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SURVEYS AND CROSSOVER

This section provides surveys of the literature in investment management or short papers exemplifying advances in finance that arise from the confluence with other fields. This section acknowledges current trends in technology, and the cross-disciplinary nature of the investment management business, while directing the reader to interesting and important recent work.

WHAT DOES THE BET AGAINST BETA STRATEGY MEAN IN A MULTI-FACTOR WORLD?

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As of February 2019, an investor had a choice to invest in 1,043 smart beta Exchange-Traded Funds (ETFs). These ETFs depend on well-established asset-pricing anomalies. This paper provides a theoretical foundation justifying their existence. Loosely speaking, the investment strategy from the anomalies is simple: bet against beta. We explain the spirit of the investment strategy in a multi-factor world. In a model with heterogeneous risk-aversion agents facing margin constraints, we answer the question: What does the bet against beta strategy mean with multiple factors? Extending Frazzini and Pedersen (2014), we show that the beta is a weighted average of the factors betas. There are two implications. First, we add to the debate between fundamental indexation and cap-weighted indexation. Second, our article answers the question: which smart beta ETFs are actually smart, theoretically?

1 Introduction

In a perfect market, investors realize above-average returns only by taking above-average risk. Risky stocks have high returns on average, and safe stocks do not. CAPM formalizes the risk–return tradeoff: the expected return depends on its β, a mathematical proxy of systematic risk. Graphically, in a CAPM Security Market Line (SML) plot, the expected return linearly increases with the portfolio's β. CAPM implications are simple—in equilibrium, in a frictionless market, every asset has a zero α.

Since in reality the market is fraught with various frictions, the CAPM implications do not hold empirically. CAPM violations are neither new nor

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surprising. Using the friction of restricted borrowing, Black (1972) and Black *et al.* (1972) were probably the first to note the SML violation. They find that the expected returns do not linearly increase with the beta; the actual SML is flatter than the one predicted by the CAPM. Haugen and Heins (1975) find an even more egregious violation: they find an inverse relationship between expected returns and beta. Extending and corroborating the results in the 1990s, Fama and French (1992) lucidly conclude that beta is dead.

The flat beta has led to a host of asset-pricing anomalies. For example, Fama and French (1992) popularize an investment strategy based on the value–growth anomaly. The strategy depends on the stylized fact that value stocks outperform growth stocks on a risk-adjusted basis. Campbell *et al.* (2008) give convincing evidence about the distress-risk anomaly; the evidence finds that highly leveraged stocks underperform low leveraged stocks. Asness *et al.* (2019) find evidence of the quality–junk anomaly; they find that high-quality portfolios—safe, profitable, growing, and well-managed stocks—outperform low-quality portfolios. In fact, they find that this extra performance is not only a US phenomenon, but also a global phenomenon: they document consistent evidence from 24 countries. Across 20 equity markets, Treasury bonds, corporate bonds, and futures, Frazzini and Pedersen (2014) find that a strategy that shorts high-beta stocks and buys low-beta stocks yields high risk-adjusted returns. Summarizing the anomaly evidence, Baker *et al.* (2011) claim:

"... among the many candidates for the greatest anomaly in finance, a particularly compelling one is the long-term success of low-volatility and low-beta stock portfolios."

Frazzini and Pedersen (2014) coin the investment implication of the SML violation as "Bet Against Beta" (BAB). Expanding on this implication, this paper answers the following question:

What does the bet against beta strategy mean in a world with multiple factors?

To answer this question, following Frazzini and Pedersen (2014), we develop a model where investors face leverage constraints. For example, some investors, like mutual funds, must hold cash due to the regulations of the Investment Act of 1940. Even hedge funds suffer from leverage constraints: they have to post margin while trading derivatives. In a world with leverage constraints, high-risk investors tilt their portfolio toward high-beta stocks. This tilting artificially increases demand for high-beta stocks which, in turn, decrease their return relative to the CAPM.

In our model, we augment the intuition by assuming that the asset returns depend on K tradable factors. The model features heterogeneous investors with mean–variance preferences. The investors can be different both in wealth and risk aversion. The modeling setup mimics the classic CAPM framework with one exception—each investor is subject to a leverage limit. With the margin constraint friction, even with tradable factors, we show that an asset's α is inversely related to its β. In a K-factor world, in an equilibrium where each investor maximizes her portfolio wealth, we show that β in BAB is approximately equal to a weighted average of the K-factor βs. In this sense, we extend the implication of Frazzini and Pedersen (2014) and answer the question: What does BAB mean in a K-factor world?

In the academic literature, Harvey *et al.* (2016), McLean and Pontiff (2016), and Hou *et al.* (2017) summarize the evidence of hundreds of factors involved with the SML violation. Some of the robust factors are readily tradable. As of February 7, 2019, according to ETF.com, there were 1,043 smart beta ETFs, while smart beta funds passed one trillion in assets under management in 2017 (Thompson, 2017). The smart ETFs span a variety of investment strategies, with 8 out of the top 10 smart funds in terms of assets under management dealing with value, growth, or dividend factors. Other top 10 smart funds, like ETF USMV, which mimic a minimum variance portfolio, have over $23 billion in assets under management. Clarke *et al.* (2011), Chiu and Jiang (2016), and Bednarek and Patel (2018) show that a minimum variance portfolio overweights low-beta assets, in essence bet against beta. The results of our paper explain both the popularity of smart ETFs and their heterogeneous investment strategies. In other words, in a multiple factor framework, we explain what makes a smart fund smart.

BAB strategy raises the obvious question: Why don't hedge funds take away the α? Said differently, is there a risk-arbitrage? The answer to these two questions depends on the impact of leverage constraints. In the limit, where most of the wealth is concentrated in the hands of hedge funds, the α vanishes to zero. But, this result depends on the assumption that hedge funds do not face leverage constraints. This assumption is probably not realistic. Even though hedge funds have more access to leverage, they still face margin constraints. After all, the leverage ratio of Long-Term Capital Management (LTCM) was on the order of 25-to-1 and not more (Rubin *et al.*, 1999). More importantly, the limit case is far from reality. The assets managed by mutual funds (or funds that are regulated in general) are significantly larger than the assets managed by hedge funds for example.[1] Consequently, the α-generating strategies exist in equilibrium. There are three practical implications:

1. The article justifies portfolio tilts away from market cap. On one hand, in an influential article, Arnott *et al.* (2005) argue for portfolios dependent on "fundamental indexation" relative to capitalization-weighted indexes. On the other hand, Malkiel (2014) argues that such tilts, by definition, creates winners and losers. Our article links the two arguments.
2. The article theoretically answers Malkiel's (2014) question: Is smart beta ETF really smart? To answer, consider the smart beta ETF with ticker IWO (iShares Russell 2000 Growth ETF). As the name suggests, this ETF consists of growth stocks. According to Morningstar.com, the IWO beta with respect to S&P 500 is 1.31. Due to its high-beta, our theory suggests that IWO is unlikely to outperform the S&P 500 index on a risk-adjusted basis.[2]
3. The article also prescribes how to construct an optimal portfolio consistent with the BAB strategy. Consider an investor not constrained by the leverage constraints. Then, it is optimal for the investor to go long portfolios low on the SML and short portfolios that are high on the SML. For investors not willing to short, they should either go long portfolio low on the SML or go long a smart beta ETF that is low on the SML.

2 The model

2.1 Roadmap

In this section, we give a theoretical foundation justifying why leverage constraints lead to bet against beta. The section comprises three parts. In the first part (Subsection 2.2), we derive portfolio implications if stock returns depend on $K \geq 1$ factors. Minor algebra shows that any arbitrary portfolio also depends on K factors and therefore the covariance between any stock and the market portfolio is a weighted average of factor betas. In the second part (Subsection 2.3), using the portfolio factor structure, we set up the investor's problem. We consider that the mean–variance investor is leverage constrained. The constraint has a simple implication. A risk-tolerant investor cannot borrow enough to buy stocks with high covariance with the market. To compensate, she overweights her portfolio with stocks that have a high covariance with the market. The demand for these stocks artificially increases. As a result, the prices of stocks increase and the future returns decrease. In the third part (Subsection 2.4), we prove the implication. We first solve for an arbitrary investor's portfolio choice considering her leverage constraint. Afterward, we aggregate the choices of all investors to calculate the aggregate demand. Last, by equating the demand with supply, we derive the expected return of each stock in equilibrium. In Subsection 2.5, we show that the beta in the bet against beta is a weighted average of factor betas.

2.2 *Factor model*

We assume a one-period model with N assets. Each asset, denoted by $n = 1, 2, \ldots, N$, has a market value of V_n and a market share of $\pi_n^* \equiv V_n / \sum_{n=1}^{N} V_n$. Since these stocks are held in positive net supply, these assets comprise the primary assets.

Stock returns follow a linear K-factor structure as in Ross (1976). That is, the return is composed of the expected return and the sum of two separate sources of random returns: factor return and idiosyncratic return:

$$\tilde{R}_n = \bar{R}_n + \sum_{k=1}^{K} b_{nk}\tilde{F}_k + \tilde{\epsilon}_n. \qquad (1)$$

For $n, m \in \{1, 2, \ldots, N\}$; and $k, l \in \{1, 2, \ldots, K\}$, we assume:

$$\mathrm{E}[\tilde{F}_k] = 0; \quad \mathrm{E}[\tilde{\epsilon}_n] = 0; \quad \mathrm{E}[\tilde{\epsilon}_n | \tilde{F}_k] = 0;$$

$$\mathrm{E}[\tilde{F}_2^k] = 1; \quad \mathrm{E}[\tilde{\epsilon}_n^2] = s_n^2; \quad \mathrm{Cov}[\tilde{\epsilon}_n, \tilde{\epsilon}_{m \neq n}] = 0;$$

$$\mathrm{Cov}[\tilde{F}_k, \tilde{F}_{l \neq k}] = 0; \quad \mathrm{Cov}[\tilde{\epsilon}_n, \tilde{F}_k] = 0.$$

The number of factors, K, is known. The mean of idiosyncratic risk conditional on each of the K factors is zero. In order to resolve rotational indeterminacy, we assume that the factors have unit variance. Additionally, factors and idiosyncratic risk are uncorrelated with each other. Standard calculations show that

$$\mathrm{Cov}[\tilde{R}_n, \tilde{R}_m] = \sum_{k=1}^{K} b_{nk} b_{mk};$$

the variance of the firm return is:

$$\begin{aligned}
\mathrm{Var}(\tilde{R}_n) &= \mathrm{Var}\left(\tilde{R}_n + \sum_{k=1}^{K} b_{nk}\tilde{F}_k + \tilde{\epsilon}_n\right) \\
&= \mathrm{Var}\left(\sum_{k=1}^{K} b_{nk}\tilde{F}_k\right) + \mathrm{Var}(\tilde{\epsilon}_n) \\
&= \sum_{k=1}^{K} b_{nk}^2 \mathrm{Var}(\tilde{F}_k) + s_n^2 = \sum_{k=1}^{K} b_{nk}^2 + s_n^2.
\end{aligned}$$

The market portfolio, as expected, also has a factor structure:

$$\begin{aligned}
\tilde{R}_{mkt} &= \sum_{n=1}^{N} \pi_n^* \tilde{R}_n \\
&= \sum_{n=1}^{N} \pi_n^* \left(\bar{R}_n + \sum_{k=1}^{K} b_{nk}\tilde{F}_k + \tilde{\epsilon}_n\right) \\
&= \bar{R}_{mkt} + \sum_{k=1}^{K} b_k \tilde{F}_k \\
&\quad + \sum_{n=1}^{N} \pi_n^* \tilde{\epsilon}_n \quad \text{with } b_k \equiv \sum_{n=1}^{N} \pi_n^* b_{nk}.
\end{aligned} \qquad (2)$$

The parameter, b_k, is the economy-wide exposure to factor $\tilde{F}_k$. Throughout this paper, we assume that b_k is not affected by individual asset attributes—it is exogenous. The factor structure also leads to a tractable expression of the covariance between asset n and the market:

$$\sigma_{n,mkt} \equiv \mathrm{Cov}[\tilde{R}_n, \tilde{R}_{mkt}]$$
$$= \mathrm{Cov}\left(\tilde{R}_n, \bar{R}_{mkt} + \sum_{k=1}^{K} b_k \tilde{F}_k + \sum_{n=1}^{N} \pi_n^* \tilde{\epsilon}_n\right)$$
$$= \sum_{k=1}^{K} b_{nk} b_k + \pi_n^* s_n^2. \quad (3)$$

In addition to the N primary assets, $K + 1$ derivative assets are also traded. Following Cox *et al.* (1985), these derivative assets, although held in zero net supply, "complete" the market. The return dynamics of the derivative assets are assumed to follow:

$$\tilde{R}_{N+k} = \bar{R}_{N+k} + \tilde{F}_k \quad \text{for } k = 1, 2, \ldots, K. \quad (4)$$

Since the returns are from traded factors, they do not depend on the idiosyncratic volatility. For example, suppose $K = 1$ and the only factor is the market factor. Then, Equation (4) states that there is a robust futures market allowing investors to trade the market factor. Finally, the $N + K + 1$ asset is a risk-free bond offering a sure return, R_f:

$$\tilde{R}_{N+K+1} = R_f. \quad (5)$$

To summarize, the return dynamics of both the primary and the secondary assets follow a factor structure. All agents understand the market structure perfectly; there is no incomplete information. Lastly, all agents have homogeneous expectations; there is no disagreement. Next, we describe the agents and their portfolio choice.

2.3 Investor portfolio choice and margin constraints

2.3.1 Preferences

There are J investors with mean–variance preferences with wealth $W(j)$ and risk-aversion coefficient $\delta(j)$. The preference of investor j is represented as:

$$U(j) = \mathrm{E}[\tilde{R}(j)W(j)] - \frac{\delta(j)}{2\, W(j)} \mathrm{Var}[\tilde{R}(j)W(j)], \quad (6)$$

where $\tilde{R}(j)$ is the random future gross return. In order to maximize utility, each investor chooses a fraction $\{x_n(j)\}$ to invest in each of the N primary assets, a fraction $\{y_k(j)\}$ to invest in each of the K derivative assets, and a fraction $\{z(j)\}$ to invest in the risk-free bond. The portfolio return, $\tilde{R}(j)$, is:

$$\tilde{R}(j) = \sum_{n=1}^{N} x_n(j)\tilde{R}_n + \sum_{k=1}^{K} y_k(j)\tilde{R}_{N+k} + z(j)R_f.$$

Using the budget constraint,

$$\sum_{n=1}^{N} x_n(j) + \sum_{k=1}^{K} y_k(j) + z(j) = 1;$$

slight algebra shows that the portfolio return of investor j also follows a factor structure.

$$\tilde{R}(j) = \bar{R}(j) + \sum_{k=1}^{K} b_k(j)\tilde{F}_k$$
$$= \sum_{n=1}^{N} x_n(j)\tilde{\epsilon}, \quad \text{with}$$
$$b_k(j) \equiv \sum_{n=1}^{N} x_n(j) b_{nk} + y_k(j); \quad (7)$$
$$\bar{R}(j) \equiv R_f + \sum_{n=1}^{N} x_n(j) R_n^e$$
$$+ \sum_{k=1}^{K} b_k(j)(\bar{R}_{N+k} - R_f); \quad (8)$$

and

$$R_n^e \equiv \bar{R}_n - R_f - \sum_{k=1}^{K} b_{nk}(\bar{R}_{N+k} - R_f). \quad (9)$$

The term $b_k(j)$ shows the factor k exposure of investor j. The term R_n^e is stock n's excess return after accounting for the factor exposure. The return variance, in turn, is:

$$\mathrm{Var}(\tilde{R}(j)) = \sum_{k=1}^{K} b_k^2(j) + \sum_{n=1}^{N} x_n^2(j) s_n^2. \quad (10)$$

Upon inspection of Equations (8) and (10), it is easier to work with $b_k(j)$ as opposed to $y_k(j)$. Also, Equation (7) shows that there is a one-to-one mapping between $b_k(j)$ and $y_k(j)$.

2.3.2 *Investor portfolio choice with margin constraints*

The market in our setup has one friction—all investors do not have access to unlimited leverage. But, we distinguish the use of leverage between primary and secondary assets. On one hand, regarding primary assets, we assume that either some investors cannot use leverage or some investors can use leverage but face margin constraints. For example, Section 18 of the Investment Company Act of 1940 places leverage limits on mutual funds.[3] On the other hand, regarding secondary assets, we assume that investors do not face margin constraints. While the lack of constraint assumption is strong, we make it for tractability. Suppose an investor buys out-of-the-money options; economically, her investment is a levered position. In the same manner, a long position in a futures market is also a levered position. Because of the mechanics of the secondary asset market, an investor has access to more leverage relative to the primary asset market. To reflect the discrepancy in leverage access between the primary and secondary markets, we allow unlimited access to leverage while trading secondary assets while restricting leverage access to primary assets. Mathematically, each investor j is subject to the constraint:

$$m(j) \sum_{n=1}^{N} x_n(j)\, W(j) \le W(j) \quad \text{or}$$

$$m(j) \sum_{n=1}^{N} x_n(j) \le 1. \quad (11)$$

This constraint requires that some multiple of the total dollars invested in primary assets, the sum $x_n(j) \times W(j)$, must be less than the agent's wealth. Consider an investor who is mandated to hold cash so that $z(j) > 0$.[4] Then, for this investor, $m(j) > 1$. Consider a retail investor or a hedge fund, who can borrow but is subject to a limit. For example, suppose that the investor can borrow up to two times her wealth. Then, $m(j)$ for the investor is 0.50. Taking the portfolio choice and the margin constraints into account, an investor j chooses $\{x_n(j)\}$ and $\{b_k(j)\}$ to maximize Equation (6) subject to Equation (11).

2.4 *Equilibrium, aggregation, market clearing, and asset-pricing implications*

Maximizing investor's utility in Equation (6), the Lagrangian of investor j is:

$$L(j) = \bar{R}(j) - \frac{\delta(j)}{2}\mathrm{Var}(\tilde{R}(j)) + \lambda(j)\left(1 - m(j)\sum_{n=1}^{N} x_n(j)\right)$$

where $\lambda(j)$ is the lagrange multiplier of the margin constraint in Equation (11). The first two components of the Lagrangian reflect the risk–return tradeoff. The last component reflects the margin constraint. The first-order condition of all investors with respect to the factor exposure, $b_k(j)$, is:

$$\frac{\partial L(j)}{\partial b_k(j)} = 0 \;\Rightarrow\; b_k(j) = \frac{\bar{R}_{N+k} - R_f}{\delta(j)}. \quad (12)$$

Equation (12) is standard. The numerator is the excess return from taking one more unit of the factor k exposure. The denominator is the marginal risk scaled by risk aversion.[5] The first-order condition of all investors with respect to the primary assets, $x_n(j)$, is:

$$\frac{\partial L(j)}{\partial x_n(j)} = 0 \;\Rightarrow\; x_n(j) = \frac{\bar{R}_n^e - \lambda(j)m(j)}{\delta(j)s_n^2}. \tag{13}$$

Equation (13) is also standard. There are two different cases to consider. First, suppose that the margin constraint does not bind and hence $\lambda(j) = 0$. Then $x_n(j)$ increases with the expected excess return and decreases with the risk-aversion coefficient and idiosyncratic volatility. Second, suppose that the margin constraint does not bind so that $\lambda(j) > 0$. Since $m(j) > 0$, $x_n(j)$ is lower than the previous case.

2.4.1 *Expected return of the secondary assets*

Let $W \equiv \sum_{j=1}^{J} W(j)$ be the market wealth; let:

$$\frac{1}{W}\sum_{j=1}^{J}\frac{W(j)}{\delta(j)} \equiv \frac{1}{\delta} \tag{14}$$

be the risk tolerance of the representative agent; and let:

$$\frac{1}{W}\sum_{j=1}^{J}\frac{W(j)m(j)\lambda(j)}{\delta(j)} \equiv \frac{\lambda}{\delta} \tag{15}$$

be the aggregate margin constraint normalized by the representative agent's risk tolerance. Multiplying Equation (12) by $W(j)$, adding over all investors, dividing by the market wealth and using Equation (14), we get:

$$\sum_{j=1}^{J}\frac{b_k(j)W(j)}{W} = \frac{\bar{R}_{N+k} - R_f}{\delta}. \tag{16}$$

Additional results about b_k can be better understood by analyzing the relationship between $y_k(j)$ and $b_k(j)$ in Equation (7). Multiplying Equation (7) by $W(j)$, adding over all investors, dividing by the market wealth, we have:

$$\frac{1}{W}\sum_{j=1}^{J} y_k(j)W(j) = \sum_{n=1}^{N} b_{nk}\left(\sum_{j=1}^{J}\frac{x_n(j)W(j)}{W}\right) - \sum_{j=1}^{J}\frac{b_k(j)W(j)}{W}.$$

Since the derivative assets are held in zero net supply and noting that the term in the parenthesis is the market share while using Equation (16), the above expression simplifies to:

$$\sum_{n=1}^{N} b_{nk}\pi_n^* = b_k = \frac{\bar{R}_{N+k} - R_f}{\delta}. \tag{17}$$

Equation (17) is expected. Since the variance of the factor is normalized to unity, the expected return of the aggregate factor k exposure, b_k, equals the excess factor return scaled by the representative agent's risk tolerance, δ.

2.4.2 *Expected return of the primary assets*

Multiplying Equations (13) by $W(j)$, adding over all investors and dividing by market wealth, the excess return can be written as:

$$\bar{R}_n^e = \pi_n^* s_n^2 \delta + \lambda; \quad \text{or}$$

$$\bar{R}_n - R_f = \sum_{k=1}^{K} b_{nk}(\bar{R}_{N+k} - R_f) + \pi_n^* s_n^2 \delta + \lambda.$$

The second equality stems from the definition of the excess return in Equation (9). Using Equation (17) and using the covariance with respect to the market return (Equation (3)), the excess expected return of asset n becomes:

$$\bar{R}_n - R_f = \delta \operatorname{Cov}(\tilde{R}_n, \tilde{R}_{mkt}) + \lambda. \qquad (18)$$

Multiplying Equation (18) by the market share, π_n^*, and adding over all primary assets, we have:

$$\delta = \frac{\bar{R}_{mkt} - R_f - \lambda}{\operatorname{Var}(\tilde{R}_{mkt})}. \qquad (19)$$

Substituting Equation (19) into Equation (18), we get the main equation of the paper:

$$\bar{R}_n - R_f = \beta_n(\bar{R}_{mkt} - R_f) + \underbrace{\lambda(1-\beta_n)}_{\equiv \alpha_n}. \qquad (20)$$

Equation (20) is the main equation of this article. The expected return is composed of two terms. The first term, consistent with CAPM, scales with the investment's beta. The first term simply states that the expected excess return of asset n is a product of the asset's βn and the market risk premium $\bar{R}_{mkt} - R_f$. The second term stems from the margin constraint. Consistent with the literature, we define the residual to be alpha: $\alpha_n \equiv \lambda(1-\beta_n)$. Equation (20) shows that an investor should create low-beta portfolios to achieve a higher risk-adjusted return.

We conclude this section by summarizing the methodology and result. In the model, an investor optimizes her portfolio while taking leverage constraints into account. Due to the constraints, bold risk-tolerant investors overweight high β stocks in their portfolio relative to the situation without constraints. The overweight, in turn, leads to a higher price and lower subsequent return. Last, it is important to note that the BAB strategy is an equilibrium result. Casual inspection implies that the BAB strategy may mean some sort of risk arbitrage. After all, in a competitive capital market, how can $\alpha \neq 0$ exist? Our article suggest that the markets are not competitive due to one fundamental constraint—investors suffer from leverage constraints. In fact, as long as sufficient investors are leverage constrained, BAB strategy is viable.

2.5 *What does bet against beta strategy mean in a multi-factor world?*

Standard calculation shows that the market variance is:

$$\sigma_{mkt}^2 \equiv \operatorname{Var}\left(\bar{R}_{mkt} + \sum_{k=1}^{K} b_k \tilde{F}_k + \sum_{n=1}^{N} \pi_n^* \tilde{\epsilon}_n\right)$$

$$= \sum_{k=1}^{K} b_k^2 + \sum_{n=1}^{N} (\pi_n^*)^2 s_n^2.$$

Using the definition, $\beta n \equiv \sigma_{n,mkt}/\sigma_{mkt}^2$, the β_n is a weighted average sum of the factor loadings:

$$\beta_n \approx \sum_{k=1}^{K} b_{nk}\gamma_k \quad \text{with } \gamma_k \equiv \frac{b_k}{\sum_{k=1}^{K} b_k^2}. \qquad (21)$$

This leads to the main proposition of this article:

Proposition1 *If asset n has more exposure to all of the factors relative to asset m, then $\alpha_n < \alpha_m$:*

$$\forall k \quad if\ b_{nk} > b_{mk} \ \Rightarrow \ \alpha_n < \alpha_m. \qquad (22)$$

Proposition 1 is the key result of the article; it shows the intuition behind the increasing popularity of smart beta ETFs. More importantly, why a smart ETF fund is smart. Proposition 1 leads to two empirical hypotheses concerning smart ETFs:

1. BAB strategy implies that the β of truly smart ETFs with respect to the market is low.
2. BAB strategy also implies that high β smart ETFs are actually dumb—they suffer from negative α in equilibrium.

We conclude this section by summarizing the recent debate regarding the link between leverage constraints and BAB strategy. Starting from Blume (1970), Black *et al.* (1972), Blume and Friend (1973), and Stambaugh (1982) find evidence of a flat SML. That is, the viability of the BAB strategy is relatively uncontroversial. But the reasons behind the viability are up for debate. In this paper, we follow Frazzini and Pedersen (2014) and use leverage constraints to motivate the BAB strategy, but acknowledge the link between BAB and leverage constraints is well established. Using incomplete information, Merton (1987) explains the flat SML. In his model, investors are unaware of the return-generating process of some stocks. As a result, they do not trade those stocks. The lack of information creates an aggregate non-tradability constraint similar to the leverage constraint in our setup. Brennan (1993) and Alankar *et al.* (2013) use tracking error constraints arising due to the principal–agent issues in asset management to explain the flat SML. Hindy (1995) and Cuoco (1997) use portfolio constraints; Garleanu and Pedersen (2011) use margin constraints; Cederburg and O'doherty (2016) use the notion of a beta measure error; Liu *et al.* (2018) use idiosyncratic volatility; Hong and Sraer (2016) use short-sale constraints and heterogeneous beliefs; and Bali *et al.* (2017) use investor preferences for lotteries to explain the flat SML.

3 Conclusion

This article contributes to a better understanding of the bet against beta investment strategy. Particularly, in a general equilibrium setting, we answer: what does the strategy mean in a multi-factor world? We show that the appropriate beta is just a weighted average of the factor betas. More importantly, we explain the reasoning behind the increasing popularity of smart beta ETFs.

Notes

[1] As of January 2017, hedge funds controlled about $3 trillion in assets.https://www.bloomberg.com/news/articles/2017-01-23/hedge-fund-assets-pass-3-trillion-in-2016-for-first-time-chart.

[2] In fact, the α, the Sharpe ratio and the Treynor ratios of IWO are lower than the S&P 500 index according to Morningstar as of March 5, 2019.

[3] https://www.sec.gov/divisions/investment/seniorsecurities-bibliography.htm.

[4] Due to the daily redeemability requirement, most mutual funds hold cash. By their charter, most institutional funds also hold cash.

[5] Recall that the volatility of the factor equals 1 by assumption.

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