrix},$$

where

$$r_T^{\text{tail}} \equiv \frac{1}{2}(\underbrace{\{R_T^{5\%\text{ otm put}} - R_T^{\text{atm put}}\}}_{\text{put return spread}} + \underbrace{\{R_T^{5\%\text{ otm call}} - R_T^{\text{atm call}}\}}_{\text{call return spread}}) \quad \text{and}$$

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{tail}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\mathbf{Z}_T^\top[R_T])\}^{-1}\mathbf{1}.$$

$$\underline{\text{Model E}} : M_T[R_T] = \mathbf{Z}_T^\top[R_T]\boldsymbol{\alpha} \quad \text{with } \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ 1 + \frac{1}{2}\{(R_{f,t}-1) + r_T^{\text{tail}}\} \end{pmatrix},$$

where

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{tail, portfolio}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\mathbf{Z}_T^\top[R_T])\}^{-1}\mathbf{1}.$$

$$\underline{\text{Model F}} : M_T[R_T] = \mathbf{Z}_T^\top[R_T]\boldsymbol{\alpha} \quad \text{with } \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ 1 + \frac{1}{2}\{(R_{f,t}-1) + (R_T^{\text{straddle}} - R_T^{\text{strangle}})\} \end{pmatrix},$$

where

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{\text{market}} \\ \alpha_{\text{straddle minus strangle, portfolio}} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\mathbf{Z}_T^\top[R_T])\}^{-1}\mathbf{1}.$$

Reported is the unconditional correlation between $M_T[R_T]R_T$ and R_T, denoted as NCC_T. We adopt a bootstrap procedure and draw $\mathbf{Z}_T[R_T]$ with replacement. Then, we reestimate $\boldsymbol{\alpha}$. The reported bootstrap percentiles and standard deviation (SD) are based on 10,000 bootstrap samples. Reported also are the p-values, in curly brackets, for the hypothesis $\text{NCC}_T \leq 0$, which represents the proportion of bootstrap replications for which the estimates of $\text{NCC}_T \leq 0$ (that is, the negative correlation condition holds).

Table I.3 Expanding sample evidence on testing the NCC (Models D, E, and F).

				Model D Minimum of M_T is 0.77				Model E Minimum of M_T is 0.78				Model F Minimum of M_T is 0.35			
	Start	End	N	NCC_T	p-val. $NCC_T \leq 0$	α_{market}	α_{tail}	NCC_T	p-val. $NCC_T \leq 0$	α_{market}	$\alpha_{tail, portfolio}$	NCC_T	p-val. $NCC_T \leq 0$	α_{market}	$\alpha_{straddle minus strangle, portfolio}$
1	1990:01	2007:12	216	**0.90**	0.00	0.97	0.02	**0.89**	0.01	0.94	0.04	**0.43**	0.00	2.27	−1.22
2	1990:01	2008:12	228	**0.89**	0.00	0.97	0.02	**0.88**	0.00	0.94	0.05	**0.43**	0.00	2.35	−1.30
3	1990:01	2009:12	240	**0.87**	0.00	0.96	0.03	**0.85**	0.01	0.93	0.05	**0.45**	0.00	2.36	−1.31
4	1990:01	2010:12	252	**0.83**	0.00	0.96	0.03	**0.81**	0.01	0.92	0.07	**0.45**	0.00	2.32	−1.27
5	1990:01	2011:12	264	**0.80**	0.00	0.95	0.03	**0.79**	0.01	0.92	0.07	**0.44**	0.00	2.25	−1.21
6	1990:01	2012:12	276	**0.74**	0.01	0.95	0.04	**0.72**	0.01	0.90	0.09	**0.43**	0.00	2.28	−1.24
7	1990:01	2013:12	288	**0.78**	0.00	0.95	0.04	**0.76**	0.00	0.91	0.08	**0.44**	0.00	2.28	−1.24
8	1990:01	2014:12	300	**0.84**	0.00	0.96	0.03	**0.83**	0.00	0.93	0.06	**0.48**	0.00	1.95	−0.92
9	1990:01	2015:12	312	**0.81**	0.00	0.95	0.03	**0.79**	0.00	0.92	0.07	**0.46**	0.00	1.98	−0.95
10	1990:01	2016:12	324	**0.78**	0.00	0.95	0.04	**0.76**	0.00	0.91	0.08	**0.45**	0.00	2.03	−1.01
11	1990:01	2017:12	336	**0.71**	0.00	0.94	0.05	**0.69**	0.00	0.90	0.09	**0.47**	0.00	1.93	−0.90
12	1990:01	2018:12	348	**0.69**	0.00	0.94	0.05	**0.67**	0.00	0.89	0.10	**0.47**	0.00	1.83	−0.82
13	1990:01	2019:12	360	**0.64**	0.00	0.94	0.06	**0.62**	0.00	0.88	0.12	**0.46**	0.00	1.86	−0.84

All reported results rely on data (13 expanding samples) from the S&P 500 equity index options market. In each model, $M_T[R_T] = \mathbf{Z}_T^\top[R_T]\boldsymbol{\alpha}$. Defining, $r_T^{tail} \equiv \frac{1}{2}(\{R_T^{5\% \text{ otm put}} - R_T^{\text{atm put}}\} + \{R_T^{5\% \text{ otm call}} - R_T^{\text{atm call}}\})$, we consider the following:

$$\underline{\text{Model D}}: \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ R_T^{tail} \end{pmatrix}, \quad \text{and we infer} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{market} \\ \alpha_{tail} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\mathbf{Z}_T^\top[R_T])\}^{-1}\,\mathbf{1},$$

$$\underline{\text{Model E}}: \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ 1 + \frac{1}{2}\{(R_{f,t} - 1) + r_T^{tail}\} \end{pmatrix}, \quad \text{and we infer} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{market} \\ \alpha_{tail,\ portfolio} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^\top[R_T])\}^{-1}\mathbf{1},$$

$$\underline{\text{Model F}}: \mathbf{Z}_T[R_T] = \begin{pmatrix} R_T \\ 1 + \frac{1}{2}\{(R_{f,t} - 1) + (R_T^{straddle} - R_T^{strangle})\} \end{pmatrix}, \quad \text{and we infer} \quad \boldsymbol{\alpha} = \begin{pmatrix} \alpha_{market} \\ \alpha_{straddle\ minus\ strangle,\ portfolio} \end{pmatrix} = \{\mathbb{E}^{\mathbb{P}}(\mathbf{Z}_T[R_T]\,\mathbf{Z}_T^\top[R_T])\}^{-1}\,\mathbf{1}.$$

Reported is the unconditional correlation between $M_T[R_T]R_T$ and R_T, denoted as NCC_T. We adopt a bootstrap procedure and draw $\mathbf{Z}_T[R_T]$ with replacement. Then, we reestimate $\boldsymbol{\alpha}$. The reported bootstrap percentiles and standard deviation (SD) are based on 10,000 bootstrap samples. Reported also are the p-values for the hypothesis $NCC_T \leq 0$, which represents the proportion of bootstrap replications for which the estimates of $NCC_T \leq 0$ (that is, the negative correlation condition holds). $\mathbb{N}$ is the number of time-series observations. *The extracted M_T for each model is positive in every state.*