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MACRO-BASED PARAMETRIC ASSET ALLOCATION

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Without doubt the financial returns of asset classes are interlinked with the economy. However, a direct link between financial returns and return-driving forces has not been discovered yet. Moreover, there exist many robust approaches for within-asset-class allocation but few advances have been made for between-asset-class allocation. To address these topics I propose a direct modeling of the weights with macroeconomic risk factors. This allows to implicitly identify capital-market dynamics and thus provides a framework which can help in the tactical asset allocation decision.

1 Introduction

There is significant evidence that the financial returns of asset classes such as stocks and bonds are interlinked with the economy. Yet, the existing approaches fail in showing directly how these macroeconomic forces drive the financial returns of asset classes. This might also be a reason why few advances have been made on the topic of between-asset-class allocation, whereas there exist many robust approaches for within-asset-class allocation. This paper addresses both topics.

By far the most prominent models in asset-class allocation are the models of Markowitz (1952) and Black and Litterman (1992). In the approach of Markowitz (1952), asset-class returns and a covariance matrix related to these returns need to be estimated for the asset allocation process. This induces the danger of adding up estimation errors and often results in an unstable asset allocation. Moreover, Jacobs *et al.* (2014) show that a simple heuristic approach such as an $1/N$ allocation can outperform Markowitz and its variations cost-efficiently. In Black and Litterman (1992), the investor updates the information implicitly revealed by the market, specifically by the CAPM model, with his own return expectations. Returns are usually assumed to follow a normal distribution.

In both approaches the investor needs to estimate the expected returns of asset classes. This is difficult and often subjective. Building on the work of Markowitz (1952), Brandt and Santa-Clara (2006) suggest modeling the allocation of assets

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directly within a single- or multiperiod setup. This comes at the cost of having to impose a strong structure on the dynamics of returns. Furthermore, the authors also establish a link to economic variables.

Brandt *et al.* (2009) generalize this approach and suggest a parametric asset allocation. Again, the optimal weights for the allocation of stocks within a given stock universe are modeled directly without the detour of determining expected returns. The input factors are the characteristics of stocks, which are market capitalization, the book-to-market ratio, and the lagged 12-month return.

The question is why not to apply this methodology to a between-asset-class problem for a portfolio consisting of, for instance, stocks, bonds, and the risk-free asset and link the return of these asset classes to macroeconomic factors. Although this seems methodologically close if not identical to Brandt and Santa-Clara (2006) and Brandt *et al.* (2009), there is one key difference to each paper respectively.

On the one hand, in terms of Brandt and Santa-Clara (2006) the approach suggested in this work does not require any specific return structure. Thus one avoids the estimation error of choosing a return-generating process, which is at best an approximation for actual returns. Putting differently, there is only one layer of estimation in the methodology proposed. On the other hand, in contrast to Brandt *et al.* (2009) the characteristics of a stock universe as, for example, the book-to-market ratio do not vary among assets, whereas characteristics for asset classes, as the term spread, do not vary among asset classes. However, varying characteristics is *the* key input to the original methodology, allowing to reduce the dimensionality problem of allocating a large number of assets which share the same type of characteristics. But for answering how factors describing the state of the economy are linked with the performance of asset classes, the reduction in dimensionality is of no use. In fact, it is obstructive, as it should not be surprising that different asset-classes are differently exposed to risk factors describing the state of the economy. Thus the importance or *loading* of each risk factor could be different for each asset class. These different loadings enter the weight which is allocated to each asset class. Hence, changes in the portfolio weights are directly triggered by changes in the risk factor and these changes are different for each asset class. This implicitly reveals capital-market dynamics and shows how they translate into portfolio weights without having to assume any return structure or model on the risk factors. The only assumptions imposed are that the weight given to each asset class is a linear function of the risk factors and that the interdependence between these risk factors and the return of asset classes is ergodic.

The challenge is to identify those economic forces that cause expected returns, and hence the weights, to change. However, there are some criteria limiting the selection: (1) data should be available for a long time series, (2) data frequency should be at least monthly, (3) there should be a limited risk to significant post-release data revisions, (4) the variables should have been discussed in the literature, and (5) the link between the variables and financial asset-class returns should be intuitive.

A lot of research has been done on the impact of economic variables on the return of stocks and bonds, as for example in Keim and Stambaugh (1986), Campbell and Shiller (1988), Fama and French (1988, 1989), Fama (1990), Campbell (1991), Hodrick (1992), Oertmann (1997), and Oertmann and Seiler (2010). Generally, the term spread, credit spread, and the ted spread seem to be consensus factors in the literature which drive asset-class returns. Another stream of literature focuses on (relative) valuations and momentum as return-driving forces, as discussed in Ilmanen (2003), Asness *et al.* (2013), and Koijen *et al.* (2013). For this I add the price–earnings ratio, the dividend yield, and momentum for stocks and bonds to the analysis. All of the variables mentioned fulfill the selection criteria listed.

Data are available for a 50-year period from the end of December 1964 to the end of December 2014 at a monthly frequency. Since a large sample of data is crucial for the methodology to work I constrain myself to US asset classes and macroeconomic series as data are most easily available there. Assuming ergodicity and taking advantage of a jackknife methodology I generate a 50-year non-overlapping out-of-sample period which greatly improves the analysis of the methodology. To ensure that the results are not spurious due to the overlap of the data in-sample I perform several robustness checks.

The in-sample period comprises a rising interest-rate regime, a falling interest-rate regime, several stock market crashes including a value bubble, periods of economic crisis, and an excess leverage period. This should ensure that consistent links between the state of the economy and the state of the cycle of financial markets are discovered.

The results of the method suggested are promising. The statistically significant risk factors have meaningful signs and are intuitive to interpret. In general, the economic performance is closely related to the statistical significance of the model. This is a strong indication that the method is able to implicitly unveil the link and impact of the risk factors to the financial return of the asset classes.

Moreover, the strategy outperforms its benchmarks, a mean–variance portfolio and several static portfolios significantly in terms of return and Sharpe Ratio in-sample and out-of-sample. Jensen's Alpha measured relatively to the benchmark is significantly positive. The Investment Ratio is sizeable apart for two out of eleven models, which show little statistical significance. The model's outperformance prevails largely when considering transaction costs or leverage costs. The approach can also be extended to more asset classes as long as the total return data series are long enough.

Related to this study is the paper by Laborda Herrero and Laborda Herrero (2009). The authors implement a hedge fund strategy consisting of up to 46 long/short portfolios of various asset-class pairs. The long/short decisions depend also on global risk factors. However, this is a two-step approach: In the first step, long–short subportfolios are built and optimized separately. These subportfolios are then combined in an expected loss exceeding value at risk methodology. In contrast to this study, I optimize the portfolio allocation at once, as otherwise the main question of how the state of the economy and the return of asset classes is linked cannot be answered. Furthermore, an out-of-sample analysis and robustness specifications are missing.

Further literature relates to this study as follows: Dahlquist and Harvey (2001) study a qualitative framework linking the business cycle to the asset allocation decision. The need to adapt the asset allocation through the business cycle was also pointed out by van Vliet and Blitz (2011). Kritzman *et al.* (2012) try to link a dynamic asset allocation to economic regime shifts using Markov-switching models. This imposes some sizeable assumptions on the model. Similarly, Guidolin and Timmermann (2007) propose a multivariate regime-switching asset allocation model. However, their model cannot beat the minimum-variance portfolio in terms of the Sharpe Ratio and it is well-known that the minimum-variance portfolio performs worse than an equal-weighted portfolio out-of-sample. Jurek and Viceira (2011) develop an analytical solution to the dynamic portfolio choice problem of an investor with power utility, who faces time-varying investment opportunities. The state variables are the price–earnings ratio, short-term nominal interest rate, the yield spread, and the ex-post real rate of return on a 30-day Treasury bill. To study the horizon effects in the allocation of value stocks, growth stocks, Treasury bills, and Treasury bonds, the authors impose a quite restrictive VAR model with homoskedastic, normally distributed shocks. Li and Sullivan (2011) argue strongly in favor of a top-down rather than a bottom-up approach considering different market environments and an array of asset classes. As an example, one approach to link a well-known risk factor – liquidity – to asset allocation was suggested by Xiong *et al.* (2013). Bera and Park (2008) discuss an asset allocation method based on the entropy principle. However, the model outperforms an equally-weighted benchmark only marginally.

This paper proceeds as follows. In Section 2, I describe the methodology and in Section 3 the statistical approach to solve the problem. For estimation I use generalized methods of moments (GMM) and an iterative optimization routine which averts local maxima. The data are described in detail in Section 4. In Section 5 I discuss the link between the economic risk factors, valuations and financial returns, the performance of the approach, and a model selection. In Section 6, transaction and leverage costs are added and in Section 7 I add an additional asset class to the model. The robustness of the approach is discussed in Section 8 and Section 9 concludes.

2 Methodology

In the following section I describe the setup for between-asset-class allocation using a parametric approach based on Brandt *et al.* (2009). At each date t there is a fixed number of investable risky asset classes, N. The investor faces the problem of allocating his funds among these asset classes at each point in time such that his conditional expected utility of the portfolio returns is maximized. The percentage allocation of his funds to asset class i at time t is denoted as $w_{i,t}$. Each asset class i has a return $r_{i,t+1}$ measured from t to $t+1$. Similarly, the portfolio return is $r_{p,t+1}$. Suppose the return of these asset classes is associated with a vector of K risk factors, x_t observed at date t. Assume the investor's constant relative risk-averse utility function $u(.)$ with risk aversion parameter ρ is time separable and given by

$$u(r_{p,t+1}) = \frac{(1+r_{p,t+1})^{1-\rho}}{1-\rho}. \tag{1}$$

The investor faces the problem of choosing the optimal portfolio weights such that his conditional expected utility of the portfolio's return is optimized

$$\max_{\{w_{i,t}\}_{i=1}^N} \mathrm{E}_t[u(r_{p,t+1})] = \max_{\{w_{i,t}\}_{i=1}^N} \mathrm{E}_t\left[u\left(\sum_{i=1}^N w_{i,t} r_{i,t+1}\right)\right]. \tag{2}$$

The asset class weight at time t for asset class i is parameterized by a function $f(.)$ given by

$$w_{i,t} = f(x_t; \theta_i). \tag{3}$$

Hence, the weights depend on K risk factors x_t and the associated K coefficients θ_i to be estimated for each asset class i. In contrast to Brandt *et al.* (2009), the investor is not interested in allocating his funds within a pure stock portfolio but in allocating his funds among asset classes such as stocks and bonds which share the same risk factors x_t but have different coefficients θ_i. It is important to emphasize that the k-th risk factor $x_{k,t}$ is the same for each asset class i. However, the parameters $\theta_{i,k}$ with which this risk factor "loads" on the weight $w_{i,t}$ of asset i can be different for each asset class. This highlights the importance of the risk factors: For example, the ted spread (spread between the three-month interbank and government rates) might have an impact on returns of stocks and bonds, though this impact might be of different magnitude.[1]

This formulation implies that not the returns but the weights given to the assets are directly related to the risk factors. The advantage is that there is no need to perform a two-stage optimization as in traditional asset allocation approaches, in which first expected returns are estimated which are then used to find the best allocation.

As in Brandt *et al.* (2009) suppose that the weighting function is linear

$$w_{i,t} = \bar{w}_{i,t} + \theta_i' \hat{x}_t, \tag{4}$$

where $\bar{w}_{i,t}$ is the asset-class–weight in a benchmark for asset class i and $\theta_i' \hat{x}_t$ captures the deviation from these benchmark weights. The benchmark weight is set to $\bar{w}_{i,t} = \frac{1}{N}$ as this is the best guess of an investor having no additional information.[2] To ensure that the portfolio weights sum up to one, a risk-free asset with return $r_{f,t+1}$ from t to $t+1$ at which the investor can borrow and lend serves as residual. There is no leverage constraint in place, hence the investor could borrow unboundedly. I add leverage costs as well as transaction costs in Section 6. Formally, at any time t the following condition must hold:

$$\sum_{i=1}^{N} w_{i,t} + w_{r_f,t} = 1, \tag{5}$$

where $w_{r_f,t} = 1 - \sum_{i=1}^{N} w_{i,t} \equiv 1 - \sum_{i=1}^{N} f(x_t; \theta_i)$ is the percentage weight of the risk-free asset at time t. This is similar to the condition that the k-th coefficient of the risk-free asset $\theta_{r_f,k} = -\sum_{i=1}^{N} \theta_{i,k}$.

As the coefficients are constant I assume that those coefficients maximize the investor's conditional expected utility not only at one given date but also for all dates. Thus, the coefficients also maximize the investor's unconditional expected utility. However, the coefficients are not constant across assets. Hence, it cannot be assumed that the risk factors fully capture the joint distribution of expected returns. This means that the allocation to each asset class in the optimal portfolio does not necessarily only depend on its risk factors but it could also depend on the historic returns of the asset class. This information must be considered by some measure of past returns. I thus include a form of price momentum into the analysis (see Section 4).

This implies that the optimization Problem (2) can be rewritten as the following unconditional optimization problem with respect to the coefficients θ_i

$$\max_{\theta_i} \mathrm{E}[u(r_{p,t+1})] = \mathrm{E}\left[u \left(\sum_{i=1}^{N} f(x_t; \theta_i) r_{i,t+1} + w_{r_f,t} r_{f,t+1} \right) \right]. \tag{6}$$

For estimating the above equation it is necessary to generate a sample analog. This requires the assumption of ergodicity of the interdependence between risk factors and the return of asset classes. In this case time averages are equal to state averages. Non-technically speaking, this implies that the interdependence between risk factors and the return of asset classes is constant over time, i.e., history repeats itself. The sample analog is given by

$$\max_{\theta_i} \frac{1}{T} \sum_{t=0}^{T-1} [u(r_{p,t+1})] = \frac{1}{T} \sum_{t=0}^{T-1} \left[u \left(\sum_{i=1}^{N} f(x_t; \theta_i) r_{i,t+1} + w_{r_f,t} r_{f,t+1} \right) \right], \tag{7}$$

where $w_{r_f,t} = 1 - \sum_{i=1}^{N} f(x_t; \theta_i)$. In the linear policy case the optimization problem is

$$\max_{\theta_i} \frac{1}{T}\sum_{t=0}^{T-1}[u(r_{p,t+1})] = \frac{1}{T}\sum_{t=0}^{T-1}\left[u\left(\sum_{i=1}^{N}(\bar{w}_{i,t} + \theta_i'\hat{x}_t)r_{i,t+1} + w_{r_f,t}r_{f,t+1}\right)\right], \quad (8)$$

where $w_{r_f,t} = 1 - \sum_{i=1}^{N} w_{i,t}$.

3 Statistical approach

To find the optimal coefficients θ, I perform a generalized methods of moments (GMM) estimation. In order to maximize the investor's utility, the estimates $\hat{\theta}$ should satisfy the first-order conditions of the maximization problem (Equation (8)) for each parameter $\theta_{i,k}$ of each risky asset class i and risk factor k. Thus $N \times K$ coefficients are estimated where N is the number of risky assets and K the number of characteristics. The first-order condition of parameter $\theta_{i,k}$ is

$$\frac{1}{T}\sum_{t=0}^{T-1} h(r_{t+1}, x_t; \theta_{i,k}) \equiv \frac{1}{T}\sum_{t=0}^{T-1}\frac{\delta u(r_{p,t+1})}{\delta\theta_{i,k}} = \frac{1}{T}\sum_{t=0}^{T-1} u'(r_{p,t+1})(\hat{x}_{k,t}(r_{i,t+1} - r_{f,t+1})) = 0 \quad (9)$$

with

$$u'(r_{p,t+1}) = u'\left(1 + \left(\sum_{i=1}^{N}(\bar{w}_{i,t} + \theta_i\hat{x}_t)r_{i,t+1}\right) + w_{r_f,t}r_{f,t+1}\right), \quad (10)$$

where

$$w_{r_f,t} = 1 - \sum_{i=1}^{N}(\bar{w}_{i,t} + \theta_i\hat{x}_t). \quad (11)$$

Following the statistical approach of Brandt *et al.* (2009), the first-order conditions can be interpreted as method of moments estimators, and specifically as moment conditions. Thus GMM can be used as estimation methodology. Furthermore, the asymptotic covariance matrix of this estimator is (Brandt *et al.*, 2009; Hansen, 1982)

$$\Sigma_\theta \equiv \mathrm{V}[\hat{\theta}] = \frac{1}{T}[G'V^{-1}G]^{-1} \quad (12)$$

with

$$G = \frac{1}{T}\sum_{t=0}^{T-1}\left.\frac{\delta h(r_{t+1}, x_t; \theta_{i,k})}{\delta\theta}\right|_{\theta=\hat{\theta}}, \quad (13)$$

where θ are the coefficients which are evaluated at the estimated coefficients, $\hat{\theta}$. The asymptotic covariance matrix is calculated numerically and the weighting matrix is the inverse of the covariance matrix.

Since the moment conditions cannot be solved analytically, an initial solution guess of θ_i is required by the numerical algorithm. As with all numerical procedures local maxima could potentially be a problem. This is the case as the solution to the numerical algorithm is not independent of the initial solution guess. However, this local maxima problem can be solved with an iterative estimation algorithm. Details on this algorithm can be found in Appendix A.

4 Data

Linking the financial asset allocation decision to macroeconomic variables is a challenging task as there is a great variety of potential variables to choose from. However, there are some natural criteria which limit the selection substantially and thus practically eliminate the risk of data mining: (1) data should be available for a long time series, (2) there should be a limited risk to significant post-release data revisions as for example for GDP, (3) the variables should have been discussed in the literature, and (4) the link between the variables and financial asset-class returns should be intuitive.

There seems to be a consensus in the literature about the driving forces of asset-class returns: the term spread, credit spread, and the ted spread. Additionally, many studies point out the importance of (relative) valuations and momentum. Therefore I add the price–earnings ratio, the dividend yield, and momentum for stocks and bonds to the analysis. All of the variables mentioned fulfill the selection criteria listed. The list does not include classic measures of risk as the S&P 500 Volatility Index or the correlation between stocks and bonds. The reason is that the Volatility Index is only available from 1990 onwards and including the correlation of stocks and bonds would induce another level of estimation error with a purely backward-looking measure. Due to the considerably longer data availability I will restrict the analysis to US markets, which one can take as proxy for the global allocation to stocks and bonds.

Data are available from December 1964 to 2014 at a monthly frequency. Thus the analysis covers 50 years, including a rising interest-rate regime (1960s and 1970s), a falling interest-rate regime (1980s and thereafter), periods of economic crisis, several stock market crashes including a value bubble (dot-com crisis), and an excess leverage period and its collapse (financial crisis). This should ensure that consistent links between the state of the economy and the state of the cycle of financial markets are uncovered. The forecast period is one month, hence data do not overlap. This emphasizes the active asset allocation approach.

For estimation of the parameters and the out-of-sample test I take advantage of the ergodicity assumption. As the coefficients are assumed to not change over time it should not make any difference over which time frame they are estimated. For example, the coefficients estimated from December 1964 to December 2013 (49 years, out-of-sample year 2014) or from December 1964 to December 2012 and from December 2013 until December 2014 (as well 49 years, out-of-sample year 2013) should be alike. This is essentially a jackknife methodology. Thus I can generate a 50-year non-overlapping out-of-sample period when moving forward the out-of-sample period through time. This enhances greatly the generality of the out-of-sample conclusions. Obviously the in-sample periods are overlapping, and thus I perform several robustness checks to ensure that the results are not erroneous in Section 8.

In the following paragraphs I describe the different variables. t refers to the last trading day of the month and the return period $t+1$ refers to an investment in an asset class at t held until the end of the following month, $t+1$.

The risk-free rate $r_{f,t+1}$ is the percentage return from t to $t+1$ of the monthly equivalent to the annualized three-month US government constant maturity rate, as provided by Bloomberg, and is known at t. It would be more appropriate to take the one-month US government constant maturity rate, but this data series is only available starting in 1990. The difference between the two annualized series is negligible.[3] The risk-free asset is an investable asset class by itself and thus the level of interest rates is already considered in the model. This also includes the level of long-term interest rates when combining the risk-free rate and the term spread described below.

The total return series are percentage returns from t to $t + 1$ taken from the CRSP database and are known at $t + 1$. For stocks ($r_{s,t+1}$) this is the total return series of the value-weighted CRSP stock market indexes (NYSE, AMEX, NASDAQ, and ARCA). For bonds ($r_{b,t+1}$), I take the total return series of the CRSP US government bonds series with maturities of 10 years. Long-term government bonds best capture duration risk. Together with the risk-free asset (equivalent to short-term government bills) the government bond yield curve is sufficiently spanned.

The term spread is the difference between the ten-year and three-month US government bond yields. The end-of-month data are monthly averages of daily Bloomberg rates. A high and positive term spread or equivalently a steep interest curve is usually associated with an economy before the peak of its business cycle. This is the time when business activity grows most strongly. As the business cycle matures, the central bank raises target rates to limit the risk of inflation. In this way the term spread can be interpreted as an indicator for economic activity. Investors allocate their funds towards risky assets when they expect economic activity to rise. The term spread, as all other risk factors, is known at t.

Another argument to invest relatively more in risky assets and specifically in stocks is when trust in companies is high. The credit spread serves as a measure of trust and is the difference between corporate bonds with a Moody's rating of BAA and AAA. The data are taken from Federal Reserve Economic Data (FRED) provided by the FED St.Louis and are monthly averages of the daily rates.

Risky assets get unattractive when the financial system is regarded unstable. This is measured by the ted spread, which is the difference between the monthly averages of the three-month interbank rate and the three-month US government rate. The ted spread is also often used as a measure for liquidity in the markets. Usually the USD Libor rate is taken as interbank rate. A drawback of this definition is that the Libor rate only starts in 1986. An alternative is the Eurodollar rate, which is available from 1971. To take advantage of a longer time series, I rely on the three-month certificates of deposit secondary market rate.[4] However, as FRED stopped reporting the series mid-2013 I take the three-month certificate of deposit secondary market rate until December 1985 and thereafter the more appropriate Libor rate.

Stocks are usually regarded attractive when they are cheap relative to their value. Two prominent value measures are the dividend yield and the price–earnings ratio. The dividend yield is the 12-month rolling difference known at t between the CRSP Stock Market Indexes including all distributions and the CRSP Stock Market Indexes excluding dividends. The price–earnings ratio is the Shiller price–earnings ratio downloaded from Professor Shiller's Website.[5] This price–earnings ratio, which is based on publicly available data, is measured for the S&P500 and is taken as proxy for the CRSP stock universe as a similar measure for the CRSP stock universe is not available. As the earnings used in the ratio are updated with a lag, I ignore the reported earnings of the three most recent months prior to t to ensure that I only take advantage of known data at t. An alternative

Table 1 Descriptive statistic data.

	$r_f\%$	$r_s\%$	$r_b\%$	term	credit	ted	DY	PE	$M_s\%$	$M_b\%$	$A_s\%$	$A_b\%$
Min	0.00	−22.54	−6.68	−2.65	0.32	0.13	0.84	6.62	−20.66	−11.73	−43.50	−8.79
25%	0.28	−1.76	−0.66	0.41	0.75	0.30	2.30	13.47	−3.97	−1.79	0.76	1.56
Med	0.42	1.26	0.43	1.60	0.93	0.48	3.05	19.83	−0.28	0.12	12.78	5.27
μ	0.41	0.89	0.60	1.51	1.05	0.69	3.26	19.70	0.00	0.00	10.38	6.74
75%	0.55	3.90	1.88	2.57	1.24	0.87	4.17	24.21	3.66	1.74	20.55	11.67
Max	1.21	16.56	10.00	4.42	3.38	4.70	7.36	44.66	27.53	11.48	70.39	37.67
σ	0.25	4.50	2.28	1.28	0.46	0.59	1.32	8.29	6.11	3.10	16.53	8.23

r_f is the risk-free rate, r_s the total stock return, r_b the total bond return, term denotes the term spread, credit denotes the credit spread, ted denotes the ted spread, DY denotes the dividend yield, PE denotes the price–earnings ratio, M_s denotes the past price measure for stocks and M_b denotes the past price measure for bonds, specifically $r_{i,(t+1)-1} - r_{i,(t+1)-2}$, A_s denotes the past price measure for stocks, and A_b denotes the past price measure for bonds where the cumulative return over the last 12 months excluding the last month is considered. The sample period is from December 1964 to December 2014 (end of month data).

to the price–earnings ratio would be the book–value ratio but this figure is not available before 1990.

As discussed in Section 2 and as indicated by the literature the weight of the asset class could potentially depend on the past returns of the asset class. Therefore I add a measure of past returns to the estimation defined as the difference between the return of the last month and the month before: $(r_{i,(t+1)-1} - r_{i,(t+1)-2})$. This can be interpreted as short momentum. As alternative specification I also consider whether the past 12-month cumulative raw return on the asset class skipping the last month's return (see Asness *et al.*, 2013) is more suitable as proxy for momentum. The application of momentum in this study can be interpreted cross-sectionally but also in a time series perspective. First, when taking more exposure in stocks, less exposure in bonds might be taken (and vice versa). This relates to a cross-sectional perspective as in Asness *et al.* (2013). However, momentum might also be interpreted as increasing an asset class' attractiveness by itself or it could be compared with storing the money in the risk-free asset. This perspective is more in line with time series momentum as in Moskowitz *et al.* (2011). Those effects could be interlinked.

Furthermore, a constant is added to the model. Table 1 shows descriptive statistics of the data in-sample and out-of-sample.

5 Results

There are two conditions which are key to the success of a model: First, the coefficients of the model should be meaningful and statistically significant. Secondly, the performance should be economically significant. I start with the discussion of the statistical perspective of the model before turning to the performance.

5.1 Statistical perspective

Table 2 shows the results of the estimation for various models with risk aversion $\rho = 5$. There are two sets of coefficients which are jointly estimated. The upper set in Table 2 refers to stocks, the lower set refers to bonds. Only with momentum in models ending with '.1' the coefficient for momentum is estimated solely for the specific asset class, i.e., momentum for stocks is only estimated for stocks and does not show up in the set of dependent variables of bonds. For completeness I also consider a constant in models (1) to (3). In total there are 15 specifications.

Table 2 Coefficients and significance levels.

		(1)	(1.1)	(2)	(2.1)	(3)	(4)	(4.1)	(5)	(5.1)	(6)	(7)	(8)	(9)	(10)	(11)
Stocks	c	0.07	0.08	0.07	0.10	0.15										
		(0.97)	*(0.96)*	*(0.98)*	*(0.97)*	*(0.94)*										
	term	0.19	0.19	0.18	0.17	0.17	0.20	0.20	0.17	0.16	0.17	0.19	0.28	0.13	0.09	
		(0.35)	*(0.36)*	*(0.46)*	*(0.46)*	*(0.43)*	*(0.29)*	*(0.29)*	*(0.38)*	*(0.39)*	*(0.37)*	*(0.21)*	*(0.10)*	*(0.50)*	*(0.64)*	
	credit	0.02	0.02	0.04	0.11	0.09	0.01	0.01	0.06	0.13	0.11		−0.26	0.52	0.29	0.38
		(0.96)	*(0.95)*	*(0.94)*	*(0.84)*	*(0.86)*	*(0.96)*	*(0.95)*	*(0.92)*	*(0.81)*	*(0.82)*		*(0.53)*	*(0.24)*	*(0.53)*	*(0.35)*
	ted	−0.56	−0.62*	−0.50	−0.60	−0.75**	−0.61**	−0.673**	−0.54*	−0.67**	−0.77	−0.81***		−0.74**	−1.05***	−0.81***
		(0.12)	*(0.09)*	*(0.19)*	*(0.11)*	*(0.04)*	*(0.03)*	*(0.02)*	*(0.10)*	*(0.04)*	*(0.01)*	*(0.00)*		*(0.01)*	*(0.00)*	*(0.00)*
	DY	0.13	0.15	0.08	0.09	0.14	0.16	0.17	0.10	0.13	0.17	0.19*	0.06		0.17	0.15
		(0.56)	*(0.51)*	*(0.85)*	*(0.82)*	*(0.52)*	*(0.21)*	*(0.17)*	*(0.61)*	*(0.50)*	*(0.17)*	*(0.07)*	*(0.63)*		*(0.16)*	*(0.23)*
	PE	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01		−0.01
		(0.79)	*(0.79)*	*(0.79)*	*(0.82)*	*(0.76)*	*(0.54)*	*(0.55)*	*(0.36)*	*(0.51)*	*(0.50)*	*(0.57)*	*(0.33)*	*(0.46)*		*(0.52)*
	M_s	1.88	2.59				2.79	3.39								
		(0.53)	*(0.39)*				*(0.37)*	*(0.26)*								
	M_b	0.25					1.15									
		(0.93)					*(0.86)*									
	A_s			0.50	0.43				0.50	0.16						
				(0.81)	*(0.84)*				*(0.76)*	*(0.92)*						
	A_b			1.91					2.29							
				(0.51)					*(0.44)*							
Bonds	c	−0.11	−0.02	0.00	0.04	0.66										
		(0.98)	*(0.99)*	*(1.00)*	*(0.99)*	*(0.87)*										
	term	0.71*	0.79**	0.73*	0.78**	0.91**	0.77**	0.88**	0.79**	0.86**	0.99**	1.00***	0.69**	1.28***	1.45***	
		(0.06)	*(0.04)*	*(0.08)*	*(0.04)*	*(0.02)*	*(0.04)*	*(0.02)*	*(0.04)*	*(0.03)*	*(0.01)*	*(0.00)*	*(0.04)*	*(0.00)*	*(0.00)*	
	credit	0.25	0.16	0.19	0.16	−0.26	0.24	−0.01	0.20	0.00	−0.34		0.20	−2.35**	−1.97*	0.48
		(0.81)	*(0.88)*	*(0.87)*	*(0.89)*	*(0.78)*	*(0.81)*	*(0.95)*	*(0.87)*	*(0.96)*	*(0.71)*		*(0.74)*	*(0.01)*	*(0.06)*	*(0.53)*
	ted	0.66	0.97	0.74	0.99	1.45	0.94	1.35	1.02	1.23	1.71*	1.96**		2.06**	3.28***	0.91
		(0.48)	*(0.32)*	*(0.46)*	*(0.33)*	*(0.15)*	*(0.29)*	*(0.14)*	*(0.31)*	*(0.22)*	*(0.07)*	*(0.01)*		*(0.03)*	*(0.00)*	*(0.22)*
	DY	−0.46	−0.55	−0.49	−0.55	−0.65	−0.56**	−0.61**	−0.57	−0.56**	−0.60**	−0.73***	−0.30		−0.62**	−0.53**
		(0.29)	*(0.22)*	*(0.56)*	*(0.25)*	*(0.14)*	*(0.02)*	*(0.01)*	*(0.11)*	*(0.03)*	*(0.01)*	*(0.00)*	*(0.14)*		*(0.01)*	*(0.02)*
	PE	0.00	0.00	0.00	−0.01	−0.02	0.00	−0.01	−0.01	−0.01	−0.01	−0.02	−0.01	−0.02		0.03
		(0.97)	*(0.96)*	*(0.97)*	*(0.92)*	*(0.81)*	*(0.87)*	*(0.85)*	*(0.84)*	*(0.78)*	*(0.75)*	*(0.62)*	*(0.85)*	*(0.52)*		*(0.32)*
	M_s	1.79					1.07									
		(0.79)					*(0.86)*									
	M_b	1.28	2.79				2.65	3.71								
		(0.89)	*(0.78)*				*(0.80)*	*(0.71)*								
	A_s			−0.13					0.46							
				(0.97)					*(0.89)*							
	A_b			0.30	0.81				−0.73	0.34						
				(0.95)	*(0.87)*				*(0.89)*	*(0.91)*						

This table shows the average estimated coefficients and in brackets and italics the average significance levels (p-values) for all models with risk aversion parameter $\rho = 5$. The coefficients were estimated with a rolling out-of-sample window of one-year length within the period December 1964 to December 2014 (end of month data), i.e., there are 50 single estimates available for each model. c denotes a constant, term denotes the term spread, credit denotes the credit spread, ted denotes the ted spread, DY denotes the dividend yield, PE denotes the price–earnings ratio, M_s denotes the past price measure for stocks and M_b denotes the past price measure for bonds, specifically $r_{i,(t+1)-1} - r_{i,(t+1)-2}$, A_s denotes the past price measure for stocks, and A_b denotes the past price measure for bonds where the cumulative return over the last 12 months excluding the last month is considered. Each dependent variable is estimated for each asset class (stocks and bonds). *, ** and *** relate to a p-value of ≤ 0.10, ≤ 0.05 and ≤ 0.01.

As I roll the out-of-sample period forward through time to take full advantage of the ergodicity assumption, I end up with a set of 50 different estimates for each coefficient (in the case of a one-year out-of-sample window). Table 2 shows the average coefficient and significance for these 50 different estimates for each coefficient. If the ergodicity assumption were to hold the estimated coefficients should be independent of the in-sample period.

To verify whether this condition is sufficiently met I calculate the standard deviation of the estimated coefficients in Table 3. The average value of Table 2 can be found for comparison underneath the figures in brackets. Apparently, those

Table 3 Standard deviation of coefficients.

		(1)	(1.1)	(2)	(2.1)	(3)	(4)	(4.1)	(5)	(5.1)	(6)	(7)	(8)	(9)	(10)	(11)
Stocks	c	0.02 *(0.07)*	0.03 *(0.08)*	0.02 *(0.07)*	0.03 *(0.10)*	0.04 *(0.15)*										
	term	0.02 *(0.19)*	0.02 *(0.19)*	0.03 *(0.18)*	0.03 *(0.17)*	0.03 *(0.17)*	0.02 *(0.20)*	0.02 *(0.20)*	0.03 *(0.17)*	0.03 *(0.16)*	0.03 *(0.17)*	0.02 *(0.19)*	0.03 *(0.28)*	0.03 *(0.13)*	0.03 *(0.09)*	
	credit	0.03 *(0.02)*	0.03 *(0.02)*	0.03 *(0.04)*	0.03 *(0.11)*	0.04 *(0.09)*	0.03 *(0.01)*	0.04 *(0.01)*	0.03 *(0.06)*	0.04 *(0.13)*	0.05 *(0.11)*		0.06 *(−0.26)*	0.07 *(0.52)*	0.06 *(0.29)*	0.06 *(0.38)*
	ted	0.03 *(−0.56)*	0.03 *(−0.63)*	0.04 *(−0.50)*	0.03 *(−0.60)*	0.03 *(−0.75)*	0.03 *(−0.61)*	0.03 *(−0.67)*	0.04 *(−0.54)*	0.04 *(−0.67)*	0.05 *(−0.77)*	0.04 *(−0.81)*		0.04 *(−0.74)*	0.05 *(−1.05)*	0.06 *(−0.81)*
	DY	0.02 *(0.13)*	0.02 *(0.15)*	0.02 *(0.08)*	0.01 *(0.09)*	0.02 *(0.14)*	0.02 *(0.16)*	0.02 *(0.17)*	0.02 *(0.10)*	0.02 *(0.13)*	0.02 *(0.17)*	0.01 *(0.19)*	0.02 *(0.06)*		0.02 *(0.17)*	0.02 *(0.15)*
	PE	0.00 *(−0.01)*	0.00 *(−0.01)*	0.00 *(−0.01)*	0.00 *(−0.01)*	0.00 *(−0.01)*	0.00 *(−0.01)*	0.00 *(−0.01)*	0.00 *(−0.01)*	0.00 *(−0.01)*	0.00 *(−0.01)*	0.00 *(−0.01)*	0.00 *(−0.01)*	0.00 *(−0.01)*		0.00 *(−0.01)*
	M_s	0.47 *(1.88)*	0.69 *(2.59)*				0.63 *(2.79)*	0.68 *(3.39)*								
	M_b	0.77 *(0.25)*					1.04 *(1.15)*									
	A_s			0.11 *(0.50)*	0.12 *(0.43)*				0.11 *(0.50)*	0.11 *(0.16)*						
	A_b			0.23 *(1.91)*					0.32 *(2.29)*							
Bonds	c	0.03 *(−0.11)*	0.06 *(−0.02)*	0.03 *(0.00)*	0.05 *(0.04)*	0.09 *(0.66)*										
	term	0.06 *(0.71)*	0.07 *(0.79)*	0.06 *(0.72)*	0.07 *(0.78)*	0.07 *(0.91)*	0.06 *(0.77)*	0.07 *(0.88)*	0.07 *(0.79)*	0.07 *(0.86)*	0.07 *(0.99)*	0.07 *(1.00)*	0.08 *(0.69)*	0.07 *(1.28)*	0.07 *(1.45)*	
	credit	0.06 *(0.25)*	0.09 *(0.16)*	0.05 *(0.19)*	0.08 *(0.16)*	0.17 *(−0.26)*	0.07 *(0.24)*	0.12 *(−0.01)*	0.06 *(0.20)*	0.10 *(−0.00)*	0.19 *(−0.34)*		0.19 *(0.20)*	0.14 *(−2.35)*	0.24 *(−1.97)*	0.25 *(0.48)*
	ted	0.10 *(0.66)*	0.11 *(0.97)*	0.09 *(0.74)*	0.11 *(0.99)*	0.11 *(1.45)*	0.10 *(0.94)*	0.11 *(1.35)*	0.11 *(1.02)*	0.12 *(1.23)*	0.11 *(1.71)*	0.15 *(1.96)*		0.14 *(2.06)*	0.13 *(3.28)*	0.12 *(0.91)*
	DY	0.03 *(−0.46)*	0.04 *(−0.55)*	0.04 *(−0.49)*	0.04 *(−0.55)*	0.06 *(−0.65)*	0.04 *(−0.56)*	0.05 *(−0.61)*	0.04 *(−0.57)*	0.05 *(−0.56)*	0.06 *(−0.60)*	0.04 *(−0.73)*	0.05 *(−0.30)*		0.05 *(−0.62)*	0.07 *(−0.53)*
	PE	0.01 *(0.00)*	0.01 *(−0.00)*	0.01 *(−0.00)*	0.01 *(−0.01)*	0.01 *(−0.02)*	0.01 *(−0.00)*	0.01 *(−0.01)*	0.01 *(−0.01)*	0.01 *(−0.01)*	0.01 *(−0.01)*	0.01 *(−0.02)*	0.01 *(−0.01)*	0.01 *(−0.02)*		0.01 *(0.03)*
	M_s	0.62 *(1.79)*					1.00 *(1.07)*									
	M_b	0.95 *(1.28)*	1.69 *(2.79)*				1.82 *(2.65)*	1.34 *(3.71)*								
	A_s			0.17 *(−0.13)*					0.25 *(0.46)*							
	A_b			0.25 *(0.30)*	0.42 *(0.81)*				0.36 *(−0.73)*	0.64 *(0.34)*						

This table shows the standard deviation of the estimated coefficients and in brackets and italics the average estimated coefficients for comparison (as depicted in Table 2) for all models with risk aversion parameter $\rho = 5$. The coefficients were estimated with an rolling out-of-sample window of one-year length within the period December 1964 to December 2014 (end of month data), i.e., there are 50 single estimates available for each model. c denotes a constant, term denotes the term spread, credit denotes the credit spread, ted denotes the ted spread, DY denotes the dividend yield, PE denotes the price–earnings ratio, M_s denotes the past price measure for stocks and M_b denotes the past price measure for bonds, specifically $r_{i,(t+1)-1} - r_{i,(t+1)-2}$, A_s denotes the past price measure for stocks, and A_b denotes the past price measure for bonds where the cumulative return over the last 12 months excluding the last month is considered. Each dependent variable is estimated for each asset class (stocks and bonds).

coefficients which are significant do not show a large standard deviation, which is about 10% of the average coefficient value. Looking at the standard deviation of the different estimates of the corresponding p-levels in Table 4, one can see that those coefficients which are significant are practically significant in any case and those which are not significant remain insignificant.

A graphical perspective confirms this observation by and large: Figure 1 looks at the 50 estimates for the coefficients for model (10) (upper subfigure) and for the corresponding p-values (lower subfigure). I chose model (10) as this is the model with the highest level of significance, which in the context of this paper means the model with the largest number of significant coefficients and highest

Table 4 Standard deviation of significance levels.

		(1)	(1.1)	(2)	(2.1)	(3)	(4)	(4.1)	(5)	(5.1)	(6)	(7)	(8)	(9)	(10)	(11)
Stocks	c	0.01 *(0.97)*	0.01 *(0.96)*	0.01 *(0.98)*	0.01 *(0.97)*	0.02 *(0.94)*										
	term	0.05 *(0.35)*	0.05 *(0.36)*	0.06 *(0.46)*	0.06 *(0.46)*	0.07 *(0.43)*	0.05 *(0.29)*	0.06 *(0.29)*	0.06 *(0.38)*	0.07 *(0.39)*	0.07 *(0.37)*	0.05 *(0.21)*	0.03 *(0.10)*	0.11 *(0.50)*	0.10 *(0.64)*	
	credit	0.03 *(0.96)*	0.03 *(0.95)*	0.03 *(0.94)*	0.04 *(0.84)*	0.04 *(0.86)*	0.03 *(0.96)*	0.05 *(0.95)*	0.03 *(0.92)*	0.05 *(0.81)*	0.05 *(0.82)*		0.09 *(0.53)*	0.06 *(0.24)*	0.07 *(0.53)*	0.05 *(0.35)*
	ted	0.04 *(0.12)*	0.03 *(0.09)*	0.05 *(0.19)*	0.03 *(0.11)*	0.02 *(0.04)*	0.03 *(0.03)*	0.01 *(0.02)*	0.04 *(0.10)*	0.02 *(0.04)*	0.01 *(0.01)*	0.00 *(0.00)*		0.01 *(0.01)*	0.00 *(0.00)*	0.00 *(0.00)*
	DY	0.05 *(0.56)*	0.05 *(0.51)*	0.03 *(0.85)*	0.03 *(0.82)*	0.05 *(0.52)*	0.06 *(0.21)*	0.06 *(0.17)*	0.07 *(0.61)*	0.06 *(0.50)*	0.05 *(0.17)*	0.03 *(0.07)*	0.08 *(0.63)*		0.06 *(0.16)*	0.08 *(0.23)*
	PE	0.04 *(0.79)*	0.04 *(0.79)*	0.03 *(0.79)*	0.03 *(0.82)*	0.04 *(0.76)*	0.10 *(0.54)*	0.10 *(0.55)*	0.07 *(0.36)*	0.09 *(0.51)*	0.10 *(0.50)*	0.10 *(0.57)*	0.07 *(0.33)*	0.10 *(0.46)*		0.10 *(0.52)*
	M_s	0.10 *(0.53)*	0.11 *(0.39)*				0.10 *(0.37)*	0.10 *(0.26)*								
	M_b	0.06 *(0.93)*					0.11 *(0.86)*									
	A_s			0.04 *(0.81)*	0.04 *(0.84)*				0.05 *(0.76)*	0.05 *(0.92)*						
	A_b			0.05 *(0.51)*					0.07 *(0.44)*							
Bonds	c	0.01 *(0.98)*	0.01 *(0.99)*	0.00 *(1.00)*	0.01 *(0.99)*	0.02 *(0.87)*										
	term	0.02 *(0.06)*	0.01 *(0.04)*	0.02 *(0.08)*	0.01 *(0.04)*	0.01 *(0.02)*	0.01 *(0.04)*	0.01 *(0.02)*	0.01 *(0.04)*	0.01 *(0.03)*	0.00 *(0.01)*	0.00 *(0.00)*	0.01 *(0.04)*	0.00 *(0.00)*	0.00 *(0.00)*	
	credit	0.04 *(0.81)*	0.06 *(0.88)*	0.03 *(0.87)*	0.05 *(0.89)*	0.07 *(0.78)*	0.05 *(0.81)*	0.06 *(0.95)*	0.04 *(0.87)*	0.05 *(0.96)*	0.08 *(0.71)*		0.12 *(0.74)*	0.01 *(0.01)*	0.08 *(0.06)*	0.12 *(0.53)*
	ted	0.06 *(0.48)*	0.05 *(0.32)*	0.06 *(0.46)*	0.05 *(0.33)*	0.03 *(0.15)*	0.05 *(0.29)*	0.03 *(0.14)*	0.05 *(0.31)*	0.04 *(0.22)*	0.02 *(0.07)*	0.01 *(0.01)*		0.01 *(0.03)*	0.00 *(0.00)*	0.06 *(0.22)*
	DY	0.04 *(0.29)*	0.03 *(0.22)*	0.03 *(0.56)*	0.03 *(0.25)*	0.03 *(0.14)*	0.01 *(0.02)*	0.01 *(0.01)*	0.02 *(0.11)*	0.01 *(0.03)*	0.01 *(0.01)*	0.00 *(0.00)*	0.05 *(0.14)*		0.00 *(0.01)*	0.01 *(0.02)*
	PE	0.03 *(0.97)*	0.03 *(0.96)*	0.03 *(0.97)*	0.04 *(0.92)*	0.04 *(0.81)*	0.11 *(0.87)*	0.11 *(0.85)*	0.10 *(0.84)*	0.11 *(0.78)*	0.10 *(0.75)*	0.09 *(0.62)*	0.11 *(0.85)*	0.12 *(0.52)*		0.08 *(0.32)*
	M_s	0.07 *(0.79)*					0.10 *(0.86)*									
	M_b	0.06 *(0.89)*	0.10 *(0.78)*				0.12 *(0.80)*	0.09 *(0.71)*								
	A_s			0.03 *(0.97)*					0.05 *(0.89)*							
	A_b			0.03 *(0.95)*	0.05 *(0.87)*				0.06 *(0.89)*	0.07 *(0.91)*						

This table shows the standard deviation of the significance levels of the coefficients estimated and in brackets and italics the average significance levels for comparison (as depicted in Table 2) for all models with risk aversion parameter $\rho = 5$. The coefficients were estimated with an rolling out-of-sample window of one-year length within the period December 1964 to December 2014 (end of month data), i.e., there are 50 single estimates available for each model. c denotes a constant, term denotes the term spread, credit denotes the credit spread, ted denotes the ted spread, DY denotes the dividend yield, PE denotes the price–earnings ratio, M_s denotes the past price measure for stocks and M_b denotes the past price measure for bonds, specifically $r_{i,(t+1)-1} - r_{i,(t+1)-2}$, A_s denotes the past price measure for stocks, and A_b denotes the past price measure for bonds where the cumulative return over the last 12 months excluding the last month is considered. Each dependent variable is estimated for each asset class (stocks and bonds).

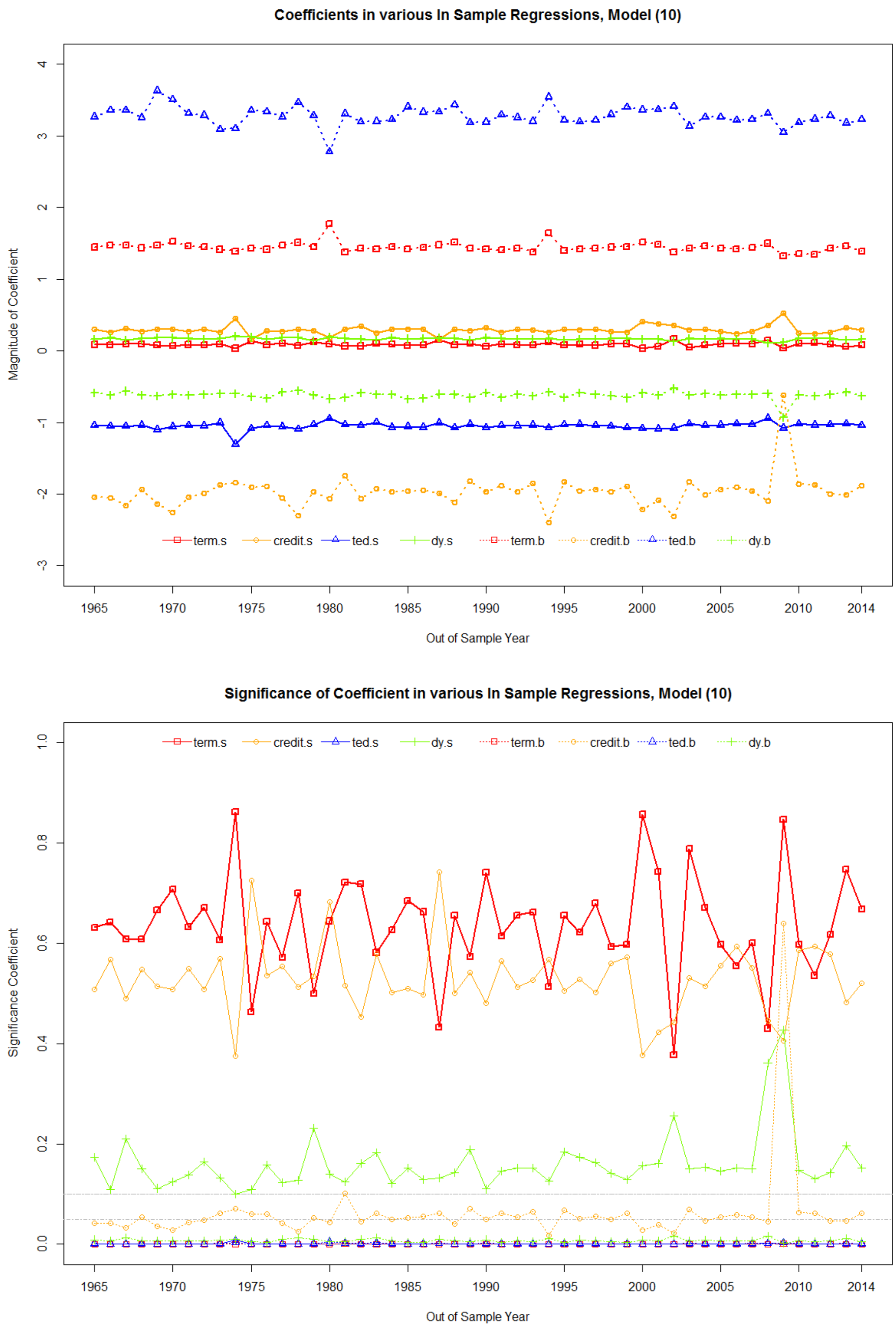

Figure 1 Estimated coefficients and significance (p-value) of model (10).

The lower horizontal gray line in the lower figure denotes the 0.05% p-value and the upper gray line denotes the 0.10% p-value.

significance levels. There is pretty much only one exceptional year with one variable: 2009, credit. This is the year where markets bottomed during the financial crisis, though the other variables are not affected. Thus there is some variation which I will discuss in Section 8, but the coefficients seem to be sufficiently stable over time.

Turning back to Table 2, the coefficients of the asset class bonds outweigh their stock-peers in terms of significance. The constant and momentum factors are not statistically significant in any specification. Moreover, the fewer dependent variables are in the model, the larger is the magnitude of the remaining variables which, partially, turn significant. This supports a parsimonious approach. I discuss the coefficients in more detail within the next paragraphs.

The term spread has a positive sign and is significant regardless of the specification for the asset class bonds. Recall that this asset class captures the performance of US treasury bonds with maturities of 10 years. Hence, long-term bonds are more attractive when long-term interest rates are high relative to short-term interest rates. Furthermore, if short-term interest rates are low this effect could feed back to the long end of the interest rate curve, bringing down yields and increasing the total return for long-term bond investors. The risk factor is not significant to stocks but has a positive sign. Therefore stocks are expected to enjoy a good return when the term spread is large. This is usually the case before the business cycle reaches its maximum.

The credit spread is positive for the asset class stocks regardless of the specification except for model (8). This is a bit counterintuitive as one would expect favorable conditions to invest in stocks when trust in companies is high and thus the credit spread is low. However, this could also be a contrarian argument, i.e., invest in stocks when markets are in higher tension and prices offer a higher risk premium. The coefficient is not of significance and the magnitude is relatively small.

Results are more mixed when looking at the asset class bonds. In most specifications credit is insignificant and is either slightly negative or slightly positive. However, when looking at models (9) and (10) there is a strong and negative relation indicating that investors would avoid bonds when the credit spread is large. Now, at the same time the coefficient for credit is relatively large when looking at stocks and comparing with the other specifications. Considering the magnitude of the coefficients the results lead to a contrarian argument: Investors would divest from long-term bonds and invest in the risk-free asset but would also buy stocks which might offer higher value during these times. This effect is more pronounced as risk aversion is lower (see Table 10 for more specific results on model (10)). The observation is also supported when comparing model (6) with models (9) and (10), where both value measures for stocks are included: The credit effect vanishes almost completely for bonds and is also largely reduced for stocks, i.e. credit picks up the interlinked value arguments of model (6) in models (9) and (10).

Another important risk factor is the ted spread. When the financial system is regarded unstable, i.e., when the ted spread is large, investors tend to avoid stocks and favor bonds: The coefficient for the ted spread is positive for bonds and negative for stocks. Hence, investors flee from risky assets into safer assets. For stocks this relationship is significant for almost all specifications and for bonds for parsimonious model specifications.

Regarding the value measures, the sign of the coefficients points out a relative valuation relation between stocks and bonds. When the dividend yield is high bonds become less attractive compared to stocks. The sign of the coefficient is negative for bonds and positive for stocks. When interpreting the dividend yield of stocks and the coupon of bonds as carry measure, as described by Koijen *et al.* (2013), a large carry in stocks relative to bonds makes bonds less attractive. The dividend yield is significant in most models for bonds unless a constant is taken into account, though it is only significant with model (7) with stocks.

Similar to the dividend yield the sign of the price–earnings ratio for bonds indicates that bonds are more attractive when stocks are expensive. For stocks the sign of the price–earnings ratio is negative, implying that it is better to buy stocks when they are cheap. The price–earnings ratio is not of statistical significance to the asset class bonds and only statistically significant at the 5% level for stocks when risk aversion is 20 and no constant is included in the model (not shown in the table). Concerning the magnitude of the coefficient the dividend yield is of more importance to the allocation than the price–earnings ratio for both asset classes. The weak statistical, but still economically meaningful impact of value measured in terms of the dividend yield and price–earnings ratio on returns has also been pointed out in the literature (see for example Goetzmann and Jorion, 1993; Campbell and Thompson, 2008).

When considering only asset class specific momentum (models (1.1), (2.1), (4.1), (5.1)), both short-term momentum (M) and long-term momentum (A) of stocks ($_s$) and bonds ($_b$) have a positive impact on the specific asset class, respectively. For stocks the magnitude of the coefficients is somewhat weaker compared to bonds especially when looking at long-term momentum. This can be explained by the lower average monthly return of bonds versus stocks and by the lower standard deviation of these returns (see Table 1). Hence, when stocks perform well (or badly), they perform better (or worse) compared to bonds. Therefore, the momentum coefficient of stocks is multiplied with a larger number than the momentum coefficient of bonds when momentum is large. Comparing the magnitude of short-term and long-term momentum, long-term momentum is of greater relevance when considering the magnitude of the input factors in Table 1.

For short-term momentum the effect is also positive when considering momentum of stocks and bonds in each asset class (models (1) and (4)). When looking at long-term momentum (models (2) and (5)) this is also the case for stocks, but not for bonds. In model (2) long-term stock momentum is negative, whereas long-term bond momentum is positive with the asset class bonds. This relation reverses when looking at model (5), where the constant is dropped.

There is some empirical evidence that the correlation between lagged stock prices to bond prices is negative while the correlation between lagged bond prices to stock prices is positive, as analyzed in Ilmanen (2003). The channel is roughly the following: Falling bond yields imply that the rate at which equities are discounted is also reduced whereas equity weakness might cause (or go hand in hand with) monetary or fiscal easing measures, which in turn causes a bond market rally, which is reflected in model (2) but not supported by model (5).

However, the observations are also consistent with other studies. In their momentum analysis Moskowitz *et al.* (2011) consider futures and regress the monthly excess return of each contract on its own lagged excess return. They find a strong and positive time series predictability for equities up to 12 months but an inconsistent relation for bond futures, where only the first month is highly positively predictable and the months thereafter are either not predictable, or even show a negative sign. Still, the Sharpe Ratio of the bond momentum strategy is around 0.5. Moreover, the performance of cross-asset class momentum of stocks is positive in Asness *et al.* (2013), but the return of bond momentum is negative or neutral for most of the time analyzed. Again, Baltas and Kosowski (2013) find positive Sharpe Ratios for bond and equity futures when looking at time series momentum.

Thus, there are some mixed perspectives with the asset class bonds when looking at long-term momentum of stocks and bonds simultaneously. All momentum coefficients are insignificant regardless of the specification which supports the assumption that macro and value fundamentals are of higher importance and would also be more consistent with some form of market efficiency.

The question is, whether these results also translate into economic significance, which I discuss in the next paragraphs.

5.2 Performance

Table 5 shows the performance of all models for the overlapping in-sample windows and the non-overlapping, 50-year-long out-of-sample period when risk aversion is set to $\rho = 5$. I report the averaged and annualized mean return ($\bar{r}$), standard deviation (σ_r), Sharpe Ratio (SR), Information Ratio relative to a standard-mean–variance optimization (IR_{MV}) and 60/40 allocation ($IR_{60/40}$), Jensen's Alpha relative to a standard-mean–variance optimization (JA_{MV}) and 60/40 allocation ($JA_{60/40}$), and Certainty Equivalent (CE). Although Jensen's Alpha is not fully appropriate to be used in an asset allocation context, I report it in this study as it is widely regarded by practitioners. The results for Jensen's Alpha are significant at least at the 5% level in all cases (not shown in the table). The Certainty

Table 5 Performance results.

	In-sample								Out-of-sample							
	$\bar{r}$	σ_r	SR	IR_{MV}	$IR_{60/40}$	JA_{MV}	$JA_{60/40}$	CE	$\bar{r}$	σ_r	SR	IR_{MV}	$IR_{60/40}$	JA_{MV}	$JA_{60/40}$	CE
(1)	19.7%	17.7%	0.83	0.25	0.67	8.6%	11.0%	12.8%	15.3%	17.8%	0.72	0.23	0.61	7.5%	7.0%	7.4%
(1.1)	21.3%	19.1%	0.86	0.32	0.70	10.0%	12.5%	13.8%	16.8%	19.2%	0.77	0.30	0.66	8.9%	8.5%	8.2%
(2)	19.6%	18.2%	0.80	0.24	0.65	8.3%	10.7%	12.6%	15.2%	18.6%	0.73	0.21	0.58	7.3%	6.9%	7.1%
(2.1)	19.8%	18.9%	0.79	0.24	0.62	8.5%	11.0%	12.5%	15.6%	19.3%	0.71	0.22	0.57	7.8%	7.4%	7.2%
(3)	21.6%	20.6%	0.81	0.31	0.66	10.3%	12.7%	13.4%	17.4%	21.6%	0.73	0.32	0.65	9.6%	9.1%	7.1%
(4)	21.3%	19.2%	0.85	0.32	0.70	10.0%	12.5%	13.6%	17.0%	19.3%	0.76	0.29	0.64	9.1%	8.7%	8.6%
(4.1)	22.7%	20.7%	0.86	0.37	0.72	11.3%	13.9%	14.4%	18.4%	21.3%	0.79	0.36	0.69	10.6%	10.1%	8.1%
(5)	20.5%	19.2%	0.81	0.28	0.66	9.1%	11.7%	13.0%	16.2%	19.8%	0.76	0.25	0.64	8.3%	7.9%	7.3%
(5.1)	20.7%	19.7%	0.80	0.27	0.64	9.4%	11.8%	12.9%	16.4%	20.5%	0.71	0.25	0.59	8.6%	8.1%	7.1%
(6)	22.1%	21.5%	0.80	0.32	0.65	10.7%	13.1%	13.5%	18.2%	23.0%	0.72	0.31	0.64	10.3%	9.8%	6.7%
(7)	23.0%	23.6%	0.76	0.33	0.62	11.5%	14.1%	13.3%	19.8%	24.3%	0.74	0.35	0.64	11.8%	11.4%	8.6%
(8)	15.7%	15.0%	0.72	0.01	0.52	5.3%	6.9%	10.3%	11.6%	16.9%	0.49	−0.09	0.33	4.1%	3.4%	4.2%
(9)	20.6%	19.7%	0.79	0.24	0.64	10.7%	11.6%	12.7%	17.0%	20.1%	0.66	0.24	0.49	10.0%	8.6%	8.7%
(10)	25.7%	26.5%	0.78	0.41	0.67	13.4%	16.3%	14.0%	22.9%	28.0%	0.82	0.47	0.71	14.6%	14.2%	8.7%
(11)	15.5%	15.5%	0.68	0.00	0.46	6.9%	7.2%	10.2%	12.3%	18.5%	0.68	−0.06	0.13	5.8%	4.5%	2.7%
MV	15.5%	16.1%	0.66					8.9%	11.1%	17.3%	0.56					3.5%
20/80	7.9%	7.4%	0.40					6.5%	7.9%	7.4%	0.54					6.6%
50/50	8.9%	9.2%	0.43					6.8%	8.9%	9.2%	0.66					7.0%
60/40	9.3%	10.2%	0.42					6.6%	9.3%	10.2%	0.66					6.9%
80/20	10.0%	12.8%	0.39					5.8%	10.0%	12.8%	0.65					6.2%

This table shows the averaged and annualized mean return ($\bar{r}$), standard deviation (σ_r), Sharpe Ratio (SR), Information Ratio relative to a standard-mean–variance optimization (IR_{MV}) and 60/40 allocation ($IR_{60/40}$), Jensen's Alpha relative to a standard-mean–variance optimization (JA_{MV}) and 60/40 allocation ($JA_{60/40}$), and Certainty Equivalent (CE) for the in-sample and out-of-sample period of all estimated models with risk aversion $\rho = 5$ with a one-year rolling out-of-sample window of one-year length within the period December 1964 to December 2014 (end of month data), i.e., there are 50 single estimates available for each model. The benchmark models are a standard mean–variance optimization (MV) with matching risk aversion coefficient $\rho = 5$ with a 10-year window for estimating the current month's optimal allocation, and a time static 20% (20/80), 50% (50/50), 60% (60/40), and 80% (80/20) stock allocation, where the remainder is invested into bonds.

Equivalent expresses a risk-free rate of return such that an investor values this return equivalently to the expected utility of the return r_p of a risky portfolio: $u(C) = \mathrm{E}(u(r_p))$, where C is the Certainty Equivalent.

To put results into perspective I compare the performance of the model with a standard mean–variance optimization (MV) with matching risk aversion. I use a 10-year window for estimating the correlations to determine the current month's optimal allocation. In case of the in-sample perspective the current month for which the asset weights are calculated is the last month of the mentioned 10-year window. For the out-of-sample performance the current month for which the asset weights are calculated is the first month after the corresponding 10-year window. Moreover I calculate the performance of static portfolios, where 20% (20/80), 50% (50/50), 60% (60/40), and 80% (80/20) are invested in stocks and the remainder in bonds. A static portfolio is a common benchmark to compare portfolios of institutional investors as mentioned for example in Barber and Wang (2013). The difference between the Sharpe Ratio and the Certainty Equivalent of the in-sample panel and the out-of-sample panel is due to averaging. For the in-sample period I observe 50 Sharpe Ratios and 50 Certainty Equivalents calculated of 49 years of return data, of which I take the average. For the out-of-sample period I observe just one Sharpe Ratio and one Certainty Equivalent calculated of 50 years of return data.

Other potential benchmarks suffer from some shortcoming in the context of this study. The standard Merton (1969) problem accounts for consumption as in Campbell and Viceira (1999). I do not consider consumption in this study as the research question is different. Without consumption the problem of Campbell and Viceira (1999) would collapse to the same problem the investor faces in this paper, i.e., optimizing returns, with the same linear structure in the weighting function for the allocation to the risky asset. The difference would be that the return process (of each risky asset) follows an autoregressive process with normally distributed shocks which depends only on investment opportunities. For the widely used asset allocation model of Black and Litterman (1992), the investor has to provide some return expectations. However, I do not account for return expectations in this study. Using this benchmark I would have to impose some structure on return expectations. Without a return prior from an investor Black and Litterman (1992) basically collapses to a mean–variance problem. Thus the mean–variance optimization and static portfolios seem to be the best benchmarks without introducing a number of assumptions to fit the other optimization procedures into the context of this paper.

Looking at Table 5 the outperformance of the models over the static portfolios is sizeable in-sample. The average return and the Sharpe Ratio are about doubled for most specifications. The Information Ratio compared to 60% stock, 40% bond portfolio is in the range 0.5–0.7. Jensen's Alpha with respect to the same portfolio is for most model specifications between 10% and 13%, with two outliers at the lower end (models (8) and (11)) and two outliers at the upper end (models (7) and (10)). Due to space restrictions I do not report the Information Ratio and Jensen's Alpha for the time static portfolios 20/80, 50/50, and 80/20. However, results do not change when considering these benchmarks. The Certainty Equivalent is about double the size when comparing the models with the static portfolio.

The mean–variance model outperforms the static portfolios in-sample but is itself clearly outperformed by all but two model specifications: (8) and (11). In both models the Information Ratio is (practically) 0. Hence, these models do not outperform the mean–variance benchmark with regard to this performance measure. Still, the Sharpe Ratio is larger as is the Certainty Equivalent and Jensen's Alpha are significantly positive. Models (8) and (11) ignore the ted and term spreads, respectively, indicating the importance of these factors. The other specifications show an Information Ratio in the range of 0.25 to 0.35, and hence about half of the Information Ratio with respect to the static portfolio.

Keeping everything else equal, adding a constant reduces the performance when comparing models (1) to (3) with models (4) to (6). The best model in-sample is the one which shows the highest level of significance in the single factors: Model (10) is the best performing model in-sample in terms of return, Information Ratio, Jensen's Alpha, and Certainty Equivalent. The only advantage of a broader model is a lower volatility and thus marginally higher Sharpe Ratio. In conclusion, it pays off to look for statistical significance and to prefer a parsimonious specification.

Turning to the out-of-sample results on the right panel of Table 5, it is apparent that the performance of the models is reduced compared to the in-sample performance. However, all models with the exception of models (8) and (11) outperform the mean–variance and the static portfolios substantially. The Sharpe Ratio is for most models between 0.7 and 0.8. This compares to a Sharpe Ratio of the mean–variance portfolio of 0.56. The Sharpe Ratio of the static portfolios ranges from 0.54 to 0.66, outperforming the mean–variance portfolio in terms of this performance measure. Only model (8) underperforms in terms of the Sharpe Ratio.

In general, the Information Ratios are in the same range for the in-sample and out-of-sample perspective for most specifications. Models (8) and (11) are the negative outliers, which even show negative Investment Ratios with respect to the mean–variance portfolio. The Certainty Equivalents of those two models are almost always outperformed by the benchmarks. This is consistent with the results of the in-sample period and is strong evidence for the importance of the term and ted spreads.

The models which show most statistical significance, models (7) and especially model (10), are the best models out-of-sample. Both models comprise the statistically significant term and ted spreads and achieve a staggering out-of-sample performance of roughly 20% and 23% per annum. Thus I conclude that there is a close link between statistical significance and economic significance.

Figure 2 depicts the performance of the out-of-sample series of model (10) graphically and compares it with the mean–variance and static portfolios. The model performs in all times at least as well as the benchmarks and in most times better, with only one exception: around 1969 the mean–variance portfolio outperforms the other portfolios. Hence at virtually all times the model does a better tactical asset allocation job compared to its peers.

Remarkable are the jumps around the years 1974, 1981, and 1982. Removing these years does not deteriorate the outperformance of model (10) in comparison to the benchmarks. The mean p.a. return would be reduced to 18.8% (before 22.9%) with a volatility of 25.0% (before 28%) and a Sharpe Ratio of 0.57 (before 0.82). Removing the same years from the mean–variance model would result in a similar 11.1% p.a. return, but a higher standard deviation of 16.9% (before 17.3%) and thus a lower Sharpe Ratio of 0.39 (before 0.56). This observation is qualitatively the same for the static portfolios. The return would be in the same range, but with a higher standard deviation, thus reducing the Sharpe Ratio.[6]

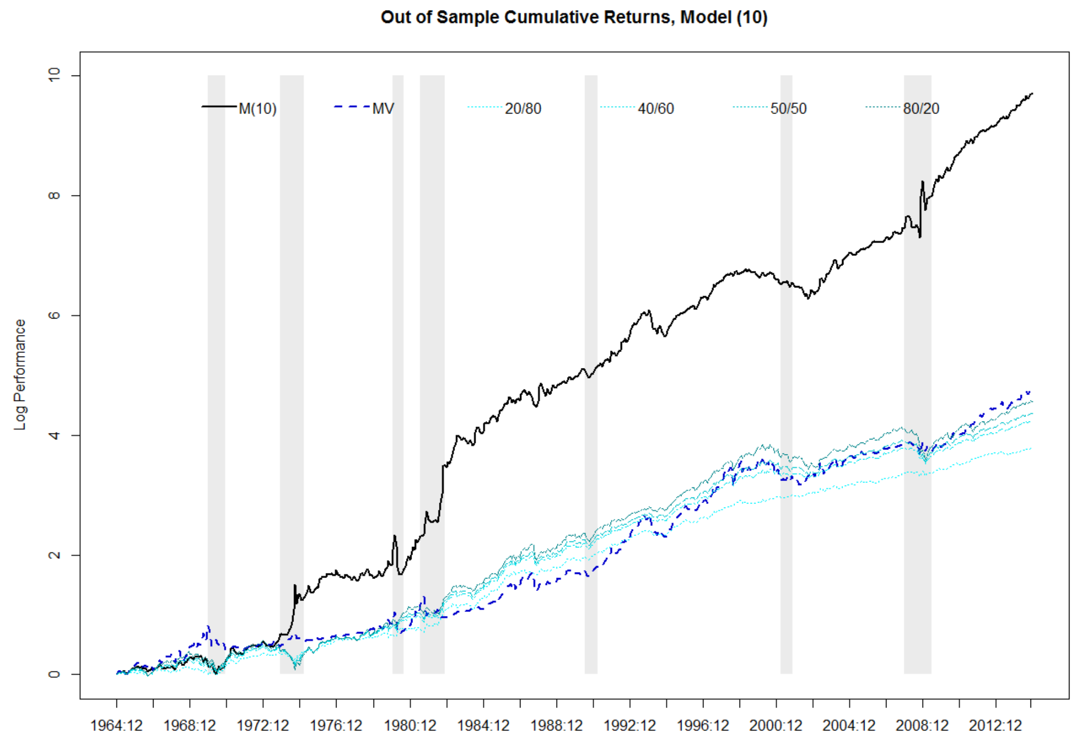

Figure 2 Performance.

The figure shows the rolling out-of-sample cumulative log return of model (10) in comparison to the mean–variance (MV) performance and the performance of and a time static 20% (20/80), 50% (50/50), 60% (60/40), and 80% (80/20) stock allocation, where the remainder is invested in bonds. The gray-shaded areas symbolize US recessions as defined by the Federal Reserve Bank of St.Louis.

Table 6 lists the mean allocation ($\bar{w}$), its standard deviation (σ_w), and the average weight change from period to period (w_c) for the in-sample and out-of-sample periods of both asset classes for all models and the benchmarks. Apparently, except the already mentioned underperforming models (8) and (11), all other models show a higher standard deviation and period-to-period change in weight in both asset classes, in-sample and out-of-sample. One can interpret this as the better performing specifications being more adaptive to the macroeconomic conditions. Although the mean allocation to stocks and bonds is in the same range for the models, the standard deviation of the allocation to bonds is much higher compared to stocks. However, this is consistent with the observations made when looking at the statistical perspective above (Section 5.1): The magnitude of the coefficients and their significance is much higher when looking at the asset class bonds compared to the asset class stocks. In sum this leads to a higher standard deviation of returns and considerably increases the return (Table 5).

Although risk aversion is always set to $\rho = 5$, the models take more leverage on average than the mean–variance specification. In-sample this amounts to roughly 30% to 40% average leverage for the models, compared with 11% average leverage in the case of the mean–variance specifications. Out-of-sample these numbers are 30% to 60%, while the mean–variance specification does on average not lever up its exposure.

The high volatility in the exposure to bonds and the overall dynamic asset allocation is best grasped by looking at Figure 3, which shows the allocation to stocks and bonds over time for model (10). Obviously, the allocation to bonds is much

Table 6 Asset class weights.

	In-sample						Out-of-sample					
	Stocks			Bonds			Stocks			Bonds		
	$\bar{w}$	σ_w	w_c	$\bar{w}$	σ_w	w_c	$\bar{w}$	σ_w	w_c	$\bar{w}$	σ_w	w_c
(1)	71%	52%	19%	68%	106%	30%	72%	49%	19%	66%	109%	29%
(1.1)	73%	56%	24%	67%	118%	32%	74%	54%	24%	66%	123%	32%
(2)	68%	55%	12%	65%	107%	25%	69%	53%	11%	63%	112%	26%
(2.1)	66%	52%	11%	67%	116%	29%	67%	50%	11%	65%	121%	30%
(3)	70%	58%	13%	68%	132%	37%	70%	57%	13%	67%	143%	38%
(4)	73%	57%	27%	67%	118%	34%	74%	55%	26%	65%	122%	34%
(4.1)	75%	61%	31%	67%	131%	40%	76%	60%	31%	66%	138%	40%
(5)	70%	60%	13%	62%	116%	30%	71%	58%	12%	61%	122%	30%
(5.1)	68%	54%	12%	66%	123%	33%	69%	53%	12%	66%	130%	34%
(6)	70%	59%	13%	68%	139%	41%	71%	60%	13%	68%	149%	42%
(7)	71%	64%	14%	64%	155%	46%	71%	64%	14%	63%	160%	46%
(8)	57%	38%	7%	68%	100%	18%	57%	38%	8%	67%	112%	19%
(9)	53%	54%	13%	95%	143%	51%	53%	55%	13%	94%	145%	51%
(10)	76%	66%	17%	86%	196%	69%	77%	68%	18%	87%	202%	69%
(11)	65%	49%	14%	55%	93%	19%	65%	55%	14%	59%	109%	19%
MV	57%	40%	5%	44%	154%	15%	57%	40%	4%	43%	156%	16%
20/80	20%	0%	1%	80%	0%	1%	20%	0%	1%	80%	0%	1%
50/50	50%	0%	2%	50%	0%	1%	50%	0%	2%	50%	0%	1%
60/40	60%	0%	2%	40%	0%	1%	60%	0%	2%	40%	0%	1%
80/20	80%	0%	3%	20%	0%	0%	80%	0%	3%	20%	0%	0%

This table shows the averaged mean allocation ($\bar{w}$), standard deviation (σ_w), and mean month to month change (w_c) of stocks and bonds for the in-sample and out-of-sample period of all estimated models with risk aversion $\rho = 5$ with a one-year rolling out-of-sample window of one-year length within the period December 1964 to December 2014 (end of month data), i.e., there are 50 single estimates available for each model. The benchmark models are a standard mean–variance optimization (MV) with matching risk aversion coefficient $\rho = 5$ with a 10-year window for estimating the current month's optimal allocation, and a time static 20% (20/80), 50% (50/50), 60% (60/40), and 80% (80/20) stock allocation, where the remainder is invested into bonds.

more dynamic than the allocation to stocks and takes on extreme exposure levels, which will not be implementable for practitioners. Thus I consider transaction and leverage costs in the next section.

6 Transaction and leverage costs

Extreme weights and an excessive turnover could easily ruin the best-performing model when including transaction and leverage costs. The real world will confront the investor with these constraints, even in such highly liquid asset classes as a broad US stock market index and 10-year US government bonds. Within this and the next sections I will focus on the statistically most significant model (10).

Methodologically transaction and leverage costs are straightforward to include in the model. For

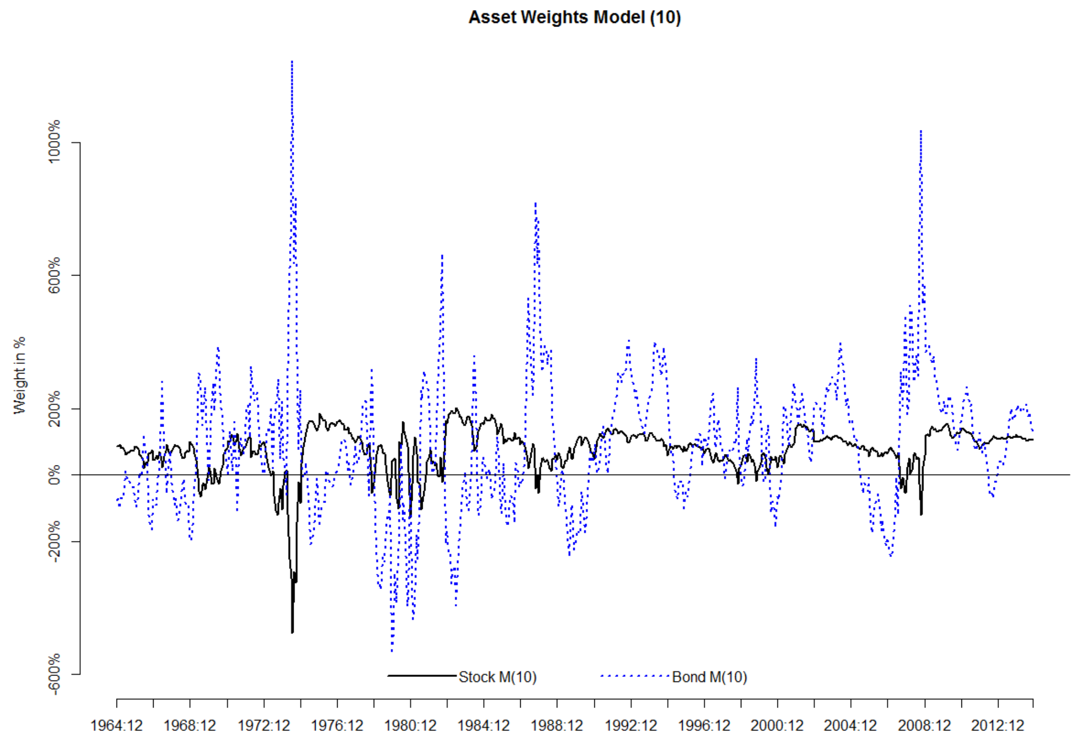

Figure 3 Asset allocation of model (10).

asset class-specific transaction costs, the absolute change in weights is multiplied with the transaction costs rate, υ_i. Suppose at time $t-1$, $w_{i,t-1}$ is allocated to asset i. From $t-1$ to t the asset i increases or decreases in value by $(1+r_{i,t})$. Hence, when no action is taken the weight of asset i changes to $w^h_{i,t} = w_{i,t-1}(1+r_{i,t})$ at time t. Therefore, the portfolio return $r_{p,t+1}$ in Equation (8) changes to

$$\max_{\theta_i} \frac{1}{T}\sum_{t=0}^{T-1}[u(r_{p,t+1})] = \frac{1}{T}\sum_{t=0}^{T-1}\left[u\left(\sum_{i=1}^{N} w_{i,t}r_{i,t+1} + w_{r_f,t}r_{f,t+1} - \upsilon_i|w_{i,t} - w^h_{i,t}|\right)\right]. \tag{14}$$

Leverage costs, ω, are defined as a premium which is paid on top of the risk-free rate. Therefore the lending and borrowing rates are different. To model leverage costs I use an indicator function: $I_\omega = 1$ if $\sum_{i=1}^{N} w_{i,t} > 1$ and $I_\omega = 0$ else. Thus Equation (8) takes the form

$$\max_{\theta_i} \frac{1}{T}\sum_{t=0}^{T-1}[u(r_{p,t+1})] = \frac{1}{T}\sum_{t=0}^{T-1}\left[u\left(\sum_{i=1}^{N} w_{i,t}r_{i,t+1} + w_{r_f,t}r_{f,t+1} - I_\omega\omega\left(\sum_{i=1}^{N} w_{i,t} - 1\right)\right)\right]. \tag{15}$$

Both approaches generate kinks in the first-order conditions and the covariance matrix. However, as the asymptotic covariance matrix is calculated numerically, the adoption of transaction and leverage costs is without risk for calculating significant values.

Table 7 Performance results for model (10) with transaction and leverage costs.

	Parameters		In-sample									Out-of-sample								
						Stocks			Bonds						Stocks			Bonds		
	υ	ω	$\bar{r}$	σ_r	SR	$\bar{w}$	σ_w	w_c	$\bar{w}$	σ_w	w_c	$\bar{r}$	σ_r	SR	$\bar{w}$	σ_w	w_c	$\bar{w}$	σ_w	w_c
(10)			26.1%	26.8%	0.79	76%	66%	17%	86%	196%	69%	25.7%	32.1%	0.70	77%	68%	18%	87%	202%	69%
(10)	0.1%		23.6%	24.7%	0.76	75%	63%	17%	81%	179%	62%	20.9%	26.4%	0.77	76%	66%	17%	82%	185%	62%
	0.5%		17.2%	18.6%	0.66	72%	53%	15%	61%	119%	37%	14.7%	21.6%	0.61	72%	59%	15%	62%	127%	37%
	1.0%		12.8%	14.3%	0.55	69%	43%	13%	41%	74%	14%	10.3%	19.4%	0.48	68%	55%	14%	42%	89%	15%
		0.1%	20.7%	21.0%	0.75	66%	56%	16%	44%	158%	54%	18.2%	22.5%	0.76	67%	59%	16%	45%	164%	54%
		0.5%	14.0%	13.0%	0.70	57%	43%	14%	2%	79%	24%	12.5%	14.7%	0.73	57%	48%	14%	5%	86%	25%
		1.0%	13.3%	12.3%	0.68	58%	43%	13%	4%	65%	19%	11.7%	13.3%	0.70	58%	45%	13%	5%	68%	19%
MV			15.5%	16.1%	0.66	57%	40%	5%	44%	154%	15%	11.1%	17.3%	0.56	57%	40%	4%	43%	156%	16%
20/80			7.9%	7.4%	0.40	20%	0%	1%	80%	0%	1%	7.9%	7.4%	0.54	20%	0%	1%	80%	0%	1%
50/50			8.9%	9.2%	0.43	50%	0%	2%	50%	0%	1%	8.9%	9.2%	0.66	50%	0%	2%	50%	0%	1%
60/40			9.3%	10.2%	0.42	60%	0%	2%	40%	0%	1%	9.3%	10.2%	0.66	60%	0%	2%	40%	0%	1%
80/20			10.0%	12.8%	0.39	80%	0%	3%	2%	0%	0%	10.0%	12.8%	0.65	80%	0%	3%	20%	0%	0%

This table shows the averaged and annualized mean return ($\bar{r}$), standard deviation (σ_r), Sharpe Ratio (SR), mean allocation ($\bar{w}$), standard deviation (σ_w), and mean month to month change (w_c) of stocks and bonds for the in-sample and out-of-sample period for a variation of different levels of transaction costs and fees of model (10), where υ denotes one-way transaction costs, ω denotes leverage costs above the risk-free rate. The risk aversion ρ is set to 5 with a one-year rolling out-of-sample window of one-year length within the period December 1964 to December 2014 (end of month data), i.e., there are 50 single estimates available for each model. The benchmark models are a standard mean–variance optimization (MV) with matching risk aversion coefficient $\rho = 5$ with a 10-year window for estimating the current month's optimal allocation, and a time static 20% (20/80), 50% (50/50), 60% (60/40), and 80% (80/20) stock allocation, where the remainder is invested in bonds.

Table 7 shows results when adding various levels of one-way transaction costs (υ) and leverage costs (ω) above the risk-free rate. For comparison, the first row depicts the results of model (10) without transaction or leverage costs. To keep the competition fierce and the table concise I compare the results of adding transaction and leverage costs with model (10) with the pure forms of the benchmarks as shown in the lower panel. Statistically, the magnitude of the coefficients reduces in size and turns insignificant (not shown in the table).

In general, transaction and leverage costs decrease the return ($\bar{r}$) and standard deviation of the returns (σ_r). In case of transaction costs the Sharpe Ratio (SR) is reduced considerably, whereas with leverage costs the Sharpe Ratio only declines slightly. In terms of the Sharpe Ratio the model outperforms the benchmarks apart from a high level of one-way transaction costs of 1% in-sample. Out-of-sample the same is true with respect to the mean–variance benchmark. However, most static portfolios outperform the model in terms of the Sharpe Ratio as soon as transaction costs rise above an elevated level of 0.5%. The static benchmarks are still outperformed in terms of average out-of-sample return.

As expected, the turnover per period (w_c) decreases along with the volatility of the exposure (σ_w) in-sample and out-of-sample as transaction costs increase. Moreover, the average allocation is reduced. Figure 4 shows the effect of transaction costs graphically with a one-way fee of 0.5% for any transaction in stocks or bonds for model (10). Especially the turnover of bonds is reduced compared to Figure 3, where no market frictions are assumed.

The effect of considering leverage costs drastically reduces the average exposure ($\bar{w}$) from about 160% in-sample to 110% with $\omega = 0.1\%$

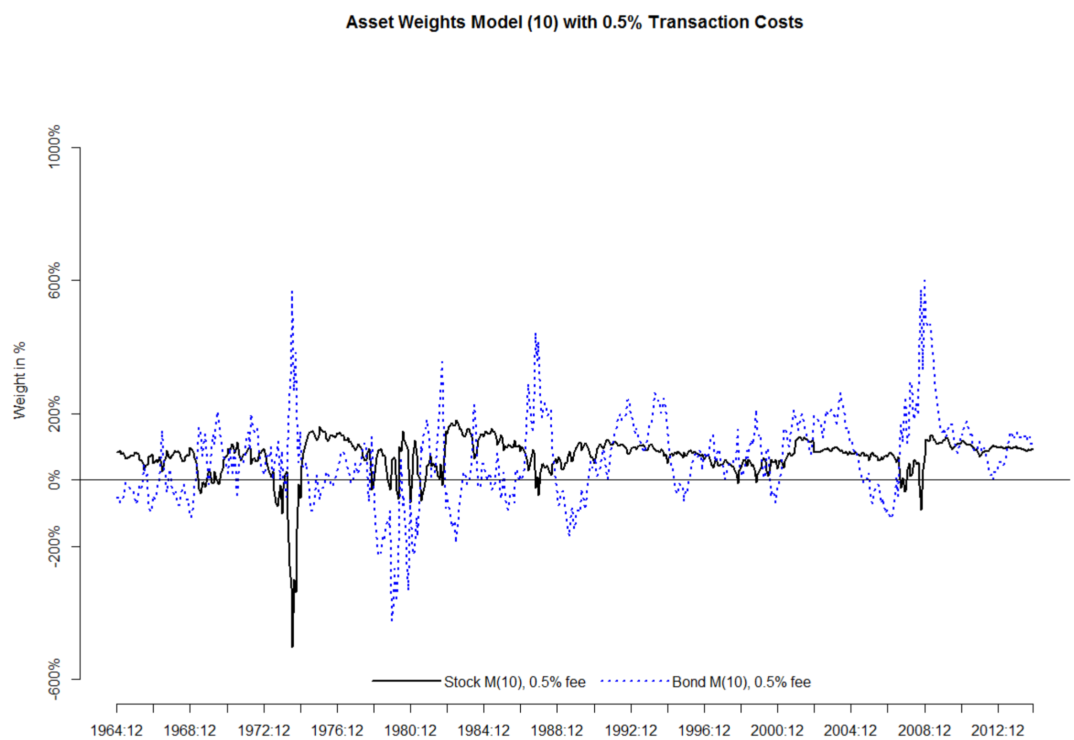

Figure 4 Asset allocation of model (10) with one-way transaction costs of 0.5%.

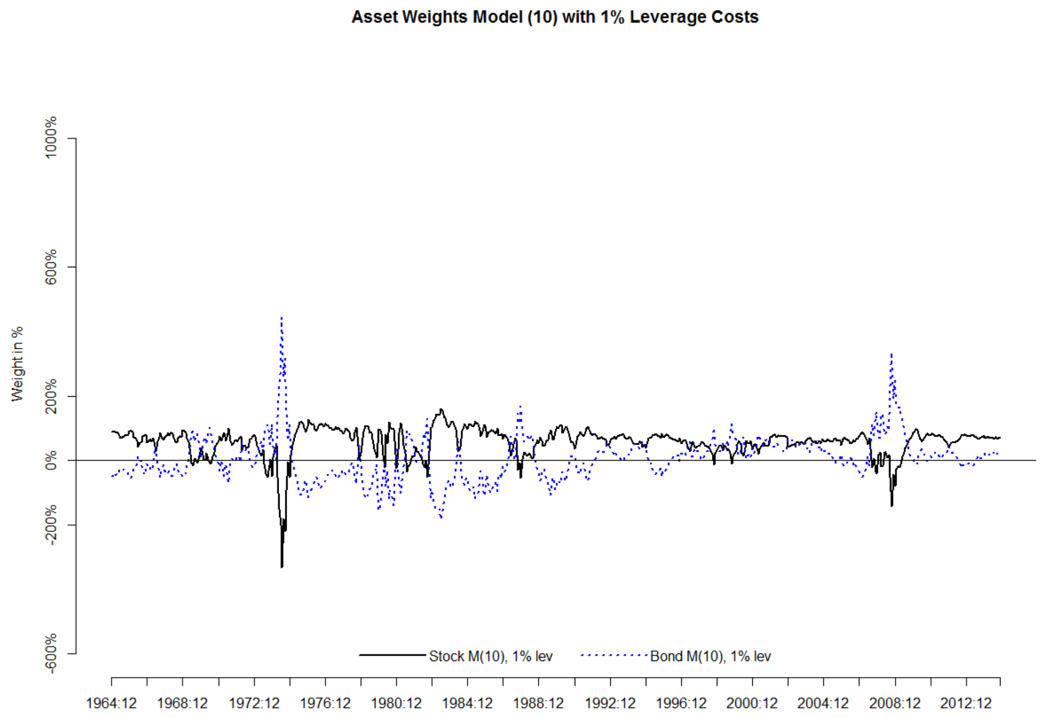

Figure 5 Asset allocation of model (10) with leverage costs of 1%.

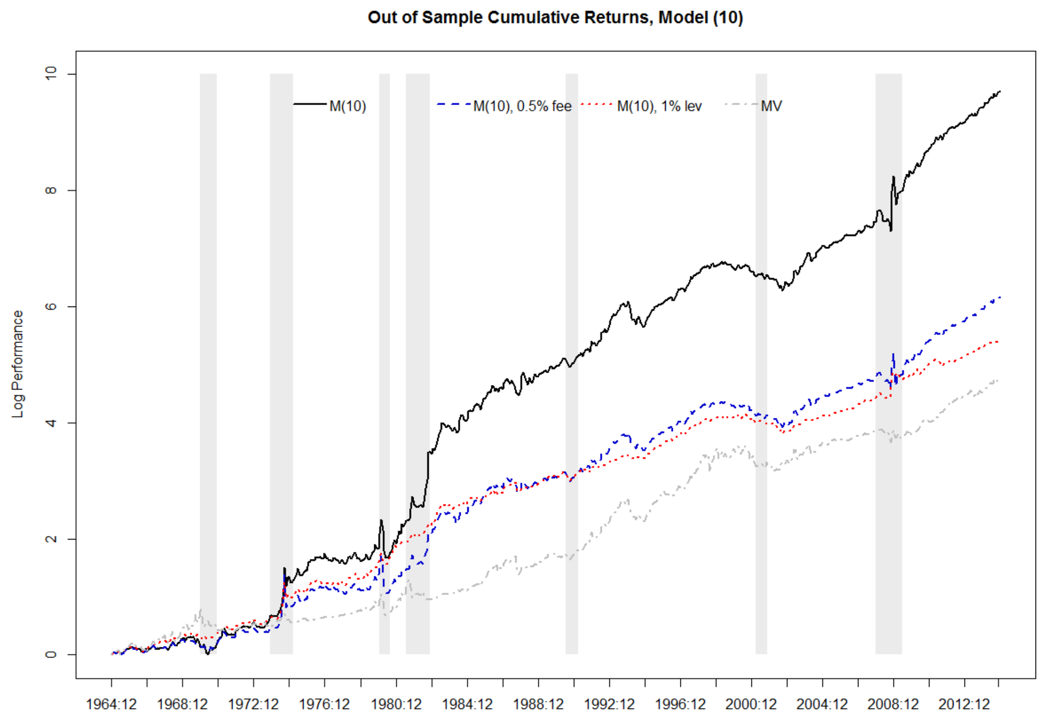

Figure 6 Performance.

The figure shows the rolling out-of-sample cumulative log return of model (10) in comparison to the same model with transaction fees (one-way 0.5%) and leverage costs (1%) to the mean–variance (MV) performance without fees or leverage costs. The gray-shaded areas symbolize US recessions as defined by the Federal Reserve Bank of St.Louis.

in-sample and out-of sample. This is also visible in Figure 5, where $\omega = 1\%$. There are still two clearly visible spikes of positive (bonds) and negative (stocks) exposures as I do not test the combination of leverage and transaction costs nor include a short sale restriction within this paper. Investors could cap these spikes to a suitable level without incurring a huge loss in overall performance.

Figure 6 shows the performance of model (10) without leverage or transaction fees, with 0.5% one-way transaction fees, with 1% leverage costs above the risk-free rate, and for comparison of the mean–variance series without transaction or leverage costs. These costs reduce the performance of the model but it still outperforms its peers which are not subject to market frictions. Thus I conclude that although the risk factors "lose" economic significance, they are still highly relevant in economic terms when including market frictions.

7 More asset classes

As most institutional investors would not limit themselves to stocks and bonds, I discuss whether the approach can be extended to more asset classes in this section by adding the Thomson Reuters Equal Weight Commodity Index to the sample.[7] The index is an investable strategy reflecting the price movement of 17 exchange-traded futures-based commodities. The commodities are equally weighted and the weights are adjusted daily. At least two and up to five contracts are bought for each commodity. The index is subjected to periodic revision by Thomson Reuters. The monthly data was downloaded

Table 8 Adding an additional asset class (commodities) to model (10). Coefficients, significance levels, and performance.

		Panel A				Panel B									
		term	ted	credit	DY			$\bar{r}$	σ_r	SR	IR_{MV}	$IR_{1/N}$	JA_{MV}	$JA_{1/N}$	CE
(10)	Stocks	0.09	0.29	−1.05***	0.17	ins	(10)	25.7%	26.5%	0.78	0.41	0.67	13.4%	16.3%	14.0%
		(0.64)	*(0.53)*	*(0.00)*	*(0.16)*	oos		22.9%	28.0%	0.82	0.47	0.71	14.6%	14.2%	8.7%
$(10)_c$		0.04	0.36	−1.02***	0.22*	ins	$(10)_c$	26.9%	26.7%	0.83	0.26	0.77	12.3%	19.9%	14.9%
		(0.83)	*(0.47)*	*(0.00)*	*(0.09)*	oos		21.9%	29.1%	0.77	0.25	0.81	12.2%	15.4%	4.7%
(10)	Bonds	1.45***	−1.97*	3.28***	−0.62**	ins	MV	15.5%	16.1%	0.66					8.9%
		(0.00)	*(0.06)*	*(0.00)*	*(0.01)*	oos		11.1%	17.3%	0.56					3.5%
$(10)_c$		1.42***	−1.65	3.07***	−0.67***	ins	MV_c	20.3%	17.5%	0.88					12.4%
		(0.00)	*(0.12)*	*(0.00)*	*(0.00)*	oos		14.0%	18.7%	0.53					5.4%
(10)	Comm.					ins	$1/N$	8.9%	9.2%	0.43					6.8%
						oos		8.9%	9.2%	0.66					7.0%
$(10)_c$		0.10	−0.01	−0.19	−0.18	ins	$1/N_c$	7.2%	7.7%	0.29					5.7%
		(0.67)	*(0.90)*	*(0.56)*	*(0.27)*	oos		7.2%	7.7%	0.50					5.8%

This table shows the average estimated coefficients in Panel A and in brackets and italics the average significance levels (p-values) for model (10) without commodities and with commodities (subscript c). Panel B shows the averaged and annualized mean return ($\bar{r}$), standard deviation (σ_r), Sharpe Ratio (SR), Information Ratio relative to a standard-mean–variance optimization (IR_{MV}) and $1/N$ allocation ($IR_{1/N}$), Jensen's Alpha relative to a standard-mean–variance optimization (JA_{MV}) and $1/N$ allocation ($JA_{1/N}$), and Certainty Equivalent (CE) for the in-sample (ins) and out-of-sample (oos) period of model (10) without and with commodities. The model is estimated with risk aversion $\rho = 5$ with a one-year rolling out-of-sample window of one-year length within the period December 1964 to December 2014 (end of month data), i.e., there are 50 single estimates available for each model. The benchmark models are a standard mean–variance optimization (MV) with matching risk aversion coefficient $\rho = 5$ with a 10-year (or data-dependent shorter) window for estimating the current month's optimal allocation, and a time static $1/N$ allocation.

from Bloomberg and is available since October 1956.[8] This is also the reason for choosing this index, as no other available alternative asset class total return index was spanning the same 50-year time frame. Most other alternative asset class indices are hardly available before the 1990s.

Adding another asset class is technically trivial. I again take model (10) as the base model to which I add the commodity index. Panel A in Table 8 shows the change of the coefficients and significance. Qualitatively and statistically there is no substantial change when comparing the results with those of stocks and bonds. Apparently, no risk factor is significant for the asset class commodities. Thus, it is no surprise that in Panel B the addition of this asset class does not improve the performance of model (10) substantially. In fact, the improvement in performance in the mean–variance model is larger. This stems from diversification gains.[9]

It is important to note that the risk factors analyzed in this study are US-based as are the asset classes stocks and bonds. However, the commodity index is on a global scale. Thus, it could either be the wrong risk factors in the approach for commodities or, what is more likely, a global term spread, credit spread, ted spread, and dividend yield combined with a global stock and bond index would provide a more suitable picture. This, however, would require a substantially larger data set, which is beyond the scope (and data accessibility) of this paper. Institutional investors who have access via their trading partners to proprietary alternative asset indices can more easily expand the approach and adapt it to their needs.

8 Robustness

In this section I discuss the robustness of the model along three dimensions: What is the effect of varying the risk aversion ρ? How does a variation of the number of out-of-sample years N_0 influence results? And does it make a difference when splitting up the in-sample and out-of-sample period in a standard static way? The last question tackles directly whether the outperformance prevails when not exploiting the ergodicity assumption. I will again focus the analysis on model (10).

Question one, the effect of varying the risk aversion, is answered when looking at Panel A and Panel B in Table 9. Panel A shows results for model (10) and Panel B for the mean–variance and the static portfolios. The results for the static portfolios are independent of risk aversion.

As risk aversion increases, the average return ($\bar{r}$) as well as volatility of returns (σ_r) decrease. Yet, the Sharpe Ratio (SR) virtually does not change. This is just what one would expect with the utility function used in this paper. The model outperforms the mean–variance benchmark in terms of the Information Ratio (IR_{MV}), Jensen's Alpha (JA_{MA}) and Certainty Equivalent (CE). The same is true with the static benchmark portfolio (60/40). Only with a risk aversion as high as $\rho = 20$ the out-of-sample performance of the model is worse in terms of the Investment Ratio and Certainty Equivalent.

Table 10 shows the coefficients and significance for model (10) with various levels of risk aversion. As can be seen in the first four columns, the magnitude of the coefficients decreases as the level of risk aversion increases, though the level of significance is virtually unchanged. A smaller magnitude translates directly into less aggressive allocations towards the risky assets.

The economic effect of varying the number of out-of-sample years N_0 from 1 to 10 is practically non-existent within the in-sample period, as can be seen in Panel A of Table 9. The out-of-sample average return rises with a comparably higher rise in volatility, leading to a lower Sharpe

Table 9 Performance results for model (10).

		Parameters		In-sample								Out-of-sample							
		ρ	N_o	$\bar{r}$	σ_r	SR	IR_{MV}	$IR_{60/40}$	JA_{MV}	$JA_{60/40}$	CE	$\bar{r}$	σ_r	SR	IR_{MV}	$IR_{60/40}$	JA_{MV}	$JA_{60/40}$	CE
Panel A																			
		2	1	57.8%	68.6%	0.77	0.49	0.74	36.9%	42.0%	28.7%	51.2%	71.7%	0.81	0.54	0.77	38.9%	36.8%	12.2%
		5	1	25.7%	26.5%	0.78	0.41	0.67	13.4%	16.3%	14.0%	22.9%	28.0%	0.82	0.47	0.71	14.6%	14.2%	8.7%
		10	1	15.2%	13.0%	0.78	0.40	0.45	6.2%	8.0%	9.2%	13.8%	13.9%	0.82	0.53	0.25	7.0%	6.8%	6.7%
(10)	Rolling	20	1	10.0%	6.8%	0.75	0.40	0.08	2.7%	3.9%	6.6%	9.3%	7.2%	0.81	0.62	−0.37	3.3%	3.3%	5.4%
		5	2	25.8%	26.6%	0.79	0.41	0.67	13.5%	16.5%	14.1%	23.9%	30.5%	0.75	0.50	0.62	15.6%	15.1%	5.0%
		5	5	26.1%	26.8%	0.79	0.42	0.68	13.7%	16.7%	14.3%	25.7%	32.1%	0.70	0.49	0.58	17.6%	16.7%	3.2%
		5	10	26.8%	27.3%	0.81	0.43	0.69	14.1%	17.4%	14.5%	26.8%	32.6%	0.70	0.49	0.57	18.7%	17.5%	−0.6%
Panel B																			
		2		25.7%	32.3%	0.64					15.4%	17.7%	34.5%	0.60					4.6%
		5		15.5%	16.1%	0.65					8.9%	11.1%	17.3%	0.56					3.5%
MV		10		10.4%	8.3%	0.66					6.9%	8.0%	8.9%	0.55					4.0%
	Rolling	20	1	7.7%	4.2%	0.66					5.9%	6.5%	4.5%	0.56					4.5%
20/80		5		7.9%	7.4%	0.40					6.5%	7.9%	7.4%	0.54					6.6%
50/50		5		8.9%	9.2%	0.43					6.8%	8.9%	9.2%	0.66					7.0%
60/40		5		9.3%	10.2%	0.42					6.6%	9.3%	10.2%	0.66					6.9%
20/80		5		10.0%	12.8%	0.39					5.8%	10.0%	12.8%	0.65					6.2%
Panel C																			
			1	25.0%	26.1%	0.76	0.39	0.65	13.1%	15.9%	13.8%	29.7%	13.0%	2.27	0.60	2.45	14.1%	19.8%	25.8%
(10)	Static	5	2	24.5%	25.7%	0.75	0.37	0.64	12.5%	15.6%	13.6%	27.6%	12.4%	2.22	1.59	2.14	17.9%	16.6%	24.0%
			5	22.6%	24.4%	0.70	0.33	0.59	11.7%	14.2%	12.8%	26.9%	12.3%	2.19	0.57	2.55	17.0%	17.8%	23.2%
			10	22.5%	22.0%	0.76	0.30	0.65	11.9%	12.8%	13.2%	34.0%	41.6%	0.78	0.63	0.69	28.5%	26.7%	1.7%
Panel D																			
			1	15.5%	16.1%	0.63					8.6%	11.1%	17.3%	2.33					24.7%
MV			2	10.4%	8.3%	0.63					6.8%	8.0%	8.9%	1.32					12.4%
			5	7.7%	4.2%	0.56					5.9%	6.5%	4.5%	1.89					6.2%
	Static	5	10	15.5%	16.1%	0.58					8.9%	11.1%	17.3%	0.85					13.4%
			1	9.3%	10.3%	0.41					6.6%	10.1%	5.6%	1.81					9.4%
60/40			2	9.2%	10.4%	0.39					6.4%	11.8%	5.8%	2.03					11.0%
			5	9.1%	10.5%	0.34					6.3%	10.9%	7.1%	1.53					9.7%
			10	9.7%	10.6%	0.37					6.9%	7.4%	8.9%	0.67					5.3%

This table shows in Panel A the averaged and annualized mean return ($\bar{r}$), standard deviation (σ_r), Sharpe Ratio (SR), Information Ratio relative to a standard-mean–variance optimization (IR_{MV}) and 60/40 allocation ($IR_{60/40}$), Jensen's Alpha relative to a standard-mean–variance optimization (JA_{MV}) and 60/40 allocation ($JA_{60/40}$), and Certainty Equivalent (CE) for the in-sample and out-of-sample period for a variation in the risk aversion parameter ρ, and the number of years in the rolling out-of-sample window N_o within the period December 1964 to December 2014 (end of month data). Panel B shows results for model (10) with a static in-sample and out-of-sample period, i.e., the last number of N_o years are out-of-sample. Panel C shows the benchmark models: A standard mean–variance optimization (MV) with a 10-year window for estimating the current month's optimal allocation, and a time static 20% (20/80), 50% (50/50), 60% (60/40), and 80% (80/20) stock allocation, where the remainder is invested in bonds.

Table 10 Coefficients and significance levels for different levels of ρ, N_0 and rolling or static in-sample window of model (10).

	ρ	2	**5**	10	20	5	5	5	5	5	5	5
	N_0	1	**1**	1	1	2	5	10	1	2	5	10
		Rolling in-sample window							Static in-sample window			
Stocks	term	0.30	**0.09**	0.02	−0.01	0.09	0.09	.10	0.08	0.05	0.09	0.15
		(0.51)	***(0.64)***	*(0.80)*	*(0.85)*	*(0.64)*	*(0.65)*	*(0.65)*	*(0.67)*	*(0.79)*	*(0.64)*	*(0.50)*
	credit	0.87	**0.29**	0.09	−0.01	0.31	.35	.41	0.29	0.33	0.20	0.60
		(0.43)	***(0.53)***	*(0.70)*	*(0.91)*	*(0.53)*	*(0.51)*	*(0.52)*	*(0.52)*	*(0.47)*	*(0.66)*	*(0.38)*
	ted	−2.56***	**−1.05*****	−0.55***	−0.31***	−1.06***	−1.08***	−1.14**	−1.04***	−1.01***	−0.96***	−0.82**
		(0.00)	***(0.00)***	*(0.00)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.01)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.01)*
	DY	0.52*	**0.17**	0.05	−0.01	0.16	0.16	0.16	0.17	0.15	0.16	−0.01
		(0.06)	***(0.16)***	*(0.41)*	*(0.17)*	*(0.17)*	*(0.23)*	*(0.23)*	*(0.15)*	*(0.20)*	*(0.16)*	*(0.96)*
Bonds	term	3.79***	**1.45*****	0.68***	0.31***	1.45***	1.47***	1.53***	1.39***	1.41***	1.20***	1.07**
		(0.00)	***(0.00)***	*(0.00)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.01)*
	credit	−5.23**	**−1.97***	−0.93*	−0.43	−1.95*	−1.90	−1.92	−1.88*	−1.92*	−1.71*	−0.08
		(0.04)	***(0.06)***	*(0.08)*	*(0.12)*	*(0.08)*	*(0.14)*	*(0.16)*	*(0.06)*	*(0.06)*	*(0.09)*	*(0.95)*
	ted	8.84***	**3.28*****	1.54***	0.71***	3.28***	3.28***	3.22**	3.23***	3.14***	3.01***	2.60**
		(0.00)	***(0.00)***	*(0.00)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.01)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.01)*
	DY	−1.46**	**−0.62****	−0.35***	−0.22***	−0.63**	−0.66**	−0.68**	−0.63**	−0.59**	−0.60**	−0.94***
		(0.01)	***(0.01)***	*(0.00)*	*(0.00)*	*(0.01)*	*(0.01)*	*(0.02)*	*(0.01)*	*(0.01)*	*(0.01)*	*(0.00)*

This table shows the average estimated coefficients and in brackets and italics the average significance levels (p-values) for model (10) with different levels of risk aversion parameter $\rho = 5$, out-of-sample period length N_0 and either rolling or static in-sample windows. In the latter case the out-of-sample period spans the last N_0 years of the data. The standard model with risk aversion $\rho = 5$ and $N_0 = 1$ is in bold. In the case of the rolling in-sample window the coefficients were estimated with an rolling out-of-sample window of N_0-year length within the period December 1964 to December 2014 (end of month data). Term denotes the term spread, credit denotes the credit spread, ted denotes the ted spread, and DY denotes the dividend yield. Each dependent variable is estimated for each asset class (stocks and bonds). *, ** and *** relate to a p-value of ≤ 0.10, ≤ 0.05, and ≤ 0.01.

Ratio. However, the Sharpe Ratio is still larger in comparison with the benchmarks in Panel B. The Investment Ratios decrease as N_0 rise, whereas Jensen's Alpha increases with N_0. Interestingly, the Certainty Equivalent turns negative with a number of out-of-sample years of 10. Note that I do not vary the out-of-sample window with the mean–variance specification, giving this series an advantage with updating its strategy at an annual rate. The main conclusion is that there is not a large difference regarding the length of the out-of-sample period, which is a first indication that it is fairly safe to assume ergodicity.

This observation is supported when looking at columns 5 to 7 in Table 10. There is very little difference in the magnitude of the coefficients and the levels of significance when varying N_0, while holding ρ constant. Only credit with bonds turns insignificant: The p-value increases from 0.06 when $N_0 = 1$ to 0.16 when $N_0 = 10$.

A mayor assumption in this paper is that the link between the financial and economic variables is ergodic. To examine this relation more closely I consider in Panel C of Table 9 the effect of not rolling the out-of-sample window forward, but having a static in-sample and out-of-sample period. The out-of-sample period is defined as the last N_0 years of the data. Panel D sets this in perspective with the mean–variance benchmark and the 60/40 portfolio. This also answers the question whether the analysis can be performed real-time without having to rely on future data as in the jackknife analysis above.

In-sample, the difference in performance compared with the rolling results is minor but increases with N_0. The out-of-sample performance is now very dependent on the respective time-frame. When $N_0 = 1$, the out-of-sample year is 2014, for $N_0 = 2$ it is 2013 and 2014 and for $N_0 = 5$ it is 2010 until 2014. Within these years stocks as well as bonds performed well as central banks lowered interest rates further and further after the financial crisis. Thus it is not surprising that the Sharpe Ratio hits levels beyond 2. When turning to $N_0 = 10$ and thus including the financial crisis, the Sharpe Ratio is markedly smaller but at levels comparable to the long-term rolling setup of Panel A. Compared to the benchmark in Panel B, the strategy outperforms in all cases and across all but one level of performance measures. Just the Certainty Equivalent of the out-of-sample period is worse in case of $N_0 = 10$.

Looking at the corresponding estimates in Table 10, the magnitude and significance of coefficients vary little when $N_0 = 1, 2$ and 5. Only when $N_0 = 10$, there is some larger variation in the magnitude. However, with the exception of credit, those coefficients which are significant are still significant at the same level and the magnitude is somehow in the same range. Thus there is some learning effect of the financial crisis. In any case, this effect is qualitatively and economically limited, implying that assuming ergodicity is not too far from reality.

The results indicate that there are persistent rules on how the economy is linked to the finance world. This has sizeable performance implications on a strategy allocating its funds accordingly. Not surprisingly, the more data are available, the better these links can be detected.

9 Conclusion

In this paper I present a framework to uncover the link between the financial return of stocks and bonds and the return-driving economic forces. The approach does not require any structure on returns or the risk factors and thus implicitly reveals capital-market dynamics and how they translate into portfolio weights. Thus the approach can also help in the tactical asset allocation problem of allocating a portfolio between asset classes. The risk factors under focus are the term spread, credit spread, and the ted spread. Additionally, factors capturing valuations and momentum are considered. As the methodology requires a long time series of data I limit the analysis to US data.

The sign of the coefficients is intuitive in economic terms and statistically significant as the specification turns parsimonious. This is also important to the economic performance of the model as the performance is linked to the statistical significance of the model. The approach performs well in economic terms in-sample and out-of-sample.

A mean–variance and several static portfolios serve as benchmarks to the model. These benchmarks are greatly outperformed in terms of the Sharpe Ratio, Investment Ratio and Certainty Equivalent in-sample and out-of-sample, whenever the statistical level of the specification is high. Jensen's Alpha is positive and statistically significant.

A drawback of the strategy, excessive leverage and large turnover, can be handled when including transaction or leverage costs. In both cases the monthly reallocation is quickly reduced without losing a lot of performance. In the case of leverage costs, the portfolio exposure quickly shrinks below or around 100% of equity. The approach can also be extended to include further asset classes.

Qualitatively and economically I find significant evidence that there are persistent rules on how the economy is linked to the financial world.

Appendix A. Iterative estimation approach

Different starting values to the numerical estimation procedure of the GMM approach result in different optimized values. This is a strong indication for a local maxima problem. To solve this problem I propose the following iterative estimation approach:

(1) In total $K \times N$ coefficients are estimated. For each of these coefficients an initial value is defined. Suppose this initial value is drawn from a pool of m_N feasible initial solution values given by the vector m for each coefficient.
(2) Define the initial solution space with all permutations p of initial coefficient vectors. For example, in the case of four coefficients and three possible values for each coefficient there are 81 initial coefficient vectors.
(3) As either K, N or m_N gets large the initial solution space grows rapidly and therefore the calculation time for all solutions. Therefore j initial solution vectors are drawn from the initial solution space, where $j \leq p$.
(4) Start the optimization procedure with j initial solution vectors. This results in j solutions which should be similar to each other. If they are similar stop the calculation. Else continue.
(5) Calculate the first and third quantile values of the solutions for each coefficient.
(6) Replace the old initial solution values with the first and third quantile as calculated and the $m_N - 2$ values equally distributed within the first and third quantile. Do this for each coefficient.
(7) Calculate the j solutions to these initial values. If they are similar, stop. Else do the following loop:
    (7.1) Calculate the first and third quantile values of the solutions for each coefficient.
    (7.2) If the difference between the first and third quantile is smaller than the difference between the first and third quantile of the previous initial solution values, replace the previous initial solution values with the corresponding first and third quantile values as calculated and

the $m_N - 2$ values equally distributed within the first and third quantile. Do this for each coefficient.

The converged solution of this optimization routine is the best with respect to utility compared to all other and previous solutions. Repeating the optimization with the same or different parameters results in practically the same final solution, i.e., less than a one-digit percentage deviation between coefficients. This has no influence on the significance or the sign of the coefficients, or on the return series. I take this as enough evidence to have found the global optimum. In this study I set $m = (-0.05, 0, 0.05)$ and $j = 100$.

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Notes

1 This is the reason why I abstain from mixing together all variables x into an index $y_t = g(x_t, \delta)$ with each weight depending with its own coefficients on the single variable y, whereas the coefficients δ would be the same for all assets. In this case it would not be possible to disentangle the effects.

2 Although not discussed in the robustness section the choice of the benchmark weight is actually of second order as θ adjusts such that the actual weights are independent of $\bar{w}$.

3 The mean of the difference is 0.1%, the maximum difference is 0.2% in March 1990, the standard deviation is 0.06%, and the correlation between the one-month and three-month US government constant maturity rate is 0.97.

4 The ted spread with the certificates of deposit rate is on average 0.1 basis points lower than the usual ted spread definition with the Libor rate. The standard deviation of this difference is 0.1. The reason for the lower rate is that certificates of deposit are insured by the Federal Deposit Insurance Corporation for banks and by the National Credit Union Administration for credit unions. However, the correlation between the Libor ted spread and certificates of deposit ted spread is very high: 0.97 for the whole time series and even higher during the financial crisis in 2008.

5 http://www.econ.yale.edu/~shiller/data/ie_data.xls.

6 P.a. return 7.7%, 9.2%, 9.7%, and 10.7%, p.a. standard deviation 7.0%, 8.8%, 9.9%, and 12.4%, p.a. Sharpe Ratio 0.45, 0.52, 0.52, and 0.49 for the 20/80, 40/60, 50/50, and 80/20 static model. This compares with Sharpe Ratios of 0.54, 0.66, 0.66, and 0.65 including years 1974, 1981, and 1982.

7 The summary statistics of the monthly returns for this series is: −18.3% for the worst month, 25%-quantile −1.70%, median 0.2%, mean 0.3%, 75%-quantile 2.2% and a maximum of 18.7%. The standard deviation of the series is 3.6%.

8 For the mean–variance optimization I use a shorter estimation window for the correlations to determine the current month's optimal allocation prior to October 1966.

9 The correlation of monthly returns between stocks and bonds for the whole sample is 0.129, for stocks and commodities 0.163, and for bonds and commodities 0.129.

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