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ILLIQUIDITY AND FACTOR RETURNS: EXPLORING THE INTERSECTION BETWEEN ILLIQUIDITY, SMALL CAP AND POPULAR FACTORS

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Factor returns are often reported as the average of factor returns among large stocks and the factor returns among small stocks. However, factor returns among small, illiquid stocks are significantly higher than those among larger, more liquid stocks, suggesting that the factor returns in the literature are exaggerated and cannot be implemented with substantial assets. Moreover, investors who are able to take greater liquidity risk can capture higher factor returns by investing in factors among small stocks.

1 Introduction

The way risk factors are traditionally calculated exaggerates expected return for the average investor. We find that the value, gross profitability, investment, net share issuance, short-term reversal, long-term reversal, momentum, and 60-day volatility factors earn significantly higher premiums when executed with illiquid, small-capitalization stocks than with liquid, large-capitalization stocks. The same does not hold for factors built from 5-year volatility and 5-year beta. The notion that some factors are influenced by liquidity risk is not new. Sadka (2006) argues that momentum returns and post-earnings-announcement drift are compensation for liquidity risk. Hong *et al.* (2000) show that momentum returns are higher among small stocks. Asness *et al.* (2000) find that, in the U.S. stock market, short-term reversal is considerably stronger in small-cap stocks than in large-cap stocks. Fama and French (1993, 2015) note the superior performance of value within small stocks.

Illiquidity, which can be proxied by market price impact measures and the low capitalization characteristic, is an important risk and return driver for equities. Amihud and Mendelson (1986), Brennan and Subrahmanyan (1996), Pastor and Stambaugh (2003), and Amihud *et al.* (2015) document an illiquidity premium for stocks using different liquidity measurements and data sets. The empirical observation that stocks exposed

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to illiquidity risk command a premium and thus deliver higher returns is intuitive.[1] The illiquidity premium has also been measured in other asset classes like private equities, options, and bonds.[2] The illiquidity premium certainly does not spur the kind of theoretical controversies surrounding the existence and rationale behind popular return premiums like value and low beta.

Indeed, what is puzzling is why illiquid stocks do not command a more robust risk premium. Brennan and Subrahmanyan (1996), Spiegel and Wang (2005), Ben-Rephael *et al.* (2013), and Lou and Shu (2014) find weak or mixed evidence for the relationship between returns and various measures of illiquidity in the equity cross-section. Vayanos (1998) explains that illiquidity premium, while positive, might be of second-order importance and hard to detect. These newer results suggest a far weaker return to illiquidity as an independent source of equity return premium. In this paper, we do not examine whether illiquidity commands a premium. Instead, we examine whether factor premiums are higher among illiquid stocks.

We extend the research on illiquidity premium by showing a nonlinear interaction between illiquidity and other popular return factors. We find that the return to illiquidity is meaningfully more significant and robust when captured as an interaction with other sources of equity return rather than as a standalone risk factor. Specifically, we demonstrate that the premium for a particular factor is substantially larger when that factor is constructed from illiquid stocks than liquid stocks. This excess premium cannot be explained by traditional linear factor models with illiquidity as an explanatory factor.

There are two other important implications from our empirical results. First, the premiums and the Sharpe ratios reported for factors constructed from the traditional Fama–French methodology exaggerate what can be captured by investors. The methodology overestimates factor returns, because the standard factors are constructed as equal weights of the factor returns among large-cap and small-cap stocks. Since the average investor in factor or smart beta strategies allocates substantially more assets to products in the liquid large-cap space, the factor premiums extracted are measurably lower. Second, performance attribution arising from traditional factor models, even when illiquidity is explicitly incorporated as a factor, will mischaracterize manager skill; traditional performance analytics will structurally favor illiquid factor strategies. Managers whose factor tilts are implemented through illiquid stocks will be assigned a larger alpha than managers whose factor tilts are implemented through liquid stocks.

Fama and French (1993) compute the value premium by averaging the value premiums of big stocks and small stocks, with the division between big and small made at the 50th percentile (median) of NYSE stock market capitalization. In computing the momentum factor, Carhart (1997) follows the same approach. The problem with this method, however, is that it gives equal weight to both small and large stocks, even though small stocks make up only about 10% of the market on a capitalization-weighted basis. Furthermore, because many institutional investors cannot invest in small stocks for regulatory and investment policy reasons, an argument can be made that small stocks should be assigned a weight lower than their market capitalization share.

2 Selection of factors

We look at 10 factors: value, gross profitability, investment, net share issuance, short-term reversal, long-term reversal, momentum, 60-day volatility, 5-year volatility, and 5-year beta. We choose these factors because they are popularly used in investment management. We exclude size since we are sorting on size as a measure of illiquidity. The construction of these factors is described in the Data and Methodology section. Following the lead of Fama and French (1993), we define the value factor as the return spread between value and growth stocks. A value stock has a high book-to-market ratio and a growth stock has a low book-to-market ratio. We use Novy-Marx's (2013) definition of gross profitability, the ratio of gross profits to assets,[3] in which we buy profitable firms and sell unprofitable firms. We use Fama and French's (2006) definition of investment, the year-over-year percent change in assets. The factor involves selling low investment firms and buying high investment ones. We use year-over-year percent change in adjusted shares outstanding as net share issuance as Pontiff and Woodgate (2008) do and buy low issuance firms and sell high issuance ones. For all financial variables, we use the most recent annual report in January and keep data constant for the remainder of the year. This means that for January 2000, we use fiscal year 1998 financial statements, because the financial statements for fiscal year 1999 will not be released until later in 2000. We keep the book-to-market value static throughout the year. That is, we do not update the market capitalization used in the ratio with each new month's market capitalization.

In line with Jegadeesh (1990), we construct short-term reversal using prior one-month returns, going long losers and going short winners. We define long-term reversal as De Bondt and Thaler (1985) but we exclude the most recent 12 months to account for momentum, also going long losers and going short winners. In keeping with Jegadeesh and Titman (1993) and Carhart (1997), we define the momentum factor as the return from buying winners and selling losers; winners are defined as stocks with the highest trailing 12-month price return, excluding the most recent month, and losers are defined as those with the lowest trailing 12-month price return, excluding the most recent month. We use price return and not total return because we want to avoid any interaction with premiums to high-dividend-yield stocks.

Frazzini and Pedersen (2014) investigate the betting against beta factor, which involves buying low beta stocks and selling high beta stocks. Ang *et al.* (2006, 2009) investigate the total and idiosyncratic volatility factors, which involve buying low volatility stocks and selling high ones. We look at the 5-year beta, 5-year volatility, and 60-day volatility, the last of which is used in French Data Library. The 5-year beta we calculate uses the market excess returns and risk-free rate from French Data Library.

3 Data and methodology

We use CRSP and Compustat data for U.S. returns and financials, respectively. All factor returns start in January 1967. Since financial data available in January of a given year is from the fiscal year 2 years prior, we begin our Compustat data in fiscal year 1965. The last month of returns for all strategies is December 2014.

We compute factor returns directly from CRSP and Compustat. We compute illiquidity as defined in Amihud (2002). Amihud (2002) illiquidity is the ratio between the absolute value of its daily return and its daily dollar trading volume. We use average illiquidity over the previous month to sort stocks.

In our analysis, in order to determine the effects of Amihud illiquidity on factor returns, we perform a double sort on Amihud illiquidity and the relevant factor variable: book-to-market sorted positively for value; gross profitability sorted positively for profitability; percentage increase in assets sorted negatively for investments; annual percent increase in adjusted shares outstanding sorted negatively for net share issuance; trailing 1-month return sorted negatively for short-term reversal; trailing 60-month return excluding the most recent 12 months sorted negatively for long-term reversal; trailing 12-month returns, excluding the most recent month, sorted positively for momentum; trailing 60-day volatility sorted negatively for 60-day volatility; trailing 5-year volatility sorted negatively for 5-year volatility; and trailing 5-year beta sorted negatively for 5-year beta. We begin by sorting each group of stocks by market capitalization and then divide each group into quintiles. Within each of the five size quintiles, we sort the stocks on the factor variable (e.g. book-to-market for value) and divide again into quintiles based on the factor variable. We choose to sort the factor variable within each size quintile to ensure that each portfolio has roughly the same number of stocks.

We define quintile cutoffs for illiquidity and the relevant factor characteristic on all stocks, and we sort by each factor characteristic within each illiquidity quintile. In addition, each month, we sort stocks on illiquidity, which in turn results in monthly rebalancing for all strategies and the ability for stocks to switch illiquidity quintiles from month to month. For beta, we lever all portfolios to have an ex post beta of 1 with respect to the French Data Library market excess return. For volatility quintiles, we lever all quintiles to have an ex post annualized volatility of 16%. We apply this adjustment to the beta- and volatility-sorted portfolios to ensure that the long and short legs of the factors are risk-matched. We then compare the factor premiums within illiquid stocks and within liquid stocks.

To test for robustness and also to make the results more directly relevant and implementable, instead of sorting on illiquidity, we also sort stocks on market capitalization. Rouwenhorst (1998) performs a similar sort for momentum and, as in our analysis, finds that momentum returns are higher among small-capitalization stocks.

For our initial size sort, we use market capitalization from the previous month. As with illiquidity, stocks can shift between size quintiles from month to month. For value, we compute book-to-market value on January 1 of each year.

For robustness, we also test our claim that factor returns are higher among small stocks using 5×5 size and factor portfolios from the French Data Library. These portfolios are formed by taking the intersection of five quintiles based on market cap and five quintiles based on the relevant sorting variable (e.g. book-to-market for value). The quintile cutoffs are formed using NYSE stocks but all NYSE, AMEX, and NASDAQ stocks are included in portfolios. We calculate factor returns within each size quintile by taking the highest expected return quintile and subtracting the lowest expected return quintile. For example, to compute value returns within each size quintile, we subtract the low book-to-market quintile from the high book-to-market quintile within that particular size quintile. We use all factors on the French Data Library that have 5×5 size and factor sorts: accrual, beta, investment, long-term reversal, momentum, net share issuance, operating profitability, short-term reversal, value, and variance. As before, for beta, we lever all portfolios to have an ex post beta of 1 with respect to the market excess return for the French Data Library. For variance, we lever all portfolios to have an ex post-annualized volatility of 16%.

4 Illiquidity and its interaction with popular factor portfolios

In this paper we proxy the illiquidity characteristic with both Amihud illiquidity measure and market capitalization. We explain in greater detail in later sections the cross-sectional correlation as well as relating these two measures to other observable liquidity characteristics such as bid–ask spread. For the first part of this paper, we measure the illiquidity of stocks using the methodology described in Amihud (2002), which measures the illiquidity score as the ratio of daily absolute stock return to daily dollar trading volume. Intuitively, the statistic seeks to measure the market price impact per dollar unit of trading. Using this measure, illiquid stocks are stocks whose price moves substantially when investors go to the market to transact.

We show our key empirical findings in Exhibit 1. We put stocks into intersecting Amihud illiquidity and factor characteristic quintiles and calculate factor returns within illiquidity quintiles.

Exhibit 1

	Factor Sharpe Ratios within illiquidity quintiles					
Factor	Liquid	2	3	4	Illiquid	Start date
Panel A						
Short-term reversal	0.06	0.51	0.73	1.05	2.33	Jan 1967
Investment	0.30	0.49	0.60	0.59	0.66	Jan 1967
Gross profitability	0.10	0.19	0.34	0.50	0.45	Jan 1967
Momentum	0.17	0.36	0.53	0.76	1.00	Jan 1967
Value	0.20	0.47	0.64	0.83	0.82	Jan 1967
5-Year beta	0.50	0.61	0.50	0.82	0.55	Jan 1967
5-Year volatility	0.26	0.42	0.49	0.62	0.18	Jan 1967
Net share issuance	0.40	0.53	0.62	0.62	1.13	Jan 1967
Long-term reversal	0.12	0.18	0.30	0.22	0.32	Jan 1967
60-Day volatility	0.37	0.61	0.77	1.17	0.82	Jan 1967

	Factor excess returns within illiquidity quintiles							
Factor	Liquid	2	3	4	Illiquid	IML	*t*-Stat	Start date
Panel B								
Short-term reversal	1.0%	8.3%	11.9%	19.2%	53.3%	52.2%	13.19	Jan 1967
Investment	3.4%	5.0%	6.7%	7.2%	8.9%	5.5%	2.08	Jan 1967
Gross profitability	1.2%	2.0%	3.6%	5.2%	5.6%	4.3%	1.78	Jan 1967
Momentum	3.5%	7.2%	10.9%	16.5%	21.6%	18.1%	4.75	Jan 1967
Value	3.1%	7.0%	9.3%	11.7%	12.6%	9.6%	3.29	Jan 1967
5-Year beta	9.4%	8.8%	7.0%	12.4%	9.0%	−0.4%	−0.25	Jan 1967
5-Year volatility	3.7%	5.7%	6.1%	8.0%	2.1%	−1.6%	−0.94	Jan 1967
Net share issuance	3.6%	4.8%	5.7%	6.3%	11.2%	7.6%	4.19	Jan 1967
Long-term reversal	1.7%	2.2%	4.1%	3.4%	6.2%	4.5%	1.50	Jan 1967
60-Day volatility	6.0%	9.4%	11.8%	19.5%	16.2%	10.2%	3.36	Jan 1967

Exhibit 1 (*Continued*)

	Factor *t*-Stats within illiquidity quintiles					
Factor	Liquid	2	3	4	Illiquid	Start date
Panel C						
Short-term reversal	0.99	3.95	5.34	7.28	13.86	Jan 1967
Investment	2.46	3.66	4.42	4.34	4.88	Jan 1967
Gross profitability	1.12	1.66	2.71	3.75	3.47	Jan 1967
Momentum	1.87	3.13	4.32	5.78	7.09	Jan 1967
Value	1.91	3.69	4.77	5.90	5.86	Jan 1967
5-Year beta	3.99	4.58	3.85	5.91	4.22	Jan 1967
5-Year volatility	2.27	3.30	3.72	4.60	1.65	Jan 1967
Net share issuance	3.01	3.94	4.49	4.55	7.78	Jan 1967
Long-term reversal	1.31	1.65	2.51	2.04	2.81	Jan 1967
60-Day volatility	3.05	4.62	5.62	8.08	6.02	Jan 1967

4.1 Major results

Exhibit 1 shows that the more illiquid variants of the well-known factor portfolios generally have meaningfully larger premiums than their more liquid counterparts. Panel A shows the Sharpe Ratios of each factor within each Amihud illiquidity quintile. For example, the first row shows the Sharpe Ratio of short-term reversal when computed within each illiquidity quintile.

Panel B shows the excess returns of each factor within each illiquidity quintile. It also shows the factor return of the most illiquid quintile minus the factor return of the least illiquid quintile (IML). Critically, this is not the illiquidity factor itself. Instead, it is the difference in a particular factor returns within the most illiquid quintile and the most liquid quintile of stocks. We find that in nearly all cases, there is a monotonic increase in excess returns when we move from more liquid factor portfolios to a less liquid factor portfolio. Five-year beta and volatility present notable exceptions to this rule. Panel C shows the *t*-Stats for the returns in Panel B. Again, we see the similar monotonic increase.

In Exhibit 2, we run an augmented Fama–French–Carhart factor model with an illiquidity factor (FFC + illiquidity). The reason why we start with the Carhart regression is that it serves as a standard to determine whether a strategy earns alpha on the four most common factors. We add the Amihud illiquidity factor to show that this result is not merely driven by the illiquidity premium.

Exhibit 2 Panel A shows alphas from the FFC + illiquidity model for the factor returns within each illiquidity quintile. For these factors, the FFC + illiquidity model-adjusted alpha often shows a monotonically increasing pattern, where the liquid factor portfolios tend to have low or negative alphas while the illiquid factor portfolios tend to have higher positive alphas. Again, the strongest exceptions to our rule of monotonicity are 5-year beta and volatility. Short-term reversal, investments, momentum, value, and net share issuance show monotonicity. Long-term reversal and 60-day volatility both show “off-by-one”

Exhibit 2

Factor	Liquid	2	3	4	Illiquid	Start date
	Alphas from Carhart 4 factor + Amihud illiquidity factor within illiquidity quintiles					
Panel A						
Short-term reversal	0.26%	0.82%	1.14%	1.75%	3.99%	Jan 1967
Investment	0.07%	0.24%	0.36%	0.49%	0.68%	Jan 1967
Gross profitability	0.47%	0.29%	0.35%	0.48%	0.55%	Jan 1967
Momentum	−0.36%	−0.07%	0.27%	0.79%	1.40%	Jan 1967
Value	0.03%	0.50%	0.67%	0.93%	1.01%	Jan 1967
5-Year beta	0.44%	0.39%	0.23%	0.71%	0.58%	Jan 1967
5-Year volatility	0.17%	0.30%	0.28%	0.46%	0.11%	Jan 1967
Net share issuance	0.22%	0.25%	0.30%	0.36%	0.85%	Jan 1967
Long-term reversal	−0.09%	−0.01%	0.23%	0.21%	0.48%	Jan 1967
60-Day volatility	0.27%	0.54%	0.63%	1.30%	1.26%	Jan 1967

Factor	Liquid	2	3	4	Illiquid	Start date
	Alphas *t*-Stats from Carhart 4 factor + Amihud illiquidity factor within illiquidity quintiles					
Panel B						
Short-term reversal	1.41	4.27	5.93	8.25	15.29	Jan 1967
Investment	0.70	2.17	2.75	3.43	4.31	Jan 1967
Gross profitability	3.97	2.44	2.89	3.86	3.63	Jan 1967
Momentum	−3.63	−0.67	2.13	4.58	6.24	Jan 1967
Value	0.27	4.60	5.33	6.09	5.47	Jan 1967
5-Year beta	2.24	2.70	1.50	4.02	2.93	Jan 1967
5-Year volatility	1.28	2.46	2.09	3.10	0.81	Jan 1967
Net share issuance	2.38	2.70	3.11	3.17	7.01	Jan 1967
Long-term reversal	−0.62	−0.04	1.45	1.21	2.13	Jan 1967
60-Day volatility	1.71	3.63	3.81	6.97	5.33	Jan 1967

monotonicity wherein switching two adjacent quintiles would result in monotonicity.

4.2 Interpretation

Our results can be consistent with both the risk-based and the behavioral interpretation of the common factor premiums. If a factor premium is driven by risk discount that risk exposure is likely to have a very nonlinear interaction with illiquidity in times of crisis. The observation that correlation increases substantially for all assets (and factors) in liquidity crisis speaks to this nonlinear interaction.

The standard narrative paints a picture where all risky assets are sold off in fire sales as investors flee to safety or voluntarily or involuntarily de-risk positions in a liquidity crunch. For many investors, a sharp decline in wealth may cause a sharp increase in risk aversion, which would result in voluntary de-risking even as assets have become meaningfully cheaper in terms of standard valuation metrics.[4] Investors with leveraged positions may be forced to de-risk involuntarily as a result of maintenance margin deficit. In a fire sale, illiquid assets are likely to decline far more than would be predicted by a linear factor model. This narrative is consistent with the empirical observation of Brennan *et al.* (2012), where much of the risk from illiquidity comes from selling in down markets.

If a factor premium is created by behavioral biases, the profit opportunity is more likely to persist if the transaction cost associated with accessing the anomaly premium is larger. Shorting illiquid stocks are orders of magnitude more expensive than shorting liquid stocks; both borrowing costs and maintenance margin for short positions are meaningfully higher for illiquid securities. Take for example Palm, which was a spinoff from 3Com during the heydays of the Tech Bubble. Palm was 95% owned by 3Com and had a float of 5% and was illiquid whether measured using bid–ask spread or Amihud illiquidity. Palm was a poster child of market inefficiency and mispricing and the favorite case study for professors lecturing on market anomalies. In 2000, Palm's share price implied a value of negative 22 billion dollars for 3Com ex-Palm.[5] However, it was nearly impossible to execute an arbitrage due to the cost of illiquidity. The cost of shorting Palm stocks was estimated as high as 60% per annum during the height of its overvaluation.[6]

Additionally, many larger investors are often precluded from investing in strategies based on illiquid stocks, which tend to have low capitalization or low float, due to policy constraints and regulatory requirements. Most pension funds cannot hold more than 5% of a company, which effectively precludes them from investing a meaningful amount of their assets into strategies that invest heavily into stocks with very small market capitalization.

5 Low capitalization as a proxy for illiquidity

To make our findings even more relevant to investors, we calculate the correlation between liquidity and market capitalization. Liu (2006) shows that illiquidity could result in the observed small-cap effect. In Exhibit 3, we show that in the cross-section, capitalization is monotonically decreasing in other measurements of illiquidity such as Amihud illiquidity and bid-ask spread. A comprehensive list of illiquidity measures can be found in Hasbrouck (2009) and Kingsley *et al.* (2014).

To demonstrate the relationship between capitalization and liquidity, we break stocks into quintiles based on most recent market cap and then average the Amihud illiquidity of each quintile. Because Amihud illiquidity is non-stationary over time, we scale each quintile's liquidity to the median quintile's liquidity for each month. We then average each quintile's liquidity from January 1950 to December 2014. The scaled illiquidity of each quintile is shown. In addition to Amihud illiquidity score, we use a second measure—the ratio of ask-minus-bid to the midpoint between bid and ask. This serves as a measure of the bid–ask spread as a percentage of the price of the stock. We only have this data with reasonable quality going back to January 1993, so we limit our sample from January 1993 to December 2014. As expected stocks in the largest (lowest) capitalization quintiles have the lowest (highest) Amihud illiquidity score and lowest (highest) bid–ask spread.

6 Small cap and its interaction with popular factors

What Exhibit 3 suggests is that the small-cap variants of the factor portfolios tested in Exhibit 1 should also deliver a higher excess return than their large-cap counterparts. In Exhibit 4, we sort the different factor portfolios along the capitalization dimension instead of the liquidity dimension.

6.1 Major results

In this paper, we forgo a comprehensive discussion on whether the small-cap effect is just an illiquidity effect.[7] We note the similarity in empirical patterns between Exhibits 1 and 4, where in Exhibit 4 the illiquidity sorting criteria are replaced with capitalization. Not reported here, for robustness we also use size break points across the entire market rather than French's NYSE break points; the results are largely similar.

The results from Exhibit 4 suggest that the small-cap factor returns are much larger than large-cap factor returns. For investors seeking to capture the small-cap premium, the portfolio created from the small-cap variant of the popular factor portfolios would offer meaningfully better bang for the buck. Conversely, investors seeking to capture other factor premiums would earn better returns by applying the factor among small stocks.

Exhibit 3

U.S. size quintiles	Scaled Amihud illiquidity (1/1950–12/2014)	Bid–ask spread (1/1993–12/2014)
Small	75.35	7.3%
2	7.02	3.2%
3	1.00	1.8%
4	0.31	1.1%
Big	0.09	0.6%

6.2 Interpretation

What makes the results from Exhibit 4 more interesting to investors is the implication on returns to factor investing. Exhibit 4 immediately suggests that the way factors are traditionally constructed can exaggerate expected return associated with factor investing for the average investor. To appreciate this point, it is useful to understand how standard factor portfolios are built, and how factor premiums are then inferred by practitioners from these factor portfolios. Standard factor portfolios are constructed by investing 50% into a dollar neutral long–short portfolio constructed from the large-cap universe and 50% into a similarly constructed long–short portfolio based on the small-cap universe.

Take for example the value factor portfolio, which is commonly referred to as HML (**H**igh book-to-market **M**inus **L**ow book-to-market). The HML portfolio is actually the average of a large-cap HML and a small-cap HML portfolio. The premium associated with the HML factor as estimated by practitioners who simply compute the average return of the HML factor is then the average of the historical premiums from the large and small HML. Historically, the estimated HML premium using data from Ken French's equity factor data library translates to a Sharpe Ratio of 0.4. However, the Sharpe Ratio of the large HML portfolio is only 0.2, whereas the Sharpe Ratio of the small HML portfolio is 0.6. As there are far more assets invested in large-cap value strategies than small-cap value strategies, the value premium as estimated by the traditional HML factor portfolio overstates the average value investor experience. This observation is equally true for other factors like profitability[8] and short-term reversal.[9]

Importantly, this substantially higher SR for small HML is not driven by the small-cap premium,

Exhibit 4

	Factor excess returns within size quintiles (Source: French Data Library)							
Factor	Small	2	3	4	Large	Diff	t-Stat	Start date
Panel A								
Accrual	3.2%	2.1%	2.8%	0.2%	4.4%	−1.3%	−0.78	July 1963
Low beta	4.8%	5.3%	5.0%	4.4%	4.1%	0.7%	0.31	July 1963
Investment	7.3%	4.5%	4.1%	2.0%	2.7%	4.6%	2.45	July 1963
Long-term reversal	5.7%	3.3%	1.6%	1.5%	1.4%	4.3%	2.31	Jan 1950
Momentum	14.7%	12.3%	10.5%	9.0%	7.0%	7.6%	3.71	Jan 1950
Net share issuance	7.3%	6.5%	5.8%	2.6%	3.0%	4.3%	2.28	July 1963
Operating profitability	3.0%	4.0%	4.0%	2.4%	1.7%	1.4%	0.66	July 1963
Short-term reversal	15.1%	8.9%	8.5%	6.8%	2.6%	12.5%	6.69	Jan 1950
Value	9.2%	5.6%	4.6%	1.8%	1.0%	8.1%	3.81	Jan 1950
Variance	15.1%	10.5%	7.7%	5.7%	2.1%	13.0%	7.13	July 1963

	Factor t-Stats within size quintiles					
Factor	Small	2	3	4	Large	Start date
Panel B						
Short-term reversal	11.48	7.79	4.60	4.15	1.86	Jan 1967
Investment	6.28	5.28	5.49	4.74	1.98	Jan 1967
Gross profitability	2.28	4.41	4.02	2.39	1.45	Jan 1967
Momentum	3.81	6.44	5.34	3.36	1.88	Jan 1967
Value	5.15	4.81	3.49	2.48	1.94	Jan 1967
Beta	4.92	5.56	5.18	5.03	3.10	Jan 1967
Volatility	5.56	5.98	4.57	4.39	2.29	Jan 1967
Net share issuance	4.06	5.60	5.37	4.41	3.11	Jan 1967
Long-term reversal	2.40	2.34	2.76	1.93	1.26	Jan 1967
60-Day volatility	6.65	9.55	7.77	5.97	2.41	Jan 1967

which is actually negligible in the sample. The higher SR is driven by an interaction between small cap and value—that is, small, value stocks carry a meaningfully larger excess return than can be explained by the value and the small-cap exposure linearly. What this means is that a large-cap value portfolio would display a negative Fama–French-3 alpha, whereas a small-cap value portfolio would display a positive Fama–French-3 alpha.[10]

7 Additional discussion

The rising popularity of factor investing and Smart Beta investing means tradable factors are increasingly used for investments rather than performance analytics and academic asset pricing research. Generally, factors are examined and used in linear models. Investment strategies are examined or created as linear combinations of known factors. However, we now show evidence that factors interact in a nonlinear way and the returns to an intersection portfolio can often be substantially larger than simple combination of the same factors.

In this paper, two highly correlated factors, illiquidity and small cap, are highlighted as factors whose historical premium is low as an independent source of returns. However, even if small-cap stock returns are not meaningfully higher, factor returns among small cap stocks are meaningfully higher than factor returns among large-cap stocks.

Additionally, it is often the case that the liquid variant of a known factor portfolio will earn a sufficiently low excess return as to often not pass the threshold for statistical significance. While the low *t*-Stat for the liquid factor portfolio does not outright reject the existence of a risk or behavioral premium, given other empirical and theoretical support, it does suggest that the effect is substantially driven by an interaction with illiquidity risk.

We should note that Harvey *et al.* (2015) recommend using 3.0 *t*-statistic thresholds for 5% significance for financial economics. We meet this heightened threshold in some but not all of our results. However, that the vast majority of the results also point in the same direction—that is, factor premiums are higher among more illiquid stocks—is further evidence of the claim.

8 Conclusion

Most factors exhibit higher returns and greater significance among illiquid stocks than among liquid stocks and among small stocks than among large stocks. These factors include value, gross profitability, investment, net share issuance, short-term reversal, long-term reversal, momentum, and 60-day volatility factors. This is no surprise. Illiquidity is a limit to arbitrage among small stocks. Many institutions cannot invest in such small stocks and therefore cannot arbitrage away the premium. Instead, they must work in the large-capitalization space where arbitrage is harder. Even among true risk factors, illiquidity will cause investors to attach a steeper premium to illiquid, high-risk stocks because they do not want to be forced to liquidate at a below-market price during adverse market conditions. We cannot disentangle these effects as their outcome is the same—factor premiums are higher among small stocks.

We are not recommending that investors avoid these factors. Instead, they must consider the factor premiums for the stocks in their investment universe—often not the full universe of stocks available in return databases. As a result, an institution unable (due to its size) or unwilling (due to its investment mandate) to invest in small stocks may be forced to accept lower factor premiums than those traditionally reported for the factors.

Notes

1 See Damodoran (2005) for a wonderful review on measuring illiquidity and frameworks for thinking about the appropriate price discount given illiquidity exposure.
2 See Strebulaev (2003) and Dimson and Hanke (2004) for evidence on bonds. See Christoffersen *et al.* (2014) for evidence on options.
3 See Ick (2005) for evidence on private equities.
4 See Campbell and Cochrane (1999).
5 See Lamont and Thaler (2001).
6 See Cherkes *et al.* (2013).
7 See Berk (1997) and Arnott *et al.* (2007) who argue that part of the outperformance of small-cap stocks over large-cap stocks is driven by valuation or price. That is, on average, small-cap stocks have cheaper prices.
8 See Fama and French (2015) on the profitability premium in small versus large stocks.
9 See Asness *et al.* (2000) on the reversal effect in small stocks.
10 This was originally observed in Fama and French (1992). We replicated this observation again using an additional 25 years of data.

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