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TILT NICKELS TO DIAMONDS: AN ORTHOGONALIZATION APPROACH

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Alternative index products often achieve improved performance at the cost of increased exposure to risk. In this study, we propose a portfolio tilting strategy that alleviates the risks inherent to alternative indices by projecting fundamental factors on risk factors to purge the influence of risk factors. We argue that more efficient indices can be built on the resulting orthogonalized fundamental factors and show that a tilted equity index using return on assets, long-term-debt, and net sales as fundamental factors outperforms the Russell 1000 index by 120 basis points from 1987 to 2014.

Despite the popularity, a major concern of alternative index products and smart beta indices is their lack of control for risk. In this paper, we propose a portfolio tilting strategy to control risk of alternative index products. We accomplish this by first obtaining orthogonalized fundamental factors to control the influence of risk factors on fundamental factors[1]. Second, we regress the constituent weights of a selected benchmark index on orthogonalized fundamental factors and risk factors to obtain regression parameters. In the last step, we adjust the regression parameters (coefficients and residuals) to develop a more efficient index. This index strategy enhances portfolio performance by connecting portfolio weights to fundamental factors without adding additional exposure to conventional risk factors. In addition, because the stock weights of the new index anchor directly to the stock weights of the benchmark index, this strategy effectively accounts for the tracking errors and turnover rates of the new index.

We subsequently compare a hypothetical equity index, the fundamental factor tilted index (FIX), with the Russell 1000 index (R1000) which is our benchmark. We use returns on assets (*ROA*), net sales (*Sales*), and long-term debt (*Debt*) as fundamental factors, and find that FIX significantly outperforms R1000 during the sample period from 1987 to 2014. FIX's average annual return is 12.4%, while R1000's is only 11.2%. The

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performance difference (1.2% per year) is statistically significant at the 1% level. Over the 28-year period, FIX outperforms R1000 for 18 years in raw returns, four-factor-adjusted returns, Sharpe ratios, and information ratios. Moreover, FIX outperforms R1000 by 57 basis points per annum in the first half of the sample period from 1987 to 2000 (largely a bull market), and by 191 basis points in the second half of the sample from 2001 to 2014 (a turbulent and bearish market), indicating that the tilted index fares better in turbulent market conditions.

1 Portfolio tilting strategy

1.1 The problem

Conventional indices, such as the S&P 500 and Russell 1000 indices, are weighted on the market capitalization of individual securities. Such indices provide investors a venue to invest in the broad equity market because of their low maintenance costs. Cap-weighted indices are considered to be optimal in an efficient capital market where securities are appropriately priced and no stock can persistently have an abnormal performance. Even so, numerous works have casted doubts on the efficiency of the market portfolio (to list a few, the reference includes Shanken, 1985; Kandel and Stambaugh, 1987; Hansen and Jagannathan, 1991, 1997). Known as alternative indices, non-cap-based indices emerge to exploit potential inefficiency of the capital market.

There are a wide array of non-cap-based indices. Fundamental indices, pioneered by Arnott *et al.* (2005), tie portfolio weights to various fundamental factors such as book value of corporate assets, cash flow, sales, and aggregate dividends. A key limitation of such fundamental factor-based indices however is that they lack control for risk. Amenc *et al.* (2011), for example, present evidence that fundamental indices are overweighted with value stocks. A non-zero correlation between fundamental factors and risk factors could increase portfolios' exposure to risk factors, resulting in a poor performance for an index after risk adjustment.[2]

1.2 Solution: Portfolio tilting strategy

We address the portfolio bias problem mentioned above using a three-step portfolio tilting strategy. In the first step, we project fundamental factors on risk factors to obtain orthogonalized fundamental factors. Associated with corporate financial status and corporate intrinsic values, fundamental factors, denoted $\underline{\boldsymbol{F}}$, are useful predictors of future stock returns, however, they are potentially correlated with risk factors, denoted $\boldsymbol{K}$. Thus, the following linear regression is performed:

$$\underline{\boldsymbol{F}} = \boldsymbol{K}\boldsymbol{\Gamma} + \boldsymbol{\eta}, \tag{1}$$

where $\boldsymbol{\Gamma}$ measures the sensitivity of fundamental factors to the risk factors. $\boldsymbol{\eta}$ is a vector of regression residuals, representing orthogonalized fundamental factors. Defining $\boldsymbol{F}$ as the regression residuals $\boldsymbol{\eta}^3$, we have the expression for $\boldsymbol{F}$:

$$\boldsymbol{F} = [1 - \boldsymbol{K}(\boldsymbol{K}'\boldsymbol{K})^{-1}\boldsymbol{K}']\underline{\boldsymbol{F}}. \tag{2}$$

In other words, $\boldsymbol{F}$ is the difference between the raw fundamental factors, $\underline{\boldsymbol{F}}$, and its projected value on $\boldsymbol{K}$.

In the second step, portfolio weights of individual stocks held by the benchmark index, $\boldsymbol{W}$, are regressed onto the *orthogonalized* fundamental factors $\boldsymbol{F}$, estimated from Equation (1) and the same set of risk factors, $\boldsymbol{K}$.

$$\boldsymbol{W} = \underbrace{\boldsymbol{F}\boldsymbol{\delta}}_{\boldsymbol{W_1}} + \underbrace{\boldsymbol{K}\gamma}_{\boldsymbol{W_2}} + \underbrace{\boldsymbol{\upsilon}}_{\boldsymbol{W_3}}. \tag{3}$$

In Equation (3), $\boldsymbol{\delta}$, termed as the fundamental beta or loadings to fundamental factors, measures the sensitivities of stock weights to orthogonalized fundamental factors. On the other hand, $\boldsymbol{\gamma}$, termed as the risk beta or loadings to risk factors, measures the sensitivities of stock weights to risk factors. $\boldsymbol{\upsilon}$ the regression residual, represents the component unrelated to fundamental and risk factors.

Denoting $\boldsymbol{W_1}, \boldsymbol{W_2}$, and $\boldsymbol{W_3}$ as constituent weights attributed to $\boldsymbol{F}$, $\boldsymbol{K}$, and $\boldsymbol{\upsilon}$, respectively, in Equation (3), we derive important properties related to these portfolio weights.

Given the independence of orthogonalized fundamental factors, $\boldsymbol{F}$, and risk factors, $\boldsymbol{K}$, we express the estimated values of $\boldsymbol{\delta}$ and $\boldsymbol{\gamma}$, respectively denoted as $\hat{\boldsymbol{\delta}}$ and $\hat{\boldsymbol{\gamma}}$, as below:

$$\hat{\boldsymbol{\delta}} = (\boldsymbol{F}'\boldsymbol{F})^{-1}(\boldsymbol{F}'\boldsymbol{W}) \tag{4}$$

$$\hat{\boldsymbol{\gamma}} = (\boldsymbol{K}'\boldsymbol{K})^{-1}(\boldsymbol{K}'\boldsymbol{W}). \tag{5}$$

According to Equations (4) and (5), $\hat{\boldsymbol{\delta}}$ is independent of $\boldsymbol{K}$ and $\hat{\boldsymbol{\gamma}}$ is independent of $\boldsymbol{F}$, suggesting that $\boldsymbol{W_1}, \boldsymbol{W_2}$, and $\boldsymbol{W_3}$ are orthogonal to each other. This gives rise to the following important conditions related to constituent weights: (i) $\boldsymbol{W_1'1} = 0$; (ii) $\boldsymbol{W_2'1} = 1$; and (iii) $\boldsymbol{W_3'1} = 0$.[4]

We construct a potentially more efficient index by adjusting these parameters, $\hat{\boldsymbol{\delta}}$, $\hat{\boldsymbol{\gamma}}$ and estimated value of $\boldsymbol{\upsilon}$, denoted as $\hat{\boldsymbol{\upsilon}}$. We use the term "tilting" to a mild adjustment of individual stock weights for the purpose of improving index performance.

Denoting $\boldsymbol{\delta}^*$, $\boldsymbol{\gamma}^*$, and $\boldsymbol{\upsilon}^*$ as portfolio sensitivities to $\boldsymbol{F}$ and $\boldsymbol{K}$, and the residual term for the tilted portfolio, we obtain stock weights of the new portfolio, $\boldsymbol{W}^*$, in each month:

$$\boldsymbol{W}^* = \underbrace{\boldsymbol{F\delta}^*}_{\boldsymbol{W_1^*}} + \underbrace{\boldsymbol{K}\gamma^*}_{\boldsymbol{W_2^*}} + \underbrace{\boldsymbol{\upsilon}^*}_{\boldsymbol{W_3^*}}. \tag{6}$$

Shown in Equation (6), $\boldsymbol{W}^*$ can be separated into three constituents, $\boldsymbol{W_1^*}, \boldsymbol{W_2^*}$, and $\boldsymbol{W_3^*}$, corresponding to portfolio exposures to $\boldsymbol{F}$, $\boldsymbol{K}$, and $\boldsymbol{\upsilon}$. We have $\boldsymbol{W_1^{*'}1} = 0$, $\boldsymbol{W_2^{*'}1} = 1$, and $\boldsymbol{W_3^{*'}1} = 0$. If $\boldsymbol{F}$ is indeed return predictive, we expect R^* $(= \boldsymbol{r}'\boldsymbol{W}^*)$ to exceed the performance of the benchmark portfolio, R $(= \boldsymbol{r}'\boldsymbol{W})$.

1.3 FIX: An application

In this section, we demonstrate the performance of the portfolio tilting strategy using a hypothetical index, namely the fundamental factor tilted index (abbreviated as FIX in the rest of this paper), using the Russell 1000 index as the benchmark.

The included fundamental factors are firms' return on assets, net sales, and long-term debt. We consider these variables as fundamental factors due to the following reasons. First, asset productivity positively predicts stock performance, therefore we use returns on assets as a fundamental factor (see Fama and French, 2006; Stambaugh and Yuan, 2016).[5] Second, firm sales are positively associated with the firm's exposure to macroeconomic conditions. Firms heavily relying on sales have to maintain their sales, which is challenging in the bear markets, and conversely, firms with relatively low sales have the advantage to shun away from such business cycle risk. Third, financial leverage negatively predicts future stock performance as leverage drives up firm insolvency (see, e.g., Hamada, 1972, Dimitrov and Jain, 2006; George and Hwang, 2009; Korteweg, 2010). We expect that, on average, by lowering portfolio exposure to tail risks, portfolios underweighting stocks of high leverage and sales outperform the market in economic downturns.

Considering the fact that these three fundamental factors differ in magnitudes, we normalize individual fundamental factors within the industry of a stock.[6] The normalized raw fundamental factors for each stock are denoted as *ROA, NS*, and *Debt*. We have $\underline{\boldsymbol{F}} = (\boldsymbol{ROA}, \boldsymbol{NS}, \boldsymbol{Debt})$ for stocks included in R1000. In each month, $\underline{\boldsymbol{F}}$ is a $1{,}000 * 3$ matrix as R1000 holds 1,000 stocks.

In Step 1, we project $\underline{\boldsymbol{F}}$ on a set of risk factors, $\boldsymbol{K}$, to construct orthogonalized fundamental factors. $\boldsymbol{K}$ includes factor loadings of Carhart's (1997) four-factor model[7] and 10 industry indicators based on the first two digits of the Global Industry Classification Standard (GICS),[8] denoted as $(\beta, \boldsymbol{s}, \boldsymbol{h}, \boldsymbol{u}, \boldsymbol{d}^1, \ldots, \boldsymbol{d}^{10})$.

To obtain the orthogonalized fundamental factors, in each month, we estimate $\hat{\beta}_i$, $\hat{s}_i$, $\hat{h}_i$, and $\hat{\mu}_i$ for an individual stock i using the recent 10-year data. A minimum of 2-year data is required for a stock to be included in the estimation.[9]

Orthogonalized fundamental factors are resulted from the following regression.

$$(\boldsymbol{ROA}, \boldsymbol{NS}, \boldsymbol{Debt}) = (\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{s}}, \hat{\boldsymbol{h}}, \hat{\boldsymbol{u}}, \times \boldsymbol{d}^1, \ldots, \boldsymbol{d}^{10})\boldsymbol{\Gamma} + \boldsymbol{\eta}, \quad (8)$$

where $\boldsymbol{\Gamma}$ is a $14 * 3$ matrix and $\boldsymbol{\eta}$ is a $1{,}000 * 3$ matrix, and residuals from Equation (8) are orthogonalized fundamental factors.

A critical condition for the tilted index to outperform the benchmark index is the positive return predictive power of fundamental factors. We perform analysis to verify this condition. Specifically, we assign stocks in R1000 into quintile groups based on one of the three fundamental factors in the previous month. We calculate each quintile's monthly return as market capitalization-weighted return of all stocks in the portfolio. We find the highest-*ROA* group outperforms the lowest-*ROA* group at annualized 2.96% during the full-sample period. However, when sorted by *NS* and Debt, the return differences between the top (Q5) and bottom (Q1) group are insignificantly negative. We also run the analysis on orthogonalized fundamental factors, which are obtained from estimation of Equation (8). The results show that the return difference between Q5 and Q1 group sorted by the orthogonalized *ROA* factor is positive with marginally significance, whereas those by orthogonalized *NS* and Debt factors are significantly negative. Overall, the findings suggest that *ROA* is positively return predictive, while *NS* and Debt are negatively return predictive. We provide details in the Appendix to conserve space.

In Step 2, we regress stock weights, $\boldsymbol{W}$, of Russell 1000 onto orthogonalized fundamental factors and selected risk factors, which gives us the sensitivity of R1000 stock weights to each of the factors included in the study, expressed in Equation (3). Differently put, stock weights of R1000 are associated with (i) exposure to pure fundamental factors, (ii) exposure to risk factors, and (iii) the idiosyncratic element.

In the final step, we tilt weight sensitivities to orthogonalized fundamental factors while matching the exposure to risk factors of the benchmark, i.e., holding index sensitivities to risk factors constant, i.e., $\boldsymbol{\delta}^* = \hat{\boldsymbol{\gamma}}$.

$$\boldsymbol{W}^* = \boldsymbol{F}\boldsymbol{\delta}^* + \boldsymbol{K}\hat{\boldsymbol{\eta}} + \boldsymbol{\upsilon}^*. \quad (9)$$

In Equation (9), portfolio sensitivities to risk factors of FIX are the same as that of R1000. $\boldsymbol{\delta}^*$ represents the elasticities of the new index weights to orthogonalized fundamental factors which takes the following form.

$$\boldsymbol{\delta}^* = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & 2/3 \end{bmatrix} \hat{\boldsymbol{\delta}}. \quad (10)$$

In Equation (10), $\hat{\boldsymbol{\delta}}$ is a vector of the estimated elasticities of R1000's stock weights to orthogonalized fundamental factors. Equation (10) suggests that the tilted index doubles the loading to adjusted *ROA* (i.e., overweighting stocks included in R1000 with high orthogonalized *ROA*), while lowering stock weight sensitivities to orthogonalized net sales and long-term debt by 1/3, underweighting stocks in R1000 having high orthogonalized *NS* and *Debt*. Moreover, $\boldsymbol{\upsilon}^*$ is 2/3 of $\hat{\boldsymbol{\upsilon}}$, estimated residuals from Equation (3).[10]

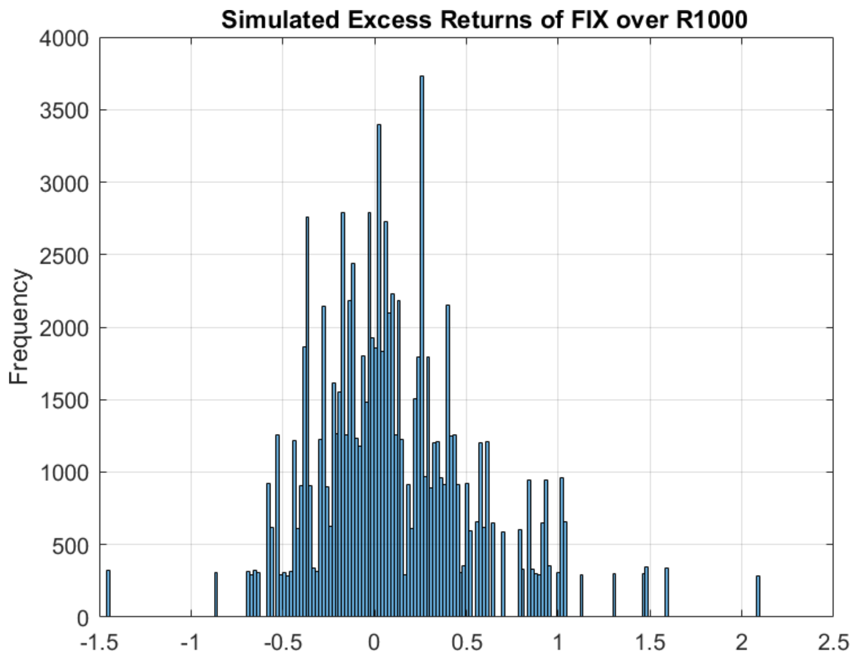

Figure 1 Simulated excess performance distribution.

This figure plots the simulated monthly return distribution between FIX and Russell 1000. We simulate the historical distribution of the monthly excess returns between FIX and R1000 with 100,000 replications. The x-axis represents the monthly excess return. The y-axis represents the frequency, the number of draws, for a specific excess return.

It is worth noting that alpha of FIX potentially comes from its lower portfolio skewness than the benchmark portfolio. Well documented in the literature, e.g., Hong and Stein (2003) and Albuquerque (2012), the aggregate stock market has a negatively skewed distribution which is related to corporate downside risk. By reducing portfolio exposure to variables cyclical to economic conditions while increasing portfolio exposure to counter-cyclical variables, the tilted index has a lower exposure to risks in bearish markets and its return distribution is expected to be less skewed. Our analysis confirms this. Specifically, we compare the skewness of FIX and R1000 and find that the skewness of FIX is -0.80, less negative than that of R1000, -0.88. Moreover, we perform the historical simulations of the excess performance between FIX and R1000 to test the out-of-the-sample evidence on the reduction in return skewness. We randomly draw FIX and the corresponding R1000 for 100,000 times with repetitions and report the differences of monthly returns of these two. Figure 1 shows the excess return distribution, which is clearly positively skewed (skewness $= 0.79$).

2 Results

2.1 Data

We obtain stock weights of R1000 for each month from Thomson Reuter's DataStream from January 1987 to December 2014. Monthly performance of FIX and the benchmark index (R1000) are based on their respective portfolio holding in the beginning of each month, then annualize it (multiplying it by 12) to obtain the annual performance. In addition, we obtain monthly stock return data from the CRSP database.

Quarterly financial statement data are from the Compustat database. In particular, *ROA* is the ratio of operating income after depreciation (Compustat item: *OIADP*) to total assets (Compustat item: *AT*). A firm's net sales are evaluated as gross sales generated by a company after the deduction of allowances for damaged (Compustat item: *SALE*). Long-term debt is the amount of debt whose maturity exceeds 1 year (Compustat item: *DLTT*).

To construct risk factors of individual stocks, we obtain factor risk premiums of *MKT*, *SMB*, *HML*, and *UMD* from Kenneth French's website.[11] Each individual stock's loadings on four-factor premiums are estimated as the coefficients of the Carhart's four-factor regressions. That is, in each month before portfolio rebalancing, we run the above regression using recent 10-year data to estimate respective factor loadings. Stock observations are excluded if we have fewer than 24 observations in a rolling regression window.

2.2 Performance

We first compute annual performance of FIX (rebalanced monthly) and R1000 and report them in Panel A of Table 1. Over the 28-year period, FIX outperforms R1000 for 18 years. Throughout the sample years from 1987 to 2014, the average annual return for FIX (i.e., monthly return of the portfolio $*$ 12) is 12.44%, while the average annual return for R1000 is 11.19%. The difference in annual performance between FIX and R1000, 1.25%, significant at the 1% level, is economically significant as well. Also documented in Table 1, interestingly, the overperformance of FIX to R1000 is 0.57% in late 1980s and 1990s (predominantly bull markets) while it is over 1.91% after year (predominantly bearish and turbulent markets). By underweighting stocks heavily loaded on leverage (i.e., deleveraging) and those heavily loaded on sales, FIX is designed to be countercyclical — it typically performs well in bad states of the stock market.[12] To better understand FIX, we further estimate the Pearson correlation of monthly return spreads between FIX and R1000 and R1000 returns. The correlation is -0.29, suggesting that the outperformance of FIX is negatively associated with R1000 returns, performance of a market index portfolio. For example, FIX underperforms R1000 in 1993, 1995, 1996, 1998, and 1999, years of the bull market while FIX outperforms R1000 in years such as 2001 through 2005, 2008 through 2011, commonly viewed as bearish or turbulent years.[13] We subsequently compare the risk-reward ratios of FIX and R1000 by assessing respective Sharpe ratios. The 1-month T-bill rate is used to measure the risk-free rate. The result shows that the Sharpe ratio for FIX, 0.58, is greater than the Sharpe ratio of R1000, 0.49– FIX continues to outperform R1000 in terms of risk-adjusted total return.

Further, we follow Carhart (1997) to conduct four-factor regressions, specified in Equation (7), to assure that FIX's performance is attributed to a risk-adjusted return, i.e., alpha, rather than a compensation to take extra risk.

In the regression, the left-hand side variable is $\boldsymbol{R_i}$, which respectively represent monthly returns of FIX in excess of the risk-free rate for FIX and R1000. The estimation allows us to compare alphas of these two portfolios.[14] Specifically, we perform the four-factor regression using all the monthly returns of FIX during the sample period to estimate the "all years" alpha. We multiply monthly alpha by 12 to obtain the simple annualized alphas. The annualized alphas in the subsample and full-sample period are reported in Panel B of Table 1. The result suggests that alphas of FIX are clearly greater than those of R1000. During the entire sample period, the annual alpha

Table 1 Returns of Fundamental Factor Tilt Index (FIX) and Russell 1000.

Year	FIX	R1000	Difference (=FIX −R1000)
Panel A. Raw returns			
1987	7.74	7.50	0.24
1988	16.34	16.13	0.21
1989	27.47	27.19	0.28
1990	−2.65	−3.39	0.74
1991	31.28	30.63	0.66
1992	9.72	9.25	0.47
1993	9.42	9.87	−0.45
1994	1.72	0.88	0.84
1995	30.85	32.44	−1.59
1996	20.55	20.81	−0.26
1997	29.86	28.85	1.01
1998	23.77	26.64	−2.87
1999	21.55	22.56	−1.01
2000	0.01	−9.05	9.06
2001	−4.63	−11.49	6.86
2002	−17.49	−21.78	4.28
2003	28.86	26.83	2.03
2004	12.71	11.18	1.53
2005	7.46	6.52	0.94
2006	14.23	14.65	−0.42
2007	5.83	6.25	−0.42
2008	−38.98	−43.74	4.76
2009	31.49	28.25	3.24
2010	20.87	16.99	3.88
2011	14.36	11.65	2.71
2012	15.04	15.83	−0.79
2013	29.23	29.77	−0.54
2014	11.94	12.58	−0.63
1987–2000	16.31	15.74	0.57/(1.99)**
2001–2014	8.19	6.29	1.90/(3.13)***
All Years	12.40	11.19	1.21/(2.65)***
Panel B. Alphas			
1987–2000	1.39***	0.24	1.15/(2.13)**
2001–2014	1 76***	0.19	1.57/(3.41)***
All years	1.46***	0.22	1.24/(2.77)***

Panel A of this table reports the annualized stock performance and their differences for the Fundamental Factor Tilt Index (FIX) and the Russell 1000 (R1000) over the sample period. All the reported numbers are in percentage. The last three rows report the averages of subsample periods (1987–2000 and 2001–2014) and the entire sample period (1987–2014). Panel B of the table reports alphas of FIX and R1000 and the alpha difference in these two portfolios for subsample periods and the full sample. The t-statistics for the return/alpha differences in subsamples and the full sample are reported in the parenthesis. ***, **, and * respectively represent the 1%, 5%, and 10% significance levels.

of FIX is 1.46%, higher than the four-factor-adjusted portfolio alpha of R1000. The reported alpha for R1000, on the other hand, is merely 0.22%. Moreover, in the first subsample (the second subsample), the annual alpha for FIX is 1.39% (1.76%) and 0.24% (0.19%) for R1000.[15] The difference is significant at the 1% level. It is worth noting that the magnitude of the alpha difference is similar to that of the raw performance difference, suggesting that FIX's performance is mainly driven by risk-adjusted performance.

2.3 *Turnover and tracking error*

In this subsection, we first evaluate the turnover rate of FIX. The turnover rate is defined as the minimum of the total amount of new securities purchased or the amount of securities sold over a 12-month period, divided by the total asset value of the fund. Reported in Panel A, Table 2, the annual turnover rates are 3.6% for R1000 and 25.9% for FIX. In other words, FIX shuffles one-quarter of its holdings in each year.

Table 2 Portfolio attributes: Risk, performance, and turnover.

	STD	SR	EX	TR	IR	TURN
Panel A: Full sample						
R1000	15.61	0.49	N/A	N/A	N/A	3.60
FIX	15.40	0.58	1.22	1.49	0.81	25.89
Panel B: 1987–2000						
R1000	15.37	0.68	N/A	N/A	N/A	4.34
FIX	15.08	0.73	0.57	1.52	0.38	26.05
Panel C: 2001–2014						
R1000	15.80	0.29	N/A	N/A	N/A	2.81
FIX	15.70	0.42	1.91	1.44	1.32	25.32

This table reports portfolio risk, tracking errors, and portfolio turnover rates of R1000 and FIX. Portfolio attributes include return volatility (*STD*), Sharpe ratio (*SR*), excess return of FIX relative to Russell 1000 (EX), tracking error (*TR*), information ratio (*IR*), and turnover rate (*TURN*). Panel A is for the result over the full-sample period (1987–2014). Panels B and C report the results of subsample periods, 1987–2000 and 2001–2014, respectively.

We assess the monthly tracking error of FIX as below:

$$TR^* = \sqrt{\frac{1}{T}\sum_{t=1}^{T}[(R_t^* - \overline{R}_t^*) - (R_t - \overline{R}_t)]^2}, \tag{11}$$

where $\overline{R}_t^*$ and $\overline{R}_t$ are the average performance of FIX and R1000 from month 1 to T (where T is 336 for the full sample). In other words, *TR* is the standard deviation of index excess returns (i.e., FIX monthly returns minus R1000 monthly returns).

Also reported in Table 2, the annualized tracking error ($\sqrt{12} * TR*$) of FIX is 1.49%; it is 1.52% in 1987–2000 and 1.44% in 2001–2014. This is much lower than the tracking error of other alternative indices. For example, Amenc *et al.* (2011) show that the annual tracking error for the fundamental index is 6.2% and that the risk-efficient index is 5.5% for the sample period from 1999 to 2014.

2.4 *Information ratio*

Next, we estimate the information ratio of FIX, which is the ratio of the monthly performance difference in FIX and R1000 to the tracking error of FIX (relative to R1000).

$$IR^* = \frac{E(R_t^* - R_t)}{\sigma(R_t^* - R_t)} = \frac{E(R_t^* - R_t)}{TR^*}. \tag{12}$$

In Table 2, we report the annualized information ratio ($\sqrt{12} * IR^*$), 0.81, in the full-sample period, indicating the dominance of FIX's performance relative to the performance of its benchmark. It also shows that the average information ratios of FIX are substantially different in two subsample periods. The average information ratio is 0.38 before 2000 will it rises up to 1.32 in the period after 2000. This confirms our earlier finding that FIX delivers much better performance relative to its benchmark when the stock market performs poorly.

The information ratios of the fundamental index and the risk-efficient index (relative to the cap-weighted S&P 500 index), reported in Amenc *et al.* (2011), are respectively 0.64 and 0.99 for the period from 2008/1999 to 2008/2011. Over the corresponding period, our estimate of the information ratio of FIX with respect to R1000 is 1.61. As such, we conclude that FIX outperforms these alternative indices in terms of the information ratio.[16]

2.5 *Decomposing portfolio performance*

2.5.1 *Performance attributions to tilts*

We perform the attribution analysis to evaluate the respective contributions of factor tilts to performance of FIX relative to R1000.

FIX's holding in individual stocks can be expressed as the sum of R1000's holding and portfolio adjustments attributable to tilts. Specifically,

$$\boldsymbol{W}^* = \boldsymbol{W} + \boldsymbol{F}(\boldsymbol{\delta}^* - \hat{\boldsymbol{\delta}}) + (d-1)\hat{\boldsymbol{\upsilon}}, \tag{13}$$

where $\boldsymbol{\delta}^*$ is provided in Equation (10) and $d = 2/3$.

We obtain the excess return FIX, i.e., the performance of FIX relative to R1000 performance, by pre-multiplying a row vector of FIX' individual security returns in both sides of the above equation.

$$R^* - R = \boldsymbol{r}'\boldsymbol{F}(\boldsymbol{\delta}^* - \hat{\boldsymbol{\delta}}) - 1/3\boldsymbol{r}'\hat{\boldsymbol{\upsilon}}. \tag{14}$$

Based on Equation (14), the excess return of FIX can be decomposed to two elements: i) performance due to tilts on fundamental betas and ii) performance due to the reduction of the residual from R1000 weight regression. We further express the middle term on the left-hand side of Equation (14), $\boldsymbol{r}'\boldsymbol{F}\ (\boldsymbol{\delta}^* - \hat{\boldsymbol{\delta}})$, as the SUM of performance attributed to tilts of portfolio sensitivities to orthogonalized *ROA*, net sales, and long-term debt.

$$\begin{aligned}\boldsymbol{r}'\boldsymbol{F}(\boldsymbol{\delta}^* - \hat{\boldsymbol{\delta}}) &= \boldsymbol{r}'\boldsymbol{F}\begin{bmatrix}1 & 0 & 0\\ 0 & -1/3 & 0\\ 0 & 0 & -1/3\end{bmatrix}\hat{\boldsymbol{\delta}}\\ &= \boldsymbol{r}'\boldsymbol{F}\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\\ &\quad\times\hat{\boldsymbol{\delta}} + \boldsymbol{r}'\boldsymbol{F}\begin{bmatrix}0 & 0 & 0\\ 0 & -1/3 & 0\\ 0 & 0 & 0\end{bmatrix}\\ &\quad\times\hat{\delta} + \boldsymbol{r}'\boldsymbol{F}\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1/3\end{bmatrix}\hat{\boldsymbol{\delta}}.\end{aligned} \tag{15}$$

Table 3 attributes FIX performance to various factors in each year and the entire sample period. The findings suggest that the performance of FIX is countercyclical; tilting typically positively contribute to the performance of FIX when R1000 has a negative return or a low positive return. See, for example, 1990, 1994, 2000, 2001, 2002, 2008, and 2014. Table 3 also shows that tilts has a negative return contribution when R1000 had favorable performance in 1992, 1993, 1995, 1996, 1998, 1999, 2006, 2009, 2012, 2013, and 2014.

Last three lines of Table 3 present results of portfolio attribution analysis in subsample periods and the full-sample period. In the entire sample period (labeled as "All Year", the annual excess return of FIX is roughly 122 basis points, (12.40%–11.18%): 10 bps from the *ROA* tilt; 25 bps from the *NS* tilt; 19 bps from the *Debt* tilt; and 55 bps from the residual tilt.[17] In the first subsample period, the average contributions of *ROA, NS*, *Debt* tilt, and residual tilt are respectively

5.5 bps, 10 bps, 7 bps, and 25 bps. In the second subsample period, the average contributions for the respective tilts are 16 bps, 25 bps, 21 bps, and 86 bps. Tilts apparently play a much greater role in the second subsample period.

2.6 *Alternative tilted indices*

Is the performance of FIX attributable to our choices of tilts? We pursue an answer in this section. To do this, we apply alternative tilting parameters (i.e., different combinations of tilting

Table 3 Attributing FIX performance to tilts.

Year	FIX	R1000	ROA	NS	Debt	Residual
1987	7.74	7.50	−0.04	0.19	−0.05	0.28
1988	16.34	16.12	−0.02	−0.11	0.00	0.64
1989	27.48	27.18	0.06	−0.01	0.01	0.02
1990	−2.72	−3.39	0.67	0.36	0.31	−0.34
1991	31.76	30.62	0.24	0.47	0.11	0.28
1992	9.66	9.24	−0.28	−0.11	0.09	0.50
1993	9.27	9.86	−0.24	−0.29	−0.28	0.42
1994	1.71	0.88	0.01	0.06	0.10	0.33
1995	31.00	32.43	−0.11	−0.39	−0.39	−0.64
1996	20.44	20.81	0.05	−0.21	0.03	−0.78
1997	30.07	28.84	0.02	0.24	0.15	0.40
1998	23.64	26.64	−0.01	−0.71	−0.25	−1.47
1999	21.80	22.56	−0.15	0.64	0.50	−0.68
2000	0.06	−9.04	0.25	1.25	0.65	4.49
2001	−4.62	−11.49	0.20	1.29	0.66	3.17
2002	−17.59	−21.77	0.39	1.67	1.02	1.34
2003	28.74	26.82	−0.22	0.02	−0.13	1.03
2004	12.54	11.17	−0.02	0.15	0.17	1.13
2005	7.25	6.51	0.15	0.01	0.25	0.72
2006	14.22	14.64	−0.06	−0.27	−0.10	0.10
2007	5.86	6.24	0.22	−0.20	0.20	−0.53
2008	−39.19	−43.73	1.32	1.60	1.79	1.24
2009	31.07	28.25	−0.50	0.28	−0.09	1.24
2010	20.72	16.98	0.02	0.94	0.77	1.68
2011	3.22	2.47	0.18	0.13	0.05	−0.02
2012	15.04	15.83	−0.27	0.11	−0.05	−0.32
2013	29.23	29.76	−0.24	−0.30	0.06	0.45
2014	11.94	12.57	0.01	0.13	−0.02	0.11
1987–2000	16.31	15.73	0.06	0.10	0.07	0.24
2001–2014	8.19	6.28	0.16	0.38	0.31	0.80
All Years	12.40	11.18	0.10	0.25	0.20	0.54

This table reports the result when FIX performance is decomposed to (i) R1000, (ii) tilts to R1000 weight sensitivities to three fundamental factors: *ROA*, *NS* and *Debt*, and (iii) the tilt to the regression residuals. All reported numbers are in percent. The last three rows report the averages of subsample periods (1987–2000 and 2001–2014) and all years (1987–2014).

Table 4 Performance and portfolio turnover of Fundamental Factor Tilt Index (FIX) and the Russell 1000 (R1000): Varying sensitivities to fundamental factors and residuals.

Port	Raw	Alpha	STD	SR	TR	IR	TURN	$\delta^*(ROA)$	$\delta^*(NS)$	$\delta^*(Debt)$
Panel A: Residual tilting parameter = 2/3										
R1000	11.19	0.22	15.61	0.49	0.00	N/A	3.60	1	1	1
X1	12.24	1.30	15.41	0.57	1.29	0.82	22.11	2	1	1
X2	12.30	1.39	15.39	0.57	1.34	0.84	24.44	3	1	1
X3	12.37	1.48	15.40	0.57	1.40	0.85	26.88	4	1	1
X4	12.32	1.37	15.39	0.57	1.39	0.81	23.96	2	2/3	1
X5	12.38	1.47	15.39	0.58	1.43	0.83	26.12	3	2/3	1
X6	12.44	1.55	15.40	0.58	1.49	0.84	28.40	4	2/3	1
X7	12.39	1.45	15.39	0.58	1.49	0.81	25.82	2	1/3	1
X8	12.45	1.54	15.38	0.58	1.52	0.83	27.80	3	1/3	1
X9	12.51	1.62	15.39	0.58	1.58	0.84	29.93	4	1/3	1
X10	12.32	1.37	15.40	0.57	1.39	0.81	23.96	2	1	2/3
X11	12.38	1.47	15.39	0.58	1.43	0.83	26.12	3	1	2/3
X12	12.44	1.55	15.39	0.58	1.49	0.84	28.40	4	1	2/3
X13(FIX)	12.39	1.45	15.40	0.58	1.49	0.81	25.82	2	2/3	2/3
X14	12.45	1.54	15.38	0.58	1.52	0.83	27.80	3	2/3	2/3
X15	12.51	1.62	15.39	0.58	1.58	0.84	29.93	4	2/3	2/3
X16	12.46	1.52	15.40	0.58	1.59	0.80	27.65	2	1/3	2/3
X17	12.52	1.61	15.38	0.58	1.62	0.82	29.45	3	1/3	2/3
X18	12.57	1.68	15.39	0.58	1.67	0.83	31.40	4	1/3	2/3
X19	12.39	1.45	15.40	0.58	1.49	0.81	25.82	2	1	1/3
X20	12.45	1.54	15.38	0.58	1.52	0.83	27.80	3	1	1/3
X21	12.51	1.62	15.39	0.58	1.58	0.84	29.93	4	1	1/3
X22	12.46	1.52	15.40	0.58	1.59	0.80	27.65	2	2/3	1/3
X23	12.52	1.61	15.38	0.59	1.62	0.82	29.45	3	2/3	1/3
X24	12.57	1.68	15.39	0.58	1.67	0.83	31.40	4	2/3	1/3
X25	12.53	1.59	15.41	0.58	1.70	0.79	29.38	2	1/3	1/3
X26	12.59	1.67	15.39	0.59	1.73	0.81	31.02	3	1/3	1/3
X27	12.64	1.74	15.39	0.59	1.77	0.82	32.85	4	1/3	1/3
Panel B: Residual tilting parameter = 1										
R1000	11.19	0.22	15.61	0.49	0.00	N/A	3.60	1	1	1
X1	12.39	1.84	15.39	0.58	1.54	0.78	28.83	2	−2/3	−2/3
X2	12.44	1.94	15.37	0.58	1.57	0.80	30.14	3	−2/3	−2/3
X3	12.48	1.97	15.37	0.58	1.62	0.80	31.70	4	−2/3	−2/3
X4	12.32	2.07	15.38	0.57	1.44	0.79	27.20	2	−1/3	−2/3
X5	12.38	2.16	15.36	0.58	1.47	0.81	28.65	3	−1/3	−2/3
X6	12.42	2.19	15.37	0.58	1.53	0.81	30.34	4	−1/3	−2/3
X7	12.26	2.28	15.37	0.57	1.34	0.80	25.50	2	0	−2/3

(Continued)

Table 4 (*Continued*)

Port	Raw	Alpha	STD	SR	TR	IR	TURN	$\delta^*(ROA)$	$\delta^*(NS)$	$\delta^*(Debt)$
X8	12.32	2.36	15.36	0.57	1.38	0.82	27.10	3	0	−2/3
X9	12.36	1.96	15.36	0.58	1.44	0.82	28.94	4	0	−2/3
X10	12.33	2.06	15.38	0.57	1.47	0.78	27.72	2	−2/3	−1/3
X11	12.39	2.15	15.36	0.58	1.51	0.80	29.13	3	−2/3	−1/3
X12	12.43	2.18	15.36	0.58	1.56	0.80	30.80	4	−2/3	−1/3
X13	12.27	2.28	15.37	0.57	1.37	0.79	26.05	2	−1/3	−1/3
X14	12.33	2.36	15.35	0.57	1.41	0.81	27.60	3	−1/3	−1/3
X15	12.37	2.34	15.35	0.58	1.47	0.81	29.41	4	−1/3	−1/3
X16	12.20	2.43	15.36	0.56	1.27	0.80	24.30	2	0	−1/3
X17	12.26	2.51	15.34	0.57	1.32	0.82	26.02	3	0	−1/3
X18	12.31	2.17	15.35	0.57	1.38	0.82	27.98	4	0	−1/3
X19	12.28	2.27	15.36	0.57	1.41	0.78	26.59	2	−2/3	0
X20	12.34	2.35	15.35	0.57	1.45	0.79	28.09	3	−2/3	0
X21	12.38	2.34	15.35	0.58	1.50	0.80	29.87	4	−2/3	0
X22	12.21	2.44	15.35	0.57	1.30	0.79	24.87	2	−1/3	0
X23	12.27	2.51	15.34	0.57	1.35	0.80	26.53	3	−1/3	0
X24	12.32	2.46	15.34	0.57	1.41	0.80	28.46	4	−1/3	0
X25	12.14	2.55	15.35	0.56	1.20	0.79	23.06	2	0	0
X26	12.20	2.63	15.33	0.57	1.25	0.81	24.94	3	0	0
X27	12.25	1.53	15.34	0.57	1.32	0.81	27.01	4	0	0

This table reports performance attributes, tracking errors, and portfolio turnovers of FIX and R1000 under alternative tilts of portfolio sensitivities to fundamental factors and the residuals. The following variables are included: the name of the portfolio (Port), raw return (Raw, in percent), Carhart alpha (alpha, in percent), standard deviation of portfolio returns (STD, in percent), Sharpe ratio (SR), tracking error (TR, in percent), information ratio (IR), and annual turnover rates (TURN, in percent). While FIX is rebalanced monthly, all the numbers are based on annual portfolio performance. The tilting parameters $\boldsymbol{\delta}^*$ are reported in the last three columns. Panel A presents the results when the tilting parameter on residuals is 2/3 (except for R1000). Panel B presents the results when the tilting parameter on residuals is 1 (except for R1000).

parameters on the residuals and on portfolio sensitivities to orthogonalized fundamental factors) to construct tilted indices. We first set a residual tilting parameter to be either 2/3 or 1, then select a tilting parameter, 2, 3, and 4, on orthogonalized *ROA* and apply a tilting parameter, 1/3, 2/3, and 1, on orthogonalized net sales and long-term debt. This results in 2 *3 *3 *3 (= 54) tilted indices.

Panel A of Table 4 reports the results for the tilted portfolios (X1, X2, ..., X27) with reduced residuals (2/3 of original residuals) in stock weights, we report the result for R1000 in the first row for comparison purpose. Consistent with our expectation, a greater tilts on orthogonalized fundamental factors (i.e., tilt on *ROA* increases from 2 to 4, and the tilts on net sales and debt decrease from 1 to 1/3) lead to greater Carhart alpha and raw performance, at a cost of greater tracking errors and portfolio turnovers. As a result, these tilted indices have similar information ratios.[18]

Alternatively, Panel B of Table 4 reports the results for tilted portfolios when the residual tilt parameter is set to be 1, i.e., holding residuals' contribution to portfolio weights in the same way as R1000, and adjusting the magnitude of tilts on portfolio sensitivities to orthogonalized fundamental factors. Like the findings reported in Panel A, performance of the titled portfolios increases as the magnitude of tilts on orthogonalized fundamental factors increases. For example, $\delta^*(ROA)$ of the first three portfolios (X1, X2, and X3) increases from 2 to 4. Raw returns (alphas) increase from 12.39% (1.84%) to 12.48% (1.97%). Turnover and tracking errors increases in $\delta^*(ROA)$ as well. This leads to similar information ratios for alternative tilted portfolios reported in Panel B.

Importantly, as reported in Panel B, larger tilts on orthogonalized fundamental factors generate substantial performance. Panel B results (holding residuals unchanged) are comparable with those reported in Panel A (with reduced residuals). To illustrate, as reported in Panel A, the raw return, Carhart alpha, Sharpe ratio, and the information ratio are respectively 12.64%, 1.70%, 0.59, and 0.82 while they are 12.25%, 2.53%, 0.57, and 0.81 in Panel B, indicating that the performance of tilted portfolios does not necessarily come from residual reduction.

In summary, the results reported in Table 4 indicate that the index performance increases when the *ROA* tilt (the sales tilt or debt tilt) is larger (smaller). The tracking error and turnover rate of the index are higher whenever a tilt is further away from 1.

2.7 Performance decomposition across industry sectors

Finally, we examine the performance of FIX in different industry sectors relative to R1000's sector performance. We include 10 industry sectors based on the first two digits of the Global Industry Classification Standard (GICS): (i) energy, (ii) materials, (iii) industrials, (iv) consumer discretionary goods, (v) consumer stable goods, (vi) health care, (vii) financial sector, (viii)

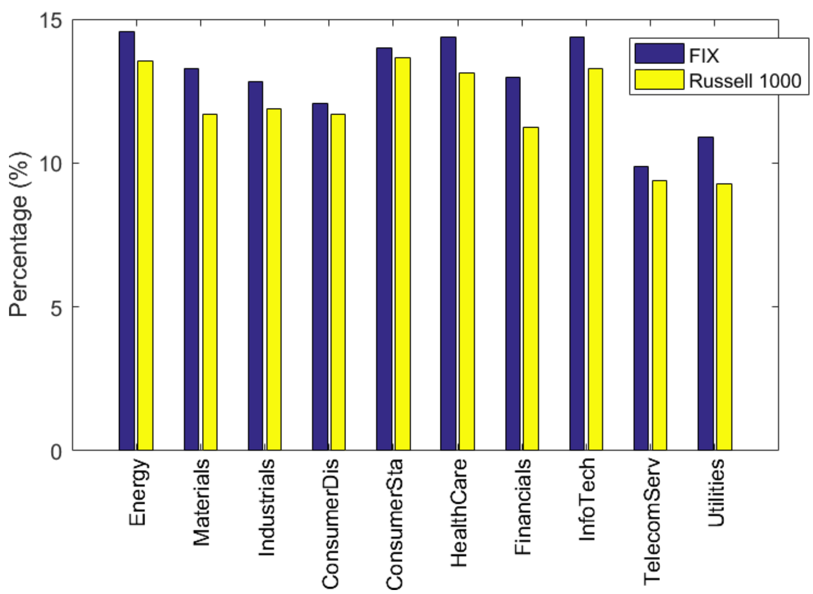

Figure 2 Sector performance of FIX and Russell 1000.

This figure plots the performance of FIX and Russell 1000 attributed to 10 industrial sectors, including (i) energy, (ii) materials, (iii) industrials, (iv) consumer discretionary goods, (v) consumer stable goods, (vi) health care, (vii) financial sector, (viii) information technology, (ix) telecom service, and (x) utilities.

information technology, (ix) telecom service, and (x) utilities.

To compare the performance between FIX and R1000 with a specific sector, we first average monthly return of each sector for FIX and R1000 based on their respective portfolio holding within a given sector, then we average the annualized sector performance respectively for FIX and R1000. Based on result plotted in Figure 2, FIX outperforms R1000 in all the 10 industry sectors, clearly indicating that FIX' performance is not driven by the performance of any specific industry.

3 Conclusions

This paper adopts a three-step methodology to address the potential risk bias of alternative indices. The first step is to obtain a set of pure fundamental factors immune from the influence of risks. In the second step, we project the portfolio weights of a specific existing index on pure fundamental factors and conventional risk factors, resulting in a vector of portfolio sensitivities on these factors. In the final step, we adjust the portfolio weights of the existing index by tilting the portfolio sensitivities obtained in the second step.

We demonstrate the performance of the tilting method through a hypothetical stock index, FIX, built on the Russell 1000 index. The new index is tilted to be more elastic to the profit factor, but less elastic to business cycle factor and financial leverage factor. The test result suggests that the new index not only delivers superior raw performance relative to the conventional cap-weighted index, but also has a significant Carhart's four-factor alpha and a higher information ratio than its benchmark. The lower exposure to financial and operating leverages allows FIX to shun corporate downside risk and makes its performance countercyclical.

Reducing index products' exposure to market risk and corporate downside risk has important practical implications. If this type of index product were to gain a substantial share of the index market, we expect that the proposed strategy would incentivize corporate executives to make more effective investment and financing decisions that could lower their exposures to risk-prone factors as the money spent on index products would be channeled to individual stocks within the index products.

Appendix

The Appendix addresses the question of whether the selected fundamental factors are return predictive. To address this question, in each month we breakdown R1000 every stock into five quintile groups based on each of the three fundamental factors; then, for each quintile, we calculate the average annualized stock return of the subsequent month weighted by market capitalization of individual stocks. The results are reported in Table A1. Panel A presents the result based on raw fundamental factors. During the full-sample period, the average annual return for the highest-*ROA* group (Q5) is 13.37% while that for the lowest-*ROA* (Q1) group is 10.41%. The *t*-statistics for the difference (2.96%) is 1.92, indicating that the Q5 group marginally outperforms the Q1 group in stock return of the subsequent month. When sorted by *NS* and *Debt*, the return differences between the top and bottom quintiles are negative, both being insignificant. Moreover, an important distinction to examine is the results of subsamples: 1987–2000 (a predominantly bullish period) and 2001 to 2014 (a predominantly bearish period). In the bullish period, the return difference between the top and bottom *ROA* sorted quintile groups is insignificant and the differences are marginally significant when *NS* and *Debt* are used as the sorting variables. On the other hand, we obtain remarkably different results in the bearish period from 2001 to 2014. Here, *NS* and *Debt* negatively and significantly predict stock returns while *ROA* positively and significantly predicts stock returns.

Subsequently, we perform contemporaneous cross-sectional regressions to obtain orthogonalized fundamental factors in each month end. Then we repeat performance analyses on the quintile portfolios sorted by each of the three orthogonalized factors. Panel B of Table A1 reports the results. Consistent with Panel A, Panel B reveals that stock returns in the subsequent month increase in the orthogonalized *ROA* factor, but decreases in orthongalized *NS* and *Debt* factors. The return predictive power is stronger in the second subsample from to 2014.

In summary, consistent with the expectation, we find that highly leverage firms and firms heavily relying on sales poorly perform in bearish markets; *ROA* appears to be a more balanced return predictor in different market conditions despite that the outperformance of high *ROA* stocks are more pronounced in bearish markets. On the other hand, *NS* and *Debt* are negatively return predictive. The findings justify our use of these variables as fundamental factors.

Table A.1 Annual stock returns across quintile groups sorted by raw and pure fundamental factors.

	1 (Low)	2	3	4	5 (High)	5–1 /(*t*-stat)
Panel A: Based on raw fundamental factors						
i. Full sample: 1987–2014						
ROA	10.41	10.83	11.02	11.04	13.37	2.96*/(1.92)
NS	11.33	11.40	12.25	12.52	11.06	−0.27/(−0.14)
Debt	11.74	13.52	11.43	12.62	10.55	−1.10/(−0.93)
ii. Subsample: 1987–2000						
ROA	16.62	15.74	14.83	14.84	18.39	1.87 /(0.77)
NS	14.08	14.62	15.96	16.73	16.34	2.26* /(1.70)
Debt	13.43	19.17	15.03	17.45	15.55	2.12*/(1.76)
iii. Subsample: 2001–2014						
ROA	4.95	5.64	7.03	6.92	7.88	2.93**/(2.08)
NS	8.48	8.19	8.30	8.13	5.46	−3.02**/(−2.15)
Debt	9.84	7.53	7.50	7.42	5.22	−4.62**/(−2.19)
Panel B: Based on orthogonalized fundamental factors						
i. Full sample: 1987–2014						
ROA	11.24	10.82	10.61	11.10	13.35	2.11*/(1.75)
NS	13.05	11.91	13.02	11.73	10.81	−2.24**/(−1.88)
Debt	12.64	12.73	13.72	11.65	10.24	−2.40**/(−1.81)
ii. Subsample I: 1987–2000						
ROA	17.37	15.30	14.42	14.88	18.71	1.34/(1.25)
NS	16.02	15.33	16.72	16.98	15.72	−0.30/(−0.15)
Debt	17.12	15.55	16.58	16.03	15.13	−2.01** (−1.56)
iii. Subsample II: 2001–2014						
ROA	4.67	6.04	6.52	7.16	7.68	3.01** /(2.03)
NS	9.73	8.35	8.91	6.04	5.56	−4.17**/(−2.54)
Debt	7.79	9.75	8.55	6.96	5.04	−2.75*/(−1.92)

This table reports the cap-weighted average annual stock performance of R1000 stocks sorted by raw and pure fundamental factors, *ROA, NS*, and *Debt*. Panel A reports the result based on raw fundamental factors. Panel B reports the result based on orthogonalized fundamental factors, i.e., the residuals when raw fundamental factors are regressed on selected risk factors. The last column reports the differences in stock returns between quintiles 5 and 1 and the associated *t*-statistics. ***, **, and * respectively represent the 1%, 5%, and 10% significance levels.

Notes

[1] Dividing stock return predictors to fundamental and risk factors is fairly common in practice. Risk factors are used in risk models (such as Barra's Risk Models) to control portfolio risk exposure. Fundamental factors are variables associated with corporate fundamental value, but not included in risk models. A firm's market value often deviates from its fundamental value due to mispricing (Stambaugh and Yuan, 2014), which gives rise to fundamental factors. Trading strategies based on fundamental factors arise to obtain risk-adjusted performance. See Bender *et al.* (2013), an MSCI publication, for more explanations.

[2] Alternatively, other indices such as risk efficient index are constructed using portfolio optimization (see, e.g., Clare *et al.*, 2013a, 2013b). However, such indices are sensitive to model inputs and the choice of utility functions. Known as heuristic indices, a large group of alternative indices like the equal-weighted indices are constructed based upon a "rule of thumb". See Chow *et al.* (2011) and Clare *et al.* (2013a, 2013b), for a review of cap-weighted and alternative indices.

[3] We use $\boldsymbol{F}$ to denote $\boldsymbol{\eta}$ because the orthogonalized fundamental factors, residuals of fundamental factors after they are projected on the risk factor space, still represent fundamental factors.

[4] We have $\boldsymbol{W}_1'\mathbf{1} = 0$ because $\boldsymbol{F}$ is the residual vector from the orthogonalization regression, Equation (1), and the sum of individual residuals at a given time is zero. The condition $\boldsymbol{W}_3'\mathbf{1} = 0$ clearly holds because regression residuals from Equation (3), $\boldsymbol{\upsilon}$, also have a zero-sum. Finally, as the sum of combined weights across individual stocks, $\boldsymbol{W}'\mathbf{1}$, is 1, we have $\boldsymbol{W}_2'\mathbf{1} = 1$.

[5] *ROA* is highly correlated with gross profits-to-assets, the measure of asset productivity used in Novy-Marx (2013). Fama and French (2015) and Hou *et al.* (2015) examine the return predictability of operating performance, proxied by returns on equity. We use returns on assets since a firm's return of assets accesses its efficiency in asset usage.

[6] To normalize a variable, in each month, we compute the industry-adjusted fundamental factor by deducting the median of the variable from the variable and scale the difference by the standard deviation of the variable across the industry. We then obtain the ranks of the standardized fundamental factors, and scale them to be a number between 0 and 1. Normalization enables us to compare values of each fundamental factor across firms.

[7] The four-factor model is expressed below:

$$\boldsymbol{R_i} = \alpha_i + \beta_i \boldsymbol{MKT} + s_i \boldsymbol{SMB} + h_i \boldsymbol{HML} + \mu_i \boldsymbol{UMD} + \boldsymbol{\varepsilon_i} \quad (7)$$

where $\boldsymbol{R_i}$ is the monthly return of an individual stock in excess of the risk-free rate (the 1-month Treasury bill rate). ***MKT*** is the market risk premium in a month t, calculated as the market return in month t minus the risk-free rate of return. ***SMB*** (small minus big) is the difference in each month between the return on small-cap stocks and large-cap stocks. ***HML*** (high minus low) is the difference in each month between the return on a portfolio of high book-to-market stocks and the return on a portfolio of low book-to-market stocks. ***UMD*** is the difference in each month between the return on a portfolio of high momentum stocks and the return on a portfolio of low momentum stocks.

[8] For information on sector breakdown, see http://en.wikipedia.org/wiki/Global_Industry_Classification_Standard.

[9] We alternatively use the 5-year window to estimate four-factor loadings and obtain consistent results. Here we use the 10-year estimation window since this horizon covers a full length of an economic cycle at least.

[10] Our choice of the tilting parameters is a compromise between controlling tracking error and performance improvement despite that the tilting parameters used here are generally for an illustration purpose. Note that, shown in Section entitled "Alternative Tilted Indices" and Table 4, Sharpe ratios and information ratios of the tilted index are largely invariant to tilting parameters, given our choices of fundamental factors.

[11] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[12] Under the classical asset-pricing paradigm, a security staying profitable in bad states is more valuable feature for securities because payoffs delivered in bad states yield greater utility.

[13] A significant number of major credit events took place after year 2001, including the Enron debacle and subsequent accounting scandals, the World Trade Center attack, GM downgrades and default, and the financial crisis.

[14] Note that this is different from the four-factor regression in the earlier Section which is performed at the individual stock level.

[15] In addition to the four-factor model, we try other risk factors. For example, Frazzini and Pedersen (2014) find that the betting-against-beta (BAB) factor

that longs leveraged low-beta assets and shorts high-beta assets produces significant positive risk-adjusted returns. Moreover, Asness *et al.* (2013) find that a quality-minus-junk (QMJ) factor that longs high-quality stocks (stocks that are safe, profitable, growing, and well managed) and shorts low-quality stocks earns significant risk-adjusted returns in the U.S. and globally across 24 countries. We include BAB and QMJ as additional factors and still find that FIX has a significant annual alpha of 0.84%.

16 Recognizing that FIX is rebalanced monthly, we perform additional analysis to ensure the robustness of the analysis. We further examine the performance of alternative FIX products under alternative rebalancing frequency: 3 months, 6 months, and 12 months. The results show a limited reduction in portfolio performance when the rebalancing frequency is reduced.

17 The aggregation of four attributed performance is 13 bps short of the performance difference between FIX and R1000. This is because a small fraction, roughly 3%, of stocks in R1000 are excluded from FIX when they are assigned a negative weight in the portfolio.

18 It can be shown that optimal portfolio adjustments (i.e., weights) are determined by choosing $\boldsymbol{x}$, a vector for the adjustments in loadings to orthogonalized fundamental factors and the residual optimal to maximize the portfolio's information ratio $\frac{\overline{A}\boldsymbol{x}}{\sqrt{\boldsymbol{x}'\Sigma\boldsymbol{x}}}$, $\sqrt{\boldsymbol{x}'\Sigma\boldsymbol{x}} \leq TR^*$ for all $\boldsymbol{x}$, where $\overline{A}$ is the average performance of R1000 attributed to orthogonalized fundamental factors and the residual; Σ is the covariance matrix among R1000 performance attributed to orthogonalized fundamental factors and the residual; and TR^* represents the target tracking error. Simple calibrations show that the information ratio of FIX is fairly close to that of the optimal portfolio when we set a reasonable TR^*.

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