Journal Of Investment Management, Vol. 17, No. 2, (2019), pp. 17–39

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A MODEL OF BOND VALUE*

EXPLAINING YIELDS WITH GROWTH AND INFLATION

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This paper looks to establish a new heuristic for investors, giving them a simple, intuitive way to relate bond yields to prevailing trends in growth and inflation. The model offers an alternative to forecasting surveys, which have been over-estimating 10-year Treasury yields for decades and continue to project yields above 4% in the long run. The model does well in in-sample and out-of-sample tests used in the literature to evaluate other measures of value. The model can be used on its own or in conjunction with other models to forecast yields and also as a benchmark to evaluate yield forecasts. The model is consistent with some of the more advanced economic models of interest rates that suggest that the low bond yields of recent years are in line with broader economic trends, rather than due to temporary factors that are likely to reverse quickly.

1 Forecasting bond yields

The prediction of future bond yields is a perennial problem in active management. The difficulty it presents is well described by Fong (1983), who states: "it is at best, extremely difficult to forecast the future direction of rates, much less their magnitude... (But,) regardless of one's conviction, rate anticipation must be dealt with if for no other reason than that it is usually the dominant source of incremental return..." for bond portfolios. In looking to anticipate yields, investors could turn to published surveys of bond yield forecasts. However, over the past two decades, forecasters have generally over-estimated bond yields. For example, every December the Livingston survey provides a 2-year forecast of the 10-year bond yield.

Livingston is a survey of professional forecasters published by the Federal Reserve Bank of Philadelphia and is considered to be of high quality. The survey's median forecast has over-estimated the 10-year yield for the past 18 consecutive years, and has only under-estimated bond yields once during the entire 24 years the survey

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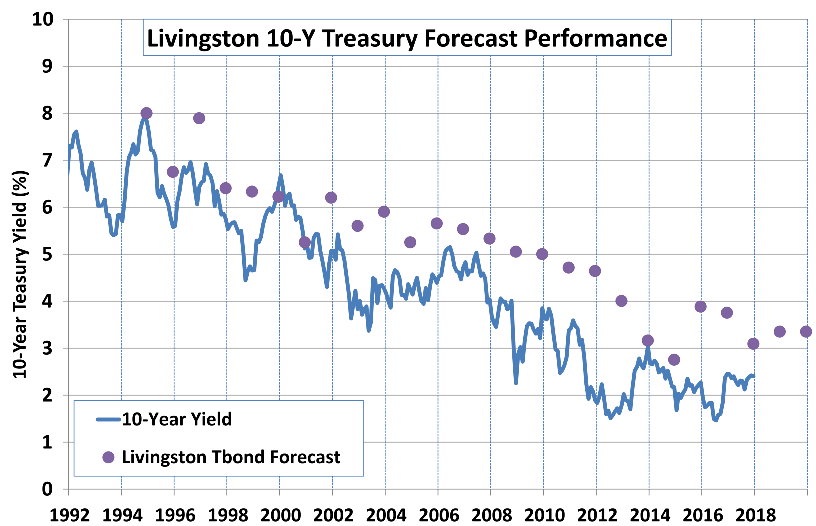

Figure 1 The figure shows Livingston 2-year forecasts of 10-year bond yields vs actual bond yields 2 years later. The figure shows a clear upward bias in forecasts.

has been conducted. On average, each forecast has over-estimated bond yields by 119 basis points, and if the forecasts were the output of a regression, its R-squared would only be 0.22.[1] As Figure 1 demonstrates, the forecasts exhibit a clear upward bias, because the purple dots (the *ex-ante* forecasts) are well above the blue line (the *ex-post* bond yields).

Over-estimating bond yields is a common theme among different forecasting surveys. For example, the President's Council of Economic Advisers (CEA) published a paper on the topic in 2015. It includes a chart to demonstrate that economists' forecasts have been substantially over-estimating yields for several decades. The chart shows that forecasters still generally expected yields to return to the 4–5% range in the long run.[2] This is consistent with many long-term projections used in the industry. For example, BNY Mellon, in its *10-Year Capital Market Return Assumptions for Calendar Year 2016* states, "In the US, we see Treasury yields rising until they reach a normalized level in six years."[3] This statement is followed by a chart showing 10-year yields rising above 4% in that time frame.

Although it does not give annual forecasts out that far, the Survey of Professional Forecasters (SPF) does give a 10-year average projection as well as annual forecasts out several years. In Figure 2, by combining long- and short-horizon forecasts from the SPF, we can see implied expected pathways in yields that are very similar to those in the CEA's chart and consistent with the BNY Mellon's 10-year projection for 2016. The SPF 2017 forecasts also imply yields rising above 4% in the out years. Like the CEA chart, we can see that the forecasts appear upwardly biased. We can also see a continuing belief that yields will eventually rise above 4%.

Given the poor performance of forecasts, investment decision-makers have good reason to be skeptical. Investment decision-makers may be able to benefit from a heuristic or rule of thumb that gives a simple expression of bond yield "value" in terms of broad, fundamental economic trends. Although interest rate models vary in their inputs, the Council of Economic Advisors (2015) lays out both inflation and economic growth (or components of economic growth, including productivity, population, and technology growth) as

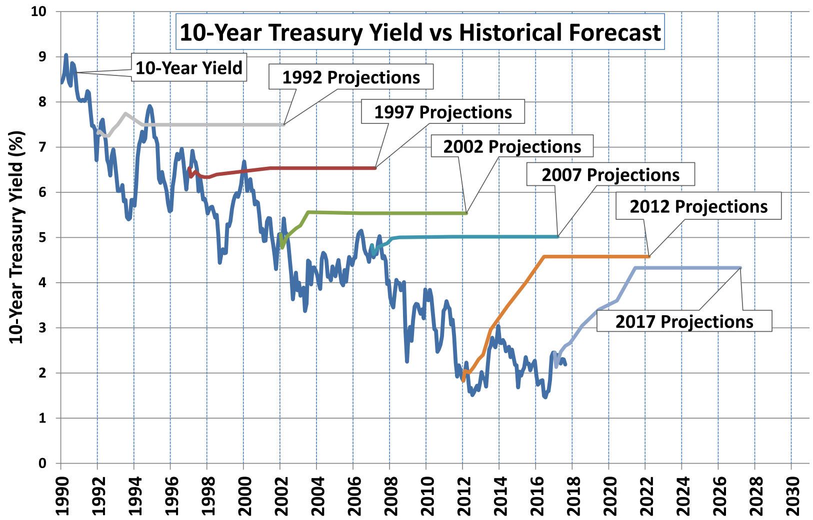

Figure 2 The figure shows Survey of Professional Forecasters' projected pathways of 10-year bond yields over 10-year horizons vs actual bond yields that occurred. It shows a clear upward bias and also shows that forecasters still expect yields to rise above 4%.

the principal drivers of interest rates supported by economic theory.[4] Moreover, in their survey of the drivers of interest rates, Rachel and Smith (2015) note that "...changes in global trend growth are probably the most commonly-cited driver of changes in real interest rates."[5] If we want to provide a simple metric, establishing an empirical link to inflation and growth would seem like a good place to start.

2 Literature review

Inflation and growth both have deep foundations in economic theory as drivers of yields. Fisher (1930) is commonly cited as linking nominal interest rates to inflation with the following approximation:

$$i \approx r + \pi \tag{1}$$

where: $i \equiv nominal\ interest\ rate$

$r \equiv real\ interest\ rate$

$\pi \equiv inflation.$

Solow (1956) presents the Solow Growth Model which relates growth to real interest rates. Ramsey (1928) relates *per capita* GDP growth to real interest rates rather than total GDP growth, but Baker *et al.* (2005) point out that the reason Ramsey uses *per capita* GDP is due to unrealistic simplifying assumptions. Although there is some disagreement in the literature as to whether the relationship should be with growth of total GDP or *per capita* GDP, a link between interest rates and growth appears well grounded in economic theory.

In the case of equity prices, there are broad measures of long-run value available. For example, Wright (2004) proposes several measures of Q based on the original concept of Tobin's Q, and also analyzes dividends and cash-flow yield. Likewise, Campbell and Shiller (1998) examine dividend–price ratios as well as what has become known as the "Cyclically Adjusted Price to Earnings Ratio" or "CAPE." The CAPE relates equity prices to trailing 10-year average real earnings. By using a 10-year average in the denominator, the method reduces the volatility in earnings coming from the business cycle. In their study, Campbell and Shiller find that both the price–dividend ratio and CAPE were predictive of future equity market prices. They used the CAPE to argue that equity prices at the time were very high relative to historical norms which they saw as an omen of low returns.

Both *Q* and CAPE are an attempt to link prevailing equity prices, which are in principal forward looking, to concrete, backward looking, fundamental measures of intrinsic value in order to allow one to judge whether prices are "too high" or "too low." In both cases, these measures may deviate significantly from their average or "fair value," for extended periods of time. Nevertheless, those investors focused on long-run returns, or long-run measures of value, find these measures useful, even if their ability to forecast equity returns over short investment horizons is limited.

Asness, Ilmanen, and Maloney (2017) examine the effectiveness of the CAPE for forecasting returns on an "out-of-sample" basis. They find "a puzzling gap between encouraging in-sample evidence and disappointing out-of-sample performance of value timing strategies" using the CAPE and other value indicators. In the case of CAPE, they find that a long-term trend toward higher valuation causes a value-weighted strategy to be under-invested, on average, over a 60-year period when equity valuations were generally rising. They show that it is possible *ex post* to remove this effect and demonstrate the benefit of the value signal if it does not lead to a bias to be underexposed in markets exhibiting a trend toward higher valuation, but note that this adjustment would be impossible to do *ex ante*. Nevertheless, they find that incorporating a momentum component into the trading strategy does allow one to achieve superior returns, so the value measure is still of benefit, even if by itself it does not ensure superior returns. In the Appendix, the trio also lay out a bond value measure akin to the measures for equities. Using expected inflation, they calculate the real interest rate of nominal 10-Year Treasuries. They use inflation forecast surveys for their expected inflation variable as far back as the series go and then use a statistical technique to approximate expectations in prior decades. In this case, they find a value timing strategy to exhibit superior returns to a buy-and-hold strategy but also that adding a momentum component improves returns still further.

Although imperfectly analogous, Taylor (1993) presents the "Taylor Rule," which attempts to describe the levels of central bank target rates. Central bankers and investors can use this framework as a relatively straightforward way of judging whether short-term rates are "too high" or "too low." This judgment is based on how these rates have typically related to prevailing economic conditions in the past. Although Taylor acknowledges a lack of consensus about the size of the coefficients for policy rules, he nevertheless sets forth a representative framework for such rules using round numbers as representative parameters. Taylor relates the Fed Funds Rate positively to the rate of inflation over the previous four quarters, the deviation of GDP from potential GDP, the deviation of inflation from a target of 2%, and a 2% constant term which is in effect a neutral short-term real interest rate. Taylor presents a chart to demonstrate how the parameters he chose for the rule do a good job of describing the level of central bank rates over the 6-year period of 1987–1992. Although deliberately simple in nature, the framework has had a profound impact. Unlike the long-term equity value measures we have discussed, however, the Taylor Rule does seek to adjust for some cyclical factors by including in its formulation both deviation from the inflation target and economic growth relative to potential.

Laubach and Williams (2003) present a model framework for determining the appropriate "natural rate of interest" or "r^*", which represents that real Fed Funds rate that is consistent with stable inflation and economic output being equal to its potential. This rate corresponds to the intercept term in feedback rules such as the Taylor Rule and allows economists to study how this parameter may vary through time. The Laubach–Williams model figures prominently in the economic literature on interest rates. The natural rate of interest (or r^*) is a cycle-neutral concept, and in this sense would be more akin to CAPE than the Taylor Rule. However, the model is extremely sophisticated and its recreation would be beyond the technical expertise of many investors.

There are different models for calculating the inflation-neutral short-term real interest rate. In an economic letter from the Federal Reserve Bank of San Francisco, Bauer and Rudebusch (2016) show three such models (including the Laubach–Williams model), charting them over the 2000 to 2016 time period. The chart shows that the different models of the neutral short-term real interest rates can be as much as 2 percentage points apart at a given point in time. Directionally, however, the chart shows broad consistency across the different measures, with all three falling considerably from 2000 to 2016, with the Laubach–Williams model falling from about 3% to about zero.

In looking for a simple rule or heuristic for defining "normal" or "fair value" at the long end, one might consider the Golden Rule, which uses the Solow Growth Model framework. Having taken the care to lay out the Golden Rule in a paper written as a fairy tale, Phelps (1961) gives a pleasant read. The paper shows that under certain assumptions, utility is maximized across generations when the real interest rate related to capital investment is equal to the potential growth rate. It was in the sense of "doing unto others as to oneself" because each generation following the rule is maximizing consumption across all generations rather than just its own generation.

The reasoning behind the Golden Rule is essentially that in equilibrium, the marginal product of capital is equal to the natural growth rate. Firms would in all likelihood expect a return at least equivalent to the rate of GDP growth when choosing to undertake an investment. Otherwise they would seek investment opportunities elsewhere. Since the marginal product of capital will also be equal to the marginal cost of capital in equilibrium, this implies that the marginal cost of capital for the economy as a whole will equal the natural rate of growth. So as long as the marginal investor can borrow at or near the risk-free rate, then it is not unreasonable to expect real long-term rates and real GDP growth to gravitate toward each other over time. This leads us to the following approximation:

$$i_{10Y} \approx g + \pi \quad (2)$$

where: $i_{10Y} \equiv$ *nominal* 10-*year government bond yield*

$g \equiv$ *real growth rate*

$\pi \equiv$ *inflation*.

Bordo and Dewald (2001) examine 13 countries and use the Golden Rule framework to argue that inflation expectations were higher during the period of 1962–1995 than during the period of 1881–1913. Looking at bond yields, they use average GDP growth rates across the two periods to isolate inflation expectations by assuming that "in longer-term equilibrium the real rates of interest would equal the growth rate of real output." They go on to note that "this is a common assumption in both theoretical and empirical studies." Examining differing periods of rising and falling inflation, they find that bond yields tend to follow inflation trends up and down, but do a poor job of predicting rising or falling inflation trends.

Another study which looks at the relationship between yields and growth is Lilico and Ficco (2012). They analyze the ability of inflation-linked bond yields to predict economic growth. They note that the Golden Rule is a "key equilibrium condition" both in economic theories of sustainable growth and corporate finance theory. However, they go on to note that the relationship between growth and real rates should not necessarily be one-to-one but that "...one should expect both that there is some relationship between the levels of the risk-free rate and the growth rate... and that, over a sufficiently long timescale, changes in the risk-free rate to be correlated with changes in the sustainable growth-rate." Indeed, Laubach and Williams (2003) estimate the relationship between potential growth and the neutral real short-term interest rate to be near, but not precisely, one.

Even in the case of inflation, there is research which would support the view that the relationship between yields and inflation is not necessarily one-to-one. For example, Feldstein and Summers (1978) argue that the relationship between bond yields and expected inflation should be greater than one due to tax distortions. A slope greater than one is also consistent with the work of Campbell *et al.* (2013). They break new ground by modeling the covariance of equities and bonds and relating it to the evolving relationship between inflation and growth. Their model finds a positive nominal bond risk premium during the early 1980s and a negative nominal bond risk premium in 2000 and especially during the 2007–2009 financial crisis. This is because bond prices tended to be positively correlated with equities during the periods of higher inflation but were negatively correlated during periods of low inflation or especially when there was a threat of deflation. During these periods the bonds provided a hedge against equity volatility. Although an over-simplification of their work, one could characterize it as finding a positive risk premium in bond yields when inflation is high and a negative risk premium when inflation is low or there is a risk of deflation, which would be consistent with a slope coefficient greater than one in this context.

Although the Golden Rule may be a common simplifying assumption, the evidence clearly suggests that it may be too simple for our purposes. Given the widespread presumption that yields should have a one-to-one relationship with growth and inflation, however, it may be rational to expect the relationship to be near one-to-one, if not exactly one-to-one.

3 Bond value model

For the purposes of creating a heuristic to explain bond yields broadly in economic terms, giving us a measure of "normal" or "fair value," let us begin with the well-established drivers of GDP growth and inflation in the following equation:

$$i_{10Y} = \beta_0 + \beta_1 * \pi + \beta_2 * g \tag{3}$$

where: $i_{10Y} \equiv nominal\ 10 - year\ government\ bond\ yield$

$g \equiv real\ growth\ rate$

$\pi \equiv inflation.$

Just as one can look at price-to-earnings ratios using both forward and backward earnings, one could try to construct a measure of fair value for yields using forward or backward measures of growth and inflation. Although forecasts of inflation and growth may appear to be an obvious input to a forward looking model, these surveys have a number of drawbacks. Ideally, one would use 10-year projections of growth and inflation to project 10-year yields, but these are infrequent and have a limited history. This would reduce the sample size, eliminate the 1980s and earlier from the field of study, and only allow for infrequent model updates. No doubt the limited availability of expectations data is a principal reason many economic studies, including Bordo and Dewald (2001) and Rachel and Smith (2015), use realized data instead. Using realized inflation and growth data would also be consistent with Bordo and Dewald's (2001) findings that bond yields tended to follow inflation trends up and down but did a poor job of anticipating "impending changes in inflation trends." Another, perhaps minor, point is that whereas past GDP and CPI are objective measures, expectations may vary, and one cannot be certain that the expectations that are driving the bond market are identical to those of public forecasters. Nevertheless, there are two forecast-based models discussed in the Appendix.

The original Taylor Rule and Laubach–Williams use 1-year trailing inflation as a proxy for expected inflation. Although an approach such as this would have the benefit of being simple, timely, and frequent, it would have the drawback of being noisy. The Taylor Rule and Laubach–Williams also use measures of potential GDP in their formulation of equilibrium interest rates. However, creating estimates of potential GDP is difficult, especially in real time, and the results one gets will depend upon the construction of the model. In considering backward looking approaches, therefore, the Campbell and Shiller's (1998) use of a 10-year moving average has appeal. It is also timely and easy to calculate, and recent trends in growth and inflation would be important components in the formulation of expectations. Plus, the length of time should largely eliminate the impact of the economic cycle. It is also not uncommon in economics to use such averages when studying the relationship between broad economic conditions and interest rates. For example, Rachel and Smith (2015) note, "One reason market measures of the global real rate may deviate from the long-run equilibrium (neutral) rate is due to cyclical factors. To sidestep this issue, we focus on very low frequency movements in the data." They use both 5- and 10-year averages in their study to avoid these problems.

As we would like a heuristic which gives a broad sense for whether current bond yields are indeed consistent with longer-term economic trends, let us use the trailing 10-year compound annual growth rate (CAGR) for both real GDP and CPI. Even though economists sometimes use "trend growth" synonymously with "potential growth," for the purposes of simplicity please allow us to refer to the 10-year trailing CAGRs of real GDP and CPI as "trend growth" and "trend inflation" in this paper. Results of using these two variables in a regression to model bond yields are displayed in Figure 3 and Table 1.

Using nothing but the compound annual growth rates of GDP and CPI, this basic econometric model explains 89%[6] of the level of 10-year bond yields since 1960 (red line in Figure 3). Because of serial correlation, it is necessary to use Newey-West standard errors to ensure we are not over-estimating our t-statistics, but even using these, we can see that all three coefficients are significant at the 99.9% level of confidence (see Table 1). Also using Newey–West standard errors, we can create a 95% confidence interval for the 10-year bond yield's "intrinsic value" (red dotted lines in Figure 3). Although there is substantial volatility within this confidence band, we can see that yields were only briefly outside this band during the sample period.

Although the red line may do a good job explaining bond yields from a long-term perspective, it may strike readers as less useful for explaining bond yields in the short run. This is naturally true of many measures of value. For those interested in forecasting yields over short time horizons (e.g., 1 year or less), one could naturally construct a forecasting model with a shorter sample

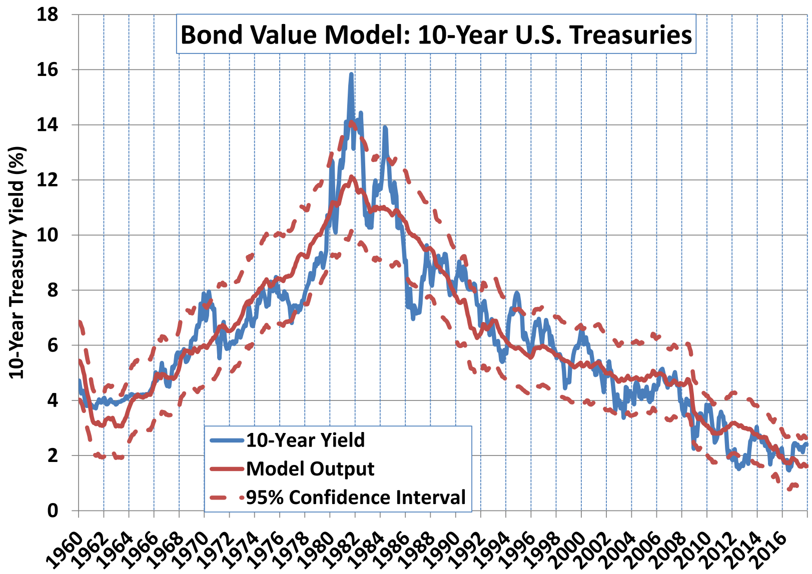

Figure 3 The figure shows the Bond Value Model's estimates of equilibrium bond yield as well as error bands, vs the actual 10-year bond yield.

Table 1 Results of bond value model.

	(Sample Period: 1960–2017)			
	Latest Value	Coefficient	T-Stat (Newey-West)	Significance Level
Trend Growth	1.43	1.1	14.7	99.9%
Trend Inflation	1.61	1.2	21.3	99.9%
Intercept	N/A	−1.8	−6.3	99.9%
R-Squared	0.89			

period, which would broaden the number of time series available for the model. Moreover, given the autocorrelation in bond yields, one might also prefer a model with an autoregressive component or to forecast changes in yields rather than levels of yields. However, for those interested in understanding broad interest rate regimes and having an intuitive sense for how bond yields today reflect broader economic trends, such models may not be completely satisfying. After all, as Achen (2001) points out "lagged dependent variables (i.e., yields of the prior month or year in this case) have no obvious causal interpretation"[7] leading researchers to omit them. He also points out that although including autoregressive components in the model can lead to a very high *R*-squared, it can also suppress the explanatory power of the independent variables (in this case growth and inflation) and can cause these independent variables to have "implausible" coefficients. Finally, if one's objective is a heuristic that tells one whether current bond yields are too high or low to be consistent with economic trends, then including an autoregressive component would undermine that objective.

Although deliberately simplistic, this model does do surprisingly well forecasting out-of-sample.

For example, using the Livingston Survey, one could combine 2-year forecasts of growth and inflation with 8-year trailing CAGRs in order to project where the CAGRs would be in 2 years' time. If one used only data available at the time to estimate the coefficients, one could use this as a forecast for bond yields. The results are the azure dots in Figure 4. Over the 24-year sample, the azure dots only over-estimate bond yields by 41 basis points on average. In contrast, the purple dots, which represent the 2-year forecasts for yields taken directly from the Livingston Survey, overestimate bond yields by 119 basis points on average. Using the methodology of Campbell and Thompson (2005) to calculate "out-of-sample" R-squareds, we get a result of 0.63 by combining the 2-year CPI and GDP forecasts with the 8-year CAGRs and using the bond value model. This compares favorably with the 0.22 we get when using the 2-year bond yield projections taken directly from the survey.

Looking at results "out-of-sample," or projected forward using information only available at the time of projection, is extremely important. This is because many models appear to be a good fit when looking at data in the sample but prove themselves utterly useless at predicting the future. Comparing the out-of-sample R-squareds, it seems clear that investors would have been better off using this bond value model approach, combined with inflation and growth forecasts available at the time, rather than using the median bond yield forecasts available. This result is surprising, as the bond value model only uses trend growth and inflation as inputs, whereas forecasters would be privy not only to this information, but also a great deal more information for use in formulating their forecasts. For investors, of course, the accuracy of projections has implications for investment results, and so using the best possible projections for making investment decisions is a matter of importance. Of course, in evaluating forecasters or forecasting models, it would be helpful to have a benchmark.

Investment decision-makers may find the bond value model to be a useful benchmark for evaluating forecasters or forecasting models. If a given forecast or model cannot do at least as well as this heuristic in out-of-sample tests, then the investor

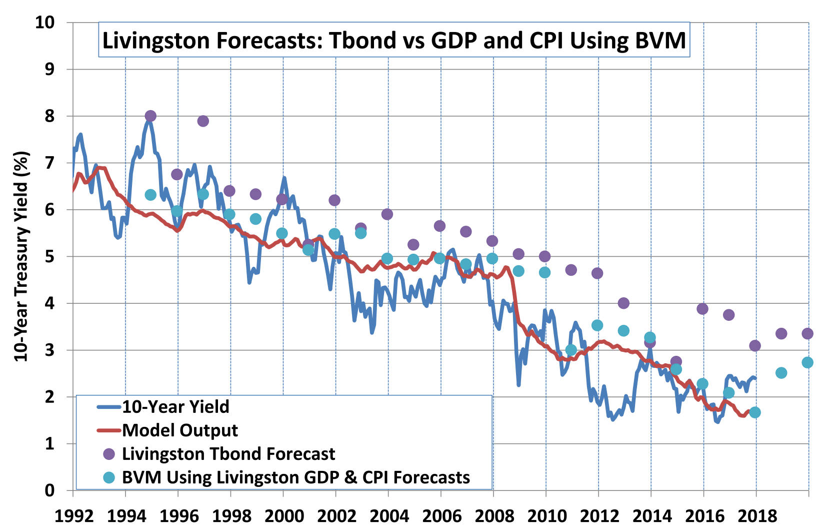

Figure 4 The figure shows that using the Bond Value Model with Livingston 2-year projections for growth and inflation (azure dots) would have given a much better forecast of 10-year yields than Livingston's actual 2-year forecasts of 10-year yield (purple dots).

may prefer not to use that forecast or model. Moreover, the bond value model can also be a useful input when formulating a forecast or as a sanity check on a forecast. By starting with a broad sense of what bond yields would be consistent with prevailing trends in growth and inflation, one can then add cyclical factors, supply and demand factors, and/or prevailing yields in order to arrive at a more comprehensive forecast. Furthermore, investors can use Newey–West consistent confidence intervals to give a sense for how far yields are likely to deviate from the model.

4 Comparisons with other measures of value

A key motivation of the bond value model outlined in this paper is to create a simple metric that gives a broad sense of value, similar to those used to assess equities. In many cases, measures of equity value are ratios of the equity price to some variables such as earnings, dividends, or cash flows. The sense of "cheap" or "rich" or "fair value" often comes from how far this ratio has deviated from its average. The bond value model outlined here generates a measure of the equilibrium yield. The sense of over- or under-valued in this case would come from how far the current bond yield is from the equilibrium yield.

As noted earlier, Campbell and Shiller (1998) study the ability of the price-to-dividend ratio and the CAPE to explain future changes in real equity prices. To demonstrate the value of these measures, they showed scatter plots of both CAPE and dividend ratios against future changes in the real price of the S&P 500 over 1- and 10-year horizons. They showed trend lines in these scatter plots and used the *R*-squared of these trends to show the percentage of variation of future equity prices explained by the ratio. They found that both measures did better at predicting the change in real equity prices over 10-year horizons than over 1-year horizons, but that CAPE's predictive power was superior to that of the dividend ratio.

In order to determine the value of this bond model as a long-term value measure analogous to those available for equities, let us use the same approach as Campbell and Shiller (1998). Fortunately, updated versions of these measures are available on Shiller's website so we can use data that match their methods exactly over the same sample period. In comparing the bond value model, we can estimate the model on a monthly basis beginning in 1980 in order to have at least 20 years of data to estimate the coefficients, and then expand the sample with each new period. Since economic data are released with a lag, let us also calculate the model with a 3-month lag. We can then compare the prevailing yield at the end of each month with bond value model's estimated yield with a 3-month lag. We can then see if the difference is negatively related to future changes in bond yields, the same way we can see a negative relationship between CAPE and future changes in real equity prices. The results over 1-, 2-, 5-, and 10-year horizons are shown in Figure 5.

Table 2 shows the results for the dividend–price ratios and CAPE as well as the bond value model over all four horizons. As we can see, the bond value model appears to do a better job of explaining future moves in bond yields over the 1-, 2-, and 5-year horizons than our two measures of equity value do explaining future changes in real equity prices. However, the dividend–price ratio does as well or better than the bond value model over the 10-year horizon and the CAPE does considerably better over the 10-year horizon. This gives us a sense that these equity value measures are somewhat longer term in nature than the bond value model, but that all have the potential of providing a useful measure of value.

As previously discussed, Asness *et al.* (2017) examine the out-of-sample performance of the CAPE. They find that using it for timing purposes fails to do as well as a simple buy-and-hold

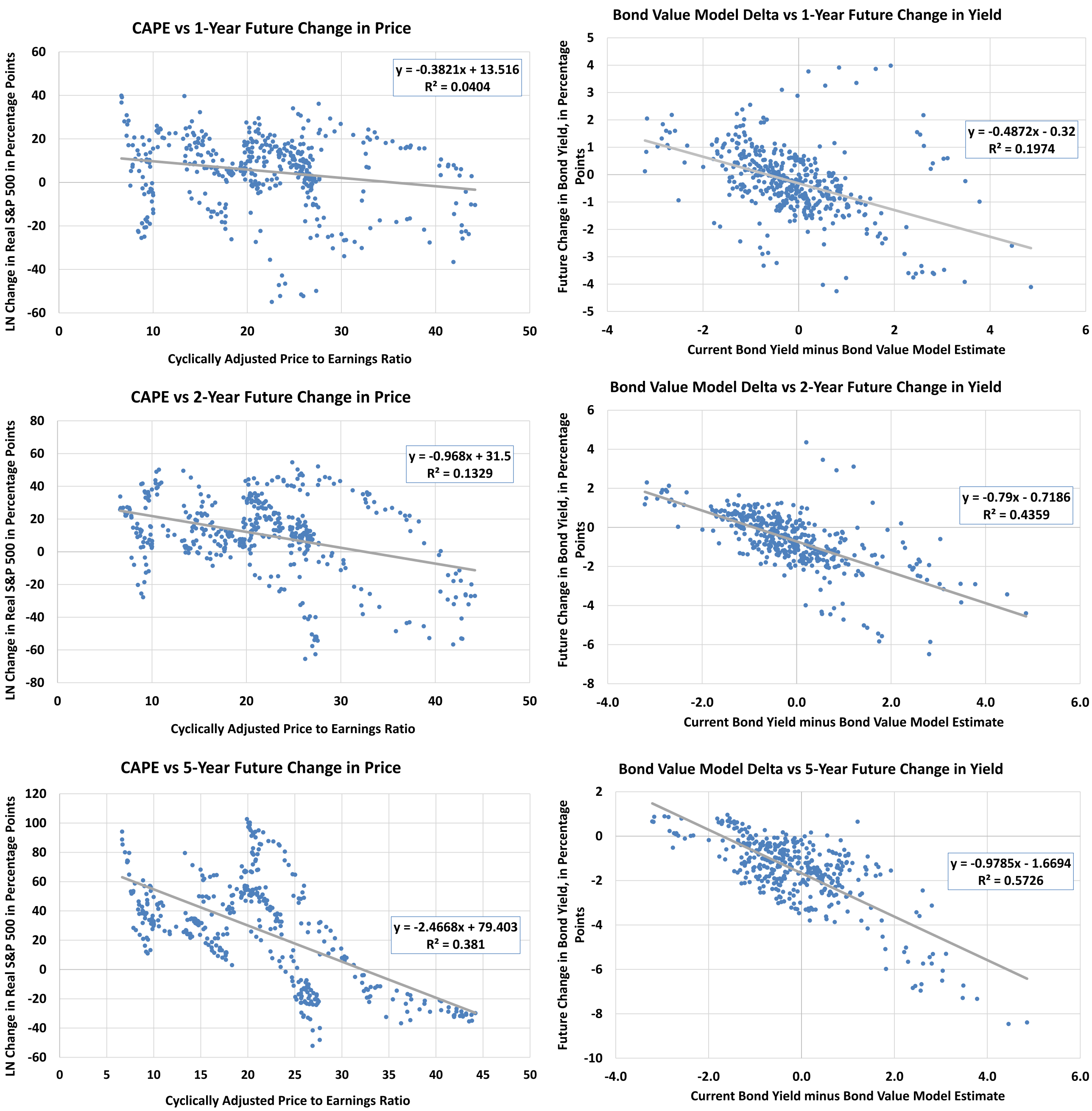

Figure 5 The figure replicates the methodology of Campbell and Shiller (1998) used to demonstrate CAPE's relationship with future changes in equity prices and then applies it also to the Bond Value Model's relationship with future changes in yields. A high R-squared is used to demonstrate predictive power. The figure shows that R-squareds over 1-, 2-, and 5-year horizons are higher for the Bond Value Model, but higher for CAPE over the 10-year horizon.

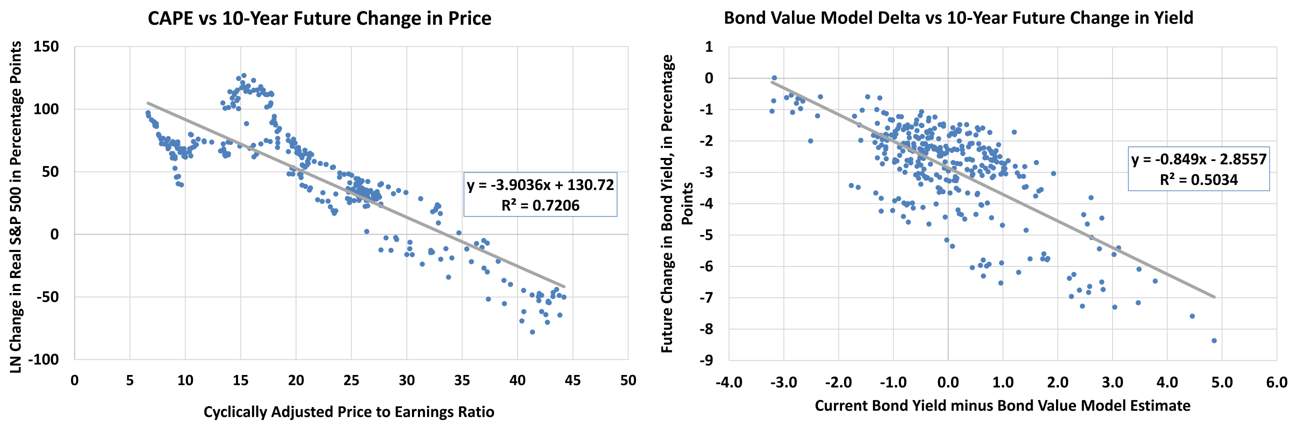

Figure 5 (*Continued*)

strategy. However, when combined with a momentum indicator, they can create a trading strategy that improves upon a buy-and-hold strategy on an out-of-sample basis. In the Appendix, they also suggest a bond value model based on real yields, calculated by subtracting expected 10-year inflation from the nominal yield. Let us therefore examine their bond value model in comparison with our own by approximating their methods using publicly available data. They compare their trading strategy with a buy-and-hold approach by varying Treasury exposure from 50 to 150% of NAV with either investing or borrowing at the cash rate to bring total NAV to 100%. Using a range from the 5th and 95th percentiles of real yields in an expanding window sample, they compare the current real yield with that range of prior yields to calculate current percent exposure on the 50 to 150% range.[8] Exposure to Treasuries caps at 150% when at the 95th percentile of real yields and is 50% when at the 5th percentile or lower.

For a series of inflation expectations, the Survey of Professional Forecasters provides 10-year inflation forecasts twice a year beginning in 1990 and also provides forecasts from BlueChip twice a year for 1979–1989, giving us an extended dataset for expected inflation. Given that we want to calculate weights based on percentiles, a higher frequency of data is necessary. We could make the semi-annual data forecast monthly either by interpolating the data or by simply using the prior forecast until the next one is published. The results for the trading strategy turn out to be the same, so let us use the method of changing our inflation expectation only when a new survey is published, since it would not be possible to interpolate between the previous and next forecast on a real-time basis. We will use an expanding window sample. Table 3 shows that the Sharpe Ratio of this trading strategy is superior to the buy-and-hold strategy over the sample period, which is consistent with Asness, Ilmanen, and Maloney's results.

To calculate the bond value model in the same way, we simply take the difference from the model and the prevailing yield in each month, as we did in the comparison for the CAPE. As we can see in Table 3, the Sharpe Ratio compares favorably with both the buy-and-hold strategy and the real yield approach suggested by Asness, Ilmanen, and Maloney. Unlike the trading strategy using CAPE, both these trading strategies outperform the buy and hold. It is interesting that both these trading strategies find more signals to underweight than over-weight 10-year Treasuries vs. Tbills, with the real yield approach having an average of 66% exposure to Treasuries (and therefore 34% average exposure to Tbills) vs. 86%

Table 2 Comparison of bond value model to equity value models.

	1-Year future change			2-Year future change			5-Year future change			10-Year future change		
	Slope coefficient			Slope coefficient			Slope coefficient			Slope coefficient		
	Value	NW-*t*-stat	R-Squared	Value	NW-*t*-stat	R-Squared	Value	NW-*t*-stat	R-Squared	Value	NW-*t*-stat	R-Squared
Campbell and Shiller Equity Value Measures												
Comparison of Dividend-Price Ratios to LN% Future Change of Real S&P 500 Price	2.716	1.70	4%	5.393	2.6	8%	15.601	6.61	28%	24.863	7.0	52%
Comparison of CAPE to LN% Future Change of Real S&P 500 Price	−0.382	−1.79	4%	−0.968	−3.2	13%	−2.467	−10.87	38%	−3.904	−12.0	72%
Bond Value Model												
Comparison of BVM Delta to Future Changes in 10 Year Bond Yield	−0.487	−3.54	20%	−0.790	−7.4	44%	−0.977	−7.45	57%	−0.849	−8.3	50%
Sample Period Begins		Apr-1980			Apr-1980			Apr-1980			Apr-1980	
Sample Period Ends		Dec-2016			Dec-2015			Dec-2012			Dec-2007	

Table 3 Out-of-sample trading strategies.

	Buy and hold	Real yield	Bond value model
Compound annual growth rate	8.26	7.66	8.17
Volatility	7.94	6.18	6.63
Sharpe ratio	0.54	0.60	0.64
Average allocation to 10-year treasuries	100%	66%	86%

*Sample Period, Jan 1982–Dec 2017.

average exposure for the bond value model. Given the trend toward lower yields over the sample period, a higher exposure may have been desirable. However, the trading strategy based on the bond value model manages to have nearly identical returns despite this handicap and has lower volatility, so it is superior from the perspective of the Sharpe Ratio over the 36-year sample period.

5 Medium to long-run implications for yields

Whether or not the low yields in recent years are temporary or more permanent has become a topic of significant debate. Clearly the view that yields were likely to rise again in the near future has been common among forecasters. The findings of the bond value model support the alternative view that yields are actually consistent with broader economic trends and are not likely to rise to the levels seen in the past, unless of course trend inflation and growth rise to the levels we have seen in the past as well. Although some investors may be skeptical that this simple model has uncovered the true nature of interest rates, it is consistent with a growing body of thought on interest rates in the economic literature.

Holston *et al.* (2017) use the Laubach–Williams methodology and find that the natural rate of interest in the United States fell to zero during the Global Financial Crisis and remained there into 2016. Although the rate they are modeling is the real Fed Funds rate consistent with a constant rate of inflation, the decline of this rate by 2 percentage points has clear implications for the 10-year bond yield. They attribute the interest rate decline to "shifts in demographics, a slowdown in trend productivity growth, and global factors affecting real interest rates." Likewise, Rachel and Smith (2015) find a secular decline in long-term real interest rates which they then try to attribute to different factors. Given their views of the sources of the change, they predict that real yields are likely to only recover a small portion of this decline and that they will settle at or slightly below 1% in the medium to long run. This coupled with a credible inflation target of 2% would lead to equilibrium bond yields around 3%.

As mentioned earlier, Bauer and Rudebusch (2016) review models for measuring the neutral short-term real rate. They also review three models for term premium. Combining the models for short rates and term premia, they find the evidence to suggest a "low new normal" for interest rates and note, "while the ongoing economic recovery and normalization of monetary policy in the United States should lead to some increases in long-term interest rates, these seem likely to be fairly moderate for the foreseeable future."

For investors seeking a way to incorporate the view that rates are likely to remain lower than in previous decades, the bond value model may prove a useful tool. In order to try to compare these other models directly with it, let us start by taking the three models of short-term interest rates that Bauer and Rudebusch (2016) show in their chart and combine them with the measures of term premium. The short-term real rate estimates are from ∼2½ to 3% in 2000 and 0–1% in 2016. The estimates of term premium are all about 1¼% in 2000 and −½% in 2016. Coupling these with inflation expectations of about 2%, we have equilibrium nominal yields of about 5¾ to 6¼% at the beginning of 2000 vs. 1½ to 2% at the beginning of 2016. The actual yield at the beginning of 2000 was 6.7%, so a bit higher than the range of these estimates, and 1.9% in 2016, which was in the middle of the range of these estimates. The bond value model estimate of equilibrium yield is 5.3% at the beginning of 2000 vs. 1.9% at the beginning of 2016. It is therefore a bit lower than the other combined models' estimates in 2000, and is about the same in 2016. So the bond value model appears consistent with these other more sophisticated models as well as the yields themselves.

For many investors, there may be significant appeal of having a simple model such as this bond value model for approximating equilibrium nominal yields in this way. This could be done either in lieu of, or in addition to, deriving the more complex models of real short-term rates and term premia, and then combining those with an expected inflation rate, in order to estimate an equilibrium yield or broad measure of value.

6 Intuition behind coefficients

That the slope coefficients for both growth and inflation are close to one is encouraging, given the strong theoretical support for the slopes being equal or close to unity. The finding that the inflation coefficient is greater than one is consistent with the Feldstein and Summers (1978) argument that tax distortions should lead the coefficient to be higher than one. It is also consistent with the findings of Campbell *et al.* (2013), whose model would be consistent with a positive nominal bond yield premium when inflation is high and a negative premium when inflation is low or when there is a chance of deflation. The estimated slope coefficient for trend CPI is 1.23, which is statistically greater than 1 at the 99.9% level of confidence using Newey–West standard errors. This gives us strong empirical evidence that the inflation coefficient is indeed greater than 1.

The intuition for the growth coefficient being greater than 1 would be similar to that of the CPI coefficient (i.e., a "growth premium"). When growth expectations are high, investors may demand more to substitute away from other investments, whereas when growth expectations are low they may prefer the default protection of Treasuries and will flee to the safety of these assets. However, in the case of growth, the coefficient estimate of 1.06 is not statistically different from 1 if we use Newey–West standard errors to calculate our *t*-statistics. This suggests that the true coefficient for growth may indeed be 1, which would be consistent with the Golden Rule.

The estimate for our constant coefficient, or intercept, is −1.8. It is statistically lower than zero with 99.9% confidence. This implies that if we went through a 10-year period of zero growth and zero inflation, yields would be negative. If the US economy were to go through such a depressed state over such a long period of time, then it is likely that there would be abundant capital with little demand for investment. Investors would in all likelihood be seeking ways to protect their principal from outright loss, rather than seeking a positive return. Although the US has never experienced such low growth and inflation trends under a fiat currency system, the experience of negative yields in Europe even with positive growth and inflation trends suggests that the intercept coefficient is not unreasonable.[9] It is worth noting, however, that unlike the other two coefficients, the 95% confidence interval for the intercept coefficient is rather wide, ranging from −1.2 to −2.4. Though the estimate is less precise than for the other two coefficients, the negative incept appears intuitive and is statistically significant at a very high level of confidence.

7 Statistical issues

The bond value model regresses 10-year government bond yields on the 10-year compound annualized growth rates of GDP in chain weighted 2009 US dollars and the Consumer Price Index for All Urban Consumers (CPI-U). Data are monthly, with monthly GDP interpolated from quarterly data. The use of overlapping time periods of such long length is appropriate for trying to quantify a relationship with long and variable lags. We use Ordinary Least Squares (OLS) regression. The model gives rise to two statistical issues: serial correlation in the residuals and stationarity. Of the two statistical issues, stationarity is the greater concern.

The presence of serial correlation in the error terms means that the estimates of the coefficients may not be efficient (i.e., it may be possible to estimate a coefficient with a lower standard error), but the coefficients are still unbiased and many econometricians view lack of efficiency as a minor concern. Serial correlation also means that the estimated standard error of the coefficients may be too low, which means that the t-statistics may give a false positive. However, this is easy to correct by using Newey–West standard errors, as we have done in this paper. This is a common practice among researchers.

If time series are not stationary, models that appear to be powerful and fit well can in fact be entirely spurious. It is therefore incumbent upon the econometrician to prove that the time series involved in the regression are either stationary or cointegrated. For purposes of completeness, this paper includes five different tests for cointegration, with two ways to optimize the test lags (both Akaike and Schwarz information criteria), yielding a total of 16 test statistics for cointegration. All 16 test statistics find in favor of cointegration at the 97.5% level of confidence or greater and 8 of the 16 at the 99.9% level of confidence or greater (see Table 4). The tests therefore give very strong evidence of cointegration, affirming that the OLS model should not be spurious for lack of stationarity.

To consider the robustness of using OLS relative to other regression models, this paper includes seven more advanced forms of regression models on the same data set (see Table 5). These other models correct for serial correlation, cointegration, or both. All of these models have similar coefficient estimates, especially for trend inflation, and all models find all coefficients statistically significant at well above the 95% level of confidence.

Given the diversity of tests used for cointegration and the diversity of more sophisticated models that have confirmed the results, we have very strong evidence that our model is statistically robust. Moreover, given the homogeneity of the model outputs, and even the coefficients, there appears to be no need to explore the benefits of models more sophisticated than OLS for quantifying our relationship. Given the desire to provide a heuristic accessible to a broad range of investors, the ability to use OLS is welcome.

8 Stability of the model

As we have mentioned, in many cases models with coefficients that seemed to fit the historical data well are useless for predicting the future because the coefficients change. Of course, even if coefficients do change somewhat, the framework of the bond value model may still be of use. After all, there is healthy debate about the

Table 4 Various tests for cointegration.

Test	Test statistic	*p*-value
Augmented Dickey Fuller, AIC Optimized		
Augmented Dicky Fuller	−5.59	0.0000
Augmented Dickey Fuller, SIC Optimized		
Augmented Dicky Fuller	−5.00	0.0000
Engler-Granger AIC Optimized		
tau-statistic	−5.59	0.0001
z-statistic	−120.73	0.0000
Engler-Granger SIC Optimized		
tau-statistic	−5.00	0.0009
z-statistic	−49.57	0.0004
Phillips-Ouliaris AIC Optimized		
tau-statistic	−4.76	0.0021
z-statistic	−43.87	0.0015
Phillips-Ouliaris SIC Optimized		
tau-statistic	−4.77	0.0021
z-statistic	−44.00	0.0014
Johansen, 12 Lags, AIC Optimized		
Trace	41.72	0.0014
Maximum Eigenvalue	33.66	0.0005
Johansen, 2 Lags, SIC Optimized		
Trace	32.83	0.0217
Maximum Eigenvalue	25.87	0.0100
ARDL Bounds Test, AIC Optimized		
F-Statistic	7.89	0.0010
ARDL Bounds Test, SIC Optimized		
F-Statistic	5.20	0.0085

Table 5 Bond value model outputs using other forms of regression.

Coefficients	Ordinary least squares	General least squares	Fully modified LS	Canonical cointegr regress	Dynamic ordinary LS	Robust least squares	ARDL AIC optimized	ARDL SIC optimized	Average other Models
Trend inflation	1.23	1.21	1.24	1.24	1.24	1.20	1.22	1.23	1.23
Trend growth	1.06	0.87	1.09	1.09	1.08	1.06	1.03	0.99	1.03
Constant	−1.80	−1.15	−1.89	−1.90	−1.86	−1.71	−1.66	−1.55	−1.69

parameters of many models. Nevertheless, as noted earlier, one could have used the model to forecast 2 years forward using Livingston growth and inflation forecasts and had considerably better results than using the actual forecasts of bond yields in the same survey. This is because the coefficients are very stable over 2-year horizons.

The green line in Figure 6 shows what projections would look like if a forecaster had perfect foresight over growth and inflation over the next 2 years. The coefficients are estimated only using data available at the time in order to project yields forward 2 years. What it shows is that at any point in time, the out-of-sample coefficients produce nearly identical results as coefficients calculated from the entire sample. This demonstrates that for making assessments about interest rates on a 2-year horizon or less, instability in the coefficients is *de minimis*.

We can also use 10-year lagged coefficients to see how useful the model would be for making 10-year forecasts. In Figure 7, we can see that for nearly two decades the model does surprisingly well. Even starting with only 20 years of data (1960–1979), we can see that coefficients calculated in January 1980 would have given almost the same estimate of yields in January 1990 as coefficients calculated using the entire sample (1960–2017). However, it does not do as well for projecting over horizons that include the Global Financial Crisis. Nevertheless, the model does not "break down" due to the financial crisis. Indeed, the model appears to actually fit the data better with the inclusion of the crisis and recovery in the sample.

As readers can see in Figure 7, beginning in 2009, the 10-year old coefficients lead to yield projections that are too high. However, the gold line, which uses 5-year lagged coefficients and also shows a divergence at the time of the crisis, fully converges on the red line by 2017. This suggests that as the data over the crisis become incorporated into the model, it may again forecast well out-of-sample for 10-year horizons. Further reason to expect a potential improvement in the

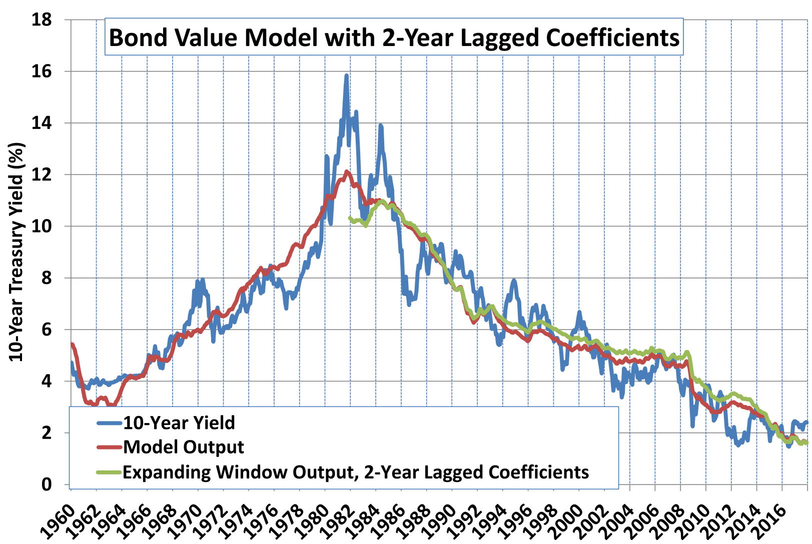

Figure 6 The figure shows full sample Bond Value Model estimates vs the same estimates using coefficients estimated with only prior data and a 2-year lag. The nearly identical results show coefficients are stable enough for 2-year forecasts.

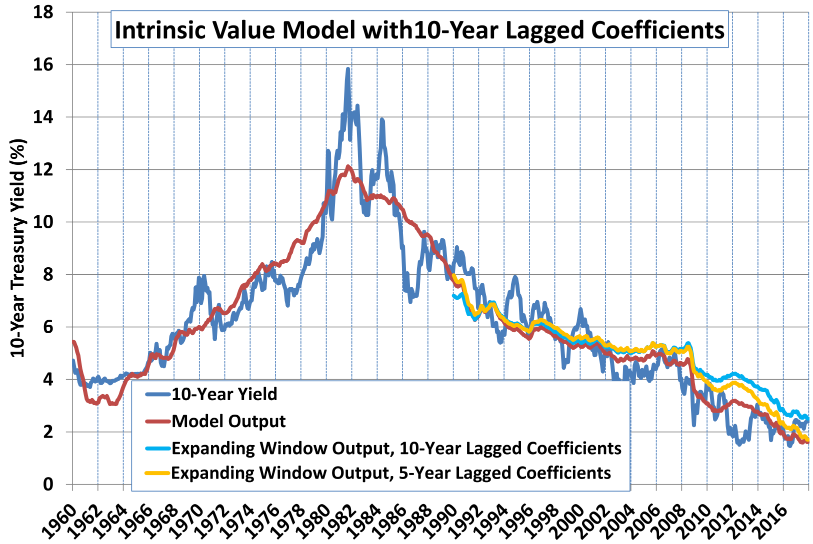

Figure 7 The figure shows full sample Bond Value Model estimates vs the same estimates using coefficients estimated with only prior data and with 5- and 10-year lags.

out-of-sample robustness of the model is that the model's statistics of fit improve with inclusion of the crisis and post-crisis years in the sample.

Table 6 compares the model using only 1960–2007 as the sample period with the full sample period of 1960–2017. One can see that the full sample has a better R-squared (0.89 vs. 0.86) and that its coefficients have tighter confidence intervals. Although the confidence interval and coefficient for trend inflation are roughly the same, the confidence interval for the growth coefficient is less than half as wide and the t-statistic rises from 4.9 to 14.7. The improvement in the

Table 6 Comparing model outputs across sample periods.

					95% Confidence interval*		
	Coefficient	St Error*	t-Statistic	Confidence	Low	High	Range
Dataset 1960–2017							
Trend inflation	1.23	0.06	21.3	99.9%	1.12	1.35	0.23
Trend growth	1.06	0.07	14.7	99.9%	0.92	1.20	0.28
Constant	−1.80	0.29	−6.3	99.9%	−2.36	−1.24	1.12
R-Squared	0.89						
Dataset 1960–2007							
Trend inflation	1.18	0.06	19.0	99.9%	1.06	1.30	0.24
Trend growth	0.74	0.15	4.9	99.9%	0.45	1.04	0.59
Constant	−0.46	0.64	−0.7	52.5%	−1.71	0.80	2.50
R-squared	0.86						

*Using Newey-West Heteroskedasticity Autocorrelation Consistent Standard Errors

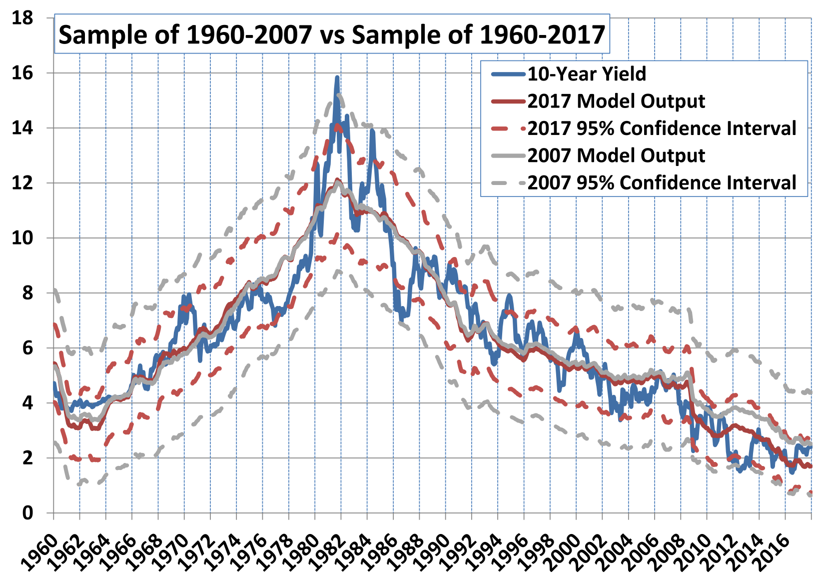

Figure 8 The figure shows the Bond Value Model calculated over 1960–2007 and 1960–2017. The Newey-West error bands are much smaller over the 1960–2017 period, indicating that including the 2008–2017 period substantially improves the precision of the model.

stability of the coefficient estimate may come from including a greater dispersion in growth outcomes in the sample. After all, prior to 2008 trend growth was always above 2% and was above 3% for 87% of the time. From 2008 onward, however, trend growth has been below 2% for 88% of the time. In Figure 8 we can see 95% confidence intervals of the equilibrium yield for both sample periods. These error bands are only half as wide for the full sample period as for the sample ending in 2007, due to the greater precision in the coefficient estimates.

Given that the 10-year out-of-sample forecasts that were too high were calculated when the Newey–West error band was twice as wide, it seems likely that future errors from using 10-year old coefficients will be smaller. With a few more years of data, it should become apparent whether the 10-year out-of-sample forecasts are again identical to the full sample forecasts. This should give more clarity about the long-term stability of the model.

9 10-Year Forecast for 10-Year Yields

We can generate a 10-year forecast for yields by combining 10-year forecasts of inflation and growth with our model. Using the Livingston Survey from December 2017 gives us a yield forecast of 3.36%, as shown in Table 7. However, some might think these forecasts a bit high. After all, for the 10 years ending in December 2017, the CAGR for GDP was just 1.43% and for CPI just 1.61%. Also, the projections for growth and inflation have exhibited an upward bias in the past, with the projections overestimating GDP by 46 basis points on average and overestimating CPI by 56 basis points on average. If we assume just half this bias exists in today's estimates, then this would give us a projected yield of 2.80%. This is considerably lower than general consensus among forecasters, which appears to be that yields will move above 4% in the next 10 years. Readers may recall, however, that the bond value model's projections would be consistent with Rachel and Smith's views on real yields rising to about 1%, as well as with the

Table 7 Using Intrinsic value model for 10-year forecast of 10-year yield.

	Coefficient	Dec-2017 10-Year Livingston forecasts Actual	Adjusted
GDP	1.06	2.18	1.95
CPI	1.23	2.34	2.06
Constant	−1.80		
Yield projections		3.39	2.80
95% Confidence Interval Projections:			
Lower bound		2.26	1.73
Upper bound		4.53	3.87

more advanced models of the natural rate, which suggest that today's low yields are mostly due to structural rather than temporary factors that are likely to reverse quickly.

10 Conclusions

The bond value model laid out in this paper gives investors a simple construct that may allow them to understand why yields are so low today, give them an intuitive way to relate yields today to other regimes of higher growth and inflation, and also help them make reasonable judgments about future yields as well as yield forecasts. The model is statistically robust and its findings of equilibrium bond yields are consistent with some of the cutting edge work which has shown a decline in the natural rate of interest and term premium in recent years. It is also consistent with longstanding economic theory on the formulation of interest rates, in that the coefficients for trend growth and inflation are close to, though not precisely, one. The model compares well with the CAPE as a long-term value measure, using the same test presented by the authors of that model at its introduction. Plus, using the same out-of-sample test Asness, Ilmanen, and Maloney apply in their paper, the bond value model compares favorably with both a buy-and-hold strategy and the alternative bond value approach they suggest. Finally, given the simple methodology, the approach can be accessible to many practitioners and requires only occasional updates to estimate the coefficients. Although no model or approach can be all things to all people, for many investors focused on long-term value, this model may be a very useful tool.

Appendix: Forecast-based models

As mentioned in the main body of this paper, one could also try to model bond yields more directly on expected future growth and inflation by using forecasting surveys. Such an approach is perhaps more consistent with the efficient markets hypothesis. Regrettably, the two models attempted for this paper have results that appear problematic. To begin, let us consider a model created using 10-year forecasts for growth and inflation. These should be the most theoretically consistent forecasts since they are for the same time horizon as the bond.

Livingston gives 10-year forecasts of GDP and CPI twice a year beginning in 1990, for a sample size of just 55. Although we have a rather high R-squared of 0.73, the coefficient for CPI is 3.00, which would appear inconsistent with economic theory (see Table A1). Moreover, the confidence intervals for the coefficient estimates are extremely wide and the intercept coefficient looks implausibly low, at −7.81.

It is also possible to gather Livingston projections of growth and inflation for 1 year forward (displayed in Table A1 as 1×2). At 1.40, the coefficient for CPI appears more reasonable than the 10-year forecast-based model. However, the coefficient for GDP is not significant and has the wrong sign. Plus, the intercept suggests that if expectations for growth and inflation were both

Table A1 Results of forecast-based models.

Variable	Coefficient	N-W *t*-stat	95% Confidence interval Low	High	Range (%)	Model Stats	
10-Year Trailing CAGRs							
GDP	1.06	14.73	0.92	1.20	27%	R-Squared	0.89
CPI	1.23	21.34	1.12	1.35	18%	Sample	696
Constant	−1.80	−6.31	−2.36	−1.24	62%	Begins	1960
10-Year Forecasts							
GDP	1.52	3.08	0.53	2.50	130%	R-Squared	0.73
CPI	3.00	9.06	2.34	3.67	44%	Sample	55
Constant	−7.81	−4.46	−11.32	−4.30	90%	Begins	1990
1 × 2-Year Forecasts							
GDP	−0.30	−0.60	−1.31	0.71	668%	R-Squared	0.74
CPI	1.40	6.23	0.94	1.85	65%	Sample	44
Constant	1.87	1.21	−1.26	4.99	335%	Begins	1974

zero, then the equilibrium bond yield would be 1.87, which looks implausibly high. As with the other forecast-based model, the confidence intervals around the coefficient estimates are very wide, denoting a very low level of precision.

The issues with these two models could stem from inadequate samples, from the surveys not being reflective of the expectations that drive the bond markets, or from some other issue or combination of issues. Whatever the cause, the model outputs do not inspire confidence and appear as an inadequate basis for making investment decisions.

Notes

[1] Using the methodology outlined by Campbell and Thompson (2005), we could have an out of sample R-squared of 0.22 if the forecasts were the output of a regression model that attempt to forecast bond yields over a 2-year horizon.

[2] Council of Economic Advisors (2015), p. 11.

[3] Rausch (2016), p. 4.

[4] Council of Economic Advisors (2015), p. 13.

[5] Rachel and Smith (2015), Executive Summary, p. 1.

[6] The interpretation of the R-squared, or coefficient of determination, is that the model explains 89% of the level of bond yields over the sample period.

[7] Parenthetical statement added.

[8] They use the expanding window sample until they have 60 years of data and use a 60-year rolling window thereafter.

[9] For example, in June of 2016, Switzerland had trend growth of 1.8% and inflation of 0.2%. The coefficients from the Bond Value Model would have implied a bond yield of 0.2% had we had those same outcomes in the United States. Nevertheless, Swiss yields fell as low as −0.7% that month, 90 basis points lower than these coefficients would have predicted. This would be consistent with an intercept coefficient at or below −1.8.

Acknowledgments

Joseph Gagnon, Ric Thomas, Ingo Walter, Nicholas Fok, Andreas Dische, and Robert Blauvelt.

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Keywords: Treasury yield; yield forecast; bond value; equilibrium yield; natural rate of interest; golden rule