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BOOK REVIEW

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GOD'S OWN ARITHMETIC: HARRY MARKOWITZ' RISK-RETURN ANALYSIS

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Every practitioner in finance should add Nobel Laureate Harry Markowitz' recent books "Risk-Return Analysis: The Theory and Practice of Rational Investing" Volume 1 and Volume 2 to their library. Read them. Re-read. Repeat.

I am going to humbly suggest the following course of readings by Professor Markowitz that you will want to review (if you have not done so already) prior to reading this book series. All of the journal articles are available online either free or for a small fee, just google the title; and all of the books are available on Amazon.

- 1952a, "Portfolio Selection", Journal of Finance, Vol. 7, No. 1, pp. 77–91;
- 1952b, "The Utility of Wealth", Journal of Political Economy, Vol. 60, pp. 151–158;
- 1956, "The Optimization of a Quadratic Function Subject to Linear Constraints", Naval Logistics Research Quarterly, Vol. 3, Issue 1–2, pp. 111–133;
- 1959 (2nd edition 1991), Portfolio Selection: Efficient Diversification of Investments, Second edition, Cambridge: Basil Blackwell, Inc.
- 1981 (with Andre Perold), "Portfolio Analysis with Factors and Scenarios". Journal of Finance, Vol. 36, No. 4 (September), pp. 871–877.
- 1987 (2nd edition 2000 with G. Peter Todd), Mean–Variance Analysis in Portfolio Choice and Capital Markets. New Hope: Frank J. Fabozzi Associates.
- 2012, "The 'Great Confusion' Concerning MPT", The IEB International Journal of Finance, Vol. 4, pp. 8–27.

Markowitz assumes the reader has high-school algebra as a basic math background. That said, these books are no simple reading for the beginning—or even the average—investor; those readers expecting the level of, say, Peter Lynch's "One Up on Wall Street" may be overwhelmed by the formulas. Even Wharton Professor Jeremy Siegel, in his book "Stocks for the Long Run", has made his information easily accessible to the layperson by avoiding mathematical formulas entirely. Professor Markowitz, on the other hand, provides a number of mathematical expositions and proofs in this books—I urge readers uncomfortable with equations to just keep reading! Skim the equations if need be, but read the text, which describes the math in pretty simple terms.

Although you will do yourself a favor by reading, thinking about and understanding the equations too.

If you insist on extreme parsimony and only want to ever purchase and read a single book by Harry Markowitz, I recommend **Markowitz 1959 (2nd edition 1991)**, **Portfolio Selection: Efficient Diversification of Investments**, **Second edition, Cambridge: Basil Blackwell.** This is the fountainhead of Modern Portfolio Theory; this book greatly expands on Markowitz' original JoF article from 1952(a) and addresses two main limitations of the original paper: First, Chapter 8 and Appendix A of Markowitz 1959 derive analytical results for the N-security case, using the Critical Line Method first presented in Markowitz (1956). I never took a linear algebra course in college; I actually learned matrix math by reading Chapter 8 of Markowitz 1959 and coding up the Critical Line Algorithm myself. Second, Chapter 13 moves beyond the static probability beliefs assumption used in Markowitz (1952a).

Markowitz (1959 (2nd edition 1991)) presents *basic portfolio concepts* in Chapters 1–6 (Parts I and II). This became the foundation of Modern Portfolio Theory.

Markowitz (1959 (2nd edition 1991)) teaches *computation* of the Critical Line Algorithm in Chapters 7–9 (Part III) with a proof in Appendix A. The computational aspects received a more thorough exposition in Markowitz (1987), and I recommend the Markowitz and Todd (2000) reissue version of Markowitz 1987 for readers who want to build their own algo.

Finally, *theoretical justification* for use of MVA is provided in Chapters 10–13 (Part IV) of Markowitz (1959 (2nd edition 1991)). The four volumes of Risk-Return Analysis (2014-present) provide a more complete and expanded discussion of these theoretical underpinnings of MVA.

Here are my reviews of the volumes in "RISK-RETURN ANALYSIS":

Risk-Return Analysis Volume 1 (2014), McGraw Hill, New York:

This is the first volume of Harry's 4-volume set, "Risk-Return Analysis". Volume 1 is essentially an in-depth exposition of Chapter 10 of Markowitz' 1959 book, "Portfolio Selection: Efficient Diversification of Investments" (which, frankly, if you have read and understand, you will not really need to read "Risk-Return Analysis" other than as a review of the research on this topic since 1959 and some extensions). The remaining three volumes of Risk-Return Analysis will likewise refresh chapters 11, 12 and 13 of Markowitz (1959). These four chapters of Markowitz (1959), and the present four-volume set explain the fundamental assumptions for the use of mean-variance analysis (MVA).

Essentially, this book justifies MVA as an application of rational decision-making under uncertainty and over time, synthesizing the approaches of von Neumann & Morgenstern (1944) "Theory of Games and Economic Behavior", Savage (1954) "The Foundations of Statistics", and Bellman (1957) "Dynamic Programming".

Volume 1 addresses the "Great Confusion" regarding the necessary and sufficient conditions for the practical use of Markowitz' MVA, first described in 1952. After the financial crisis of 2008, some investors lost faith in the diversification benefits of modern portfolio theory, and this book addresses and corrects these mistaken beliefs. Markowitz clarifies that the premises of Expected Utility and the premises of MVA are identical, and discusses how mean-variance approximations to expected utility have been emphasized in a variety of publications by Markowitz and others since 1979. For example, after 2008, some investors have questioned the use of Gaussian return distributions in MVA. First, the financial dislocation of 2008 does not indicate non-normality; second, even if it did, normal return distributions are not necessary conditions for the use of MVA, as pointed out in Markowitz (1959) and re-emphasized in the present book, "Risk Return Analysis". Markowitz clarifies that while the normal distribution is SUFFICIENT to justify MVA as being approximately equivalent to maximization of Expected Utility across a wide variety of risk-averse utility functions, normal distributions are not NECESSARY. I will just also mention that diversification is not, and has never been presented by Markowitz as, *protection from loss*; rather, it is presented as *a method of investing rationally.*

Volume 1 of Risk-Return Analysis has 5 chapters:

Chapter 1) *Rational investors should maximize their expected utility*; this chapter compares standard deviation vs. maximum loss as a measure of risk; it addresses rational decision-making vs. human decision-making and uses Weber's law to explain the human decision problems described by Kahneman & Tversky (behavioral finance) and reviews Allais' Nobel prize-winning paradox; Markowitz presents the two axiom systems from Markowitz (1959) in 4 axioms at the end of chapter 1.

Chapter 2) *MVA approximates EU*; this chapter compares utility of return to utility of wealth; Markowitz corrects the mistake of Loistl (1976) who criticized MVA; the chapter reviews the Levy & Markowitz (1979) paper on approximating EU with MVA; Markowitz discusses and agrees with Simaan's (1993) result that including a risk-free asset in the portfolio choices for highly risk averse investors is helpful; the chapter also discusses MVA utility with common stocks vs. call options and reviews several other relevant works including Young & Trent (1969), Dexter, Yu & Ziemba (1980), Kroll, Levy & Markowitz (1984), Hakansson (1971), Grauer (1986) and Pulley (1983).

Chapter 3) *MVA approximates the geometric mean*; this chapter discusses differences between geometric and arithmetic mean and explains why MVA requires arithmetic rather than geometric means as input; Markowitz presents empirical asset class results and real equity returns of 16 countries over the 20th century; he compares six methods of estimating geometric means from arithmetic means, and recommends use of three equations: equation 10f, the "HL" approximation named for Henry Latane, equation 10b, "QE" from Chapter 6 of Markowitz (1959), and equation 10e, "NLN" for near-lognormal.

Chapter 4) *Risk measures*; this chapter evaluates five alternative risk measures for potential use in MVA: variance, mean absolute deviation, semi-variance, value at risk and conditional value at risk. Markowitz uses the empirical data from Dimson, Marsh and Staunton's "Triumph of the Optimists" (Which I also highly recommend) to evaluate these measures and finds that variance is the superior measure of risk. Marketing people may find Figure 4.11, "Maximum Loss by 16 Equity Markets During the Twentieth Century" useful in terms of outcomes to avoid. The conclusion of this chapter bears direct quotation: "…proponents of alternative risk measures need to get beyond their current line of argument, which goes roughly as follows: return distributions are not normal; therefore mean-variance is inapplicable; therefore my risk measure is best."

Chapter 5) *Return distributions*; in this chapter, Markowitz explores a variety of return distributions (with co-authors Anthony Tessitore, Ansel Tessitore and Nilfur Usmen), evaluating the same empirical data from the prior chapter using Bayes' rule. If you like histograms (as I do), Figure 5.3 is for you! The chapter concludes that Canada, the U.S., and Australia appear to generate normally distributed returns, while most other countries appear to have infinite density functions (Pearson class IV distributions). All however, may be easily accommodated by MVA.

The book concludes with some practical recommendations; essentially, investors can maximize their utility of wealth by employing MVA. To do so,

1. calculate efficient frontiers as described in Markowitz (1959),
2. select portfolios from the frontiers,
3. compute return series for the portfolios,
4. find the distribution using Bayes' rule that likely generated the return series,
5. select one of the likely distributions to serve as the portfolio distribution.

I am going to simplify this [what may seem like convoluted and circular to the uninitiated] process for the financial practitioner: Follow steps 1 and 2, the market will perform step 3 for you, and the rest is academic (though interesting).

Step 1) How can you calculate the frontier? Most commercial optimizers I have found over the past quarter century employ nonparametric methods to estimate optimal portfolios. Markowitz' CLA is the only parametric method, aside from Wolfe's (1959) simplex method, available. Personally I like CLA the best, because it is simple, straightforward and robust. Open source methods of Markowitz' CLA are available, I recommend that you read Mean–Variance Analysis in Portfolio Choice and Capital Markets which provides Peter Todd's VBA code that you can implement in MS Excel. Also there is an excellent introductory article by Clarence Kwan, "A Simple Spreadsheet-Based Exposition of the Markowitz Critical Line Method for Portfolio Selection" in Spreadsheets in Education (2007) vol 2, Issue 3, Article 2 available free online if you google it. Do yourself a big favor, and walk through this example and build it in your own spreadsheet. I did that and then added some VBA to automate the procedure for large, dense matrices. Finally if you'd like to build a Python implementation, David Bailey and Marcos Lopez de Prado have a great paper with Python code for CLA in Algorithms (2013), volume 6, pp. 169–196, "An Open-Source Implementation of the Critical-Line Algorithm for Portfolio Optimization". Marcos also gives an excellent discussion of the Critical Line Algo in a presentation available on vimeo at vimeo.com/61426601.

Step 2) OK, now you have the efficient portfolios, calculated as in step 1 above. Which portfolio do you select? Well, you can just choose one consistent with the level of risk you are willing to take. Since there will be estimation errors in your ex-ante inputs, you may want to account for that somehow, such as by using shrinkage estimators; or the resampling method described in Richard and Robert Michaud's book, "Efficient Asset Management"; or you could use a simpler method, by taking the average weight of all the assets included in all of the portfolios along the efficient frontier you have calculated—I show how to do this in my paper, "Portfolio Selection: How to Construct and Use the Critical-Implied Reference Portfolio" available on SSRN.

Summary: If you want an "Introduction to Investing" book, this may be over your head. However if you are already familiar with mean variance analysis and efficient frontiers, and would like an update based on the latest thinking and research of its inventor, Harry Markowitz, buy this book. You will find it to be a valuable addition to your knowledge base.

Risk-Return Analysis Volume 2 (2016), McGraw Hill, New York:

This is the second volume of Harry's planned 4-volume set, "Risk-Return Analysis". This book is essentially an in-depth exposition of Chapter 11, "Utility Analysis Over Time", in Markowitz' 1959 book, "Portfolio Selection: Efficient Diversification of Investments". The previous volume (Volume 1) covered Chapter 10, "The Expected Utility Maxim", and the remaining two volumes will likewise refresh chapters 12 "Probability Beliefs", and 13 "Applications to Portfolio Selection" of Markowitz (1959). These four chapters of Markowitz (1959), and the present four-volume set explain the fundamental assumptions for the use of mean-variance analysis (MVA). Essentially, this book (Volume 2) explores MVA beyond the single-period choice framework that was covered in Volume 1, into a multi-period setting, still assuming known odds (a given return distribution). Volume 3 will move both single-period and multi-period analyses into a setting with unknown odds—investor choice under uncertainty. Volume 4 will wrap up the series with important practical and theoretical considerations not addressed in the first three volumes.

Volume 2 of Risk-Return Analysis includes the following 7 chapters:

Chapter 6) "*The Portfolio Selection Context*"; this chapter introduces the current volume as an evolution from Volume 1, which focused on single period choice with known odds, to a focus on multi-period choice still with known odds. Rational Decision Makers vs. Human Decision Makers, first introduced in chapter 1 of Volume 1, are briefly reviewed; the joint distribution of current period return and future period opportunities is introduced as a game (expounded upon in more detail in Chapter 11); The traditional concept of a single utility-maximizing "Investor" is expanded into a "Stakeholder" in order to accommodate multi-person households and institutional investors aggregating the interests of many investors; the critical concept of diversification is briefly reviewed; and a preview of chapters 7–12 is provided.

Chapter 7) "*Modeling Dynamic Systems*"; this chapter introduces simulation in financial analysis, covering Decision Support Systems (DSS) and Optimization Simulators. Examples covered include the GuidedChoice DSS; the Kim and Markowitz (1989) Simulator; the Markowitz and van Dijk (2003) heuristic algorithm; the Blay and Markowitz "Tax Cognizant Portfolio Analysis" simulator (TCPA) developed for the book's commercial sponsor, 1st Global; the Jacobs Levy and Markowitz (2004) JLMSim re-optimizing simulator; and the Markowitz (1991) Game of Life simulator. The chapter reviews the EAS-E DSS approach used in Markowitz' preferred programming language, SIMSCRIPT. A history and detailed description of EAS-E and SIMSCRIPT is followed by a description of GuidedChoice's commercial implementation of a DSS including Markowitz' 1991 Game of Life simulator. This led to commercial products that are similar to Bill Sharpe's Financial Engines, providing savings and consumption advice to investors participating in 401(k) plans. The chapter concludes with further detail and discussion of the history and attributes of SIMSCRIPT.

Chapter 8) "*Game Theory and Dynamic Programming*", introduces optimization. Beginning with von Neumann and Morgenstern (1944) game theory, then introducing Bellman's Dynamic Programming with an example of a tic-tac-toe solver, then generalizes Dynamic Programming as an application of conditional expected values with a coin-flipping, card-playing, dice shooting, beer drinking example. Two types of games are described, those in which players can calculate conditional probability distributions of current possible states of the world without guessing other players' strategies, and those in which they cannot. (Got that?) The latter type of game is beyond the scope of the book, as it requires Nash equilibrium approaches to solve. Markowitz' mind is beautiful, but thankfully he spares readers incredible difficulty by focusing exclusively on the former type of game. The chapter concludes with an introduction to the Curse of Dimensionality, in which the number of state variables in a state space is typically too high for optimizers to solve without the use of heuristics.

Chapter 9) "*The Mossin-Samuelson Model*", covers a dynamic investment model in which utility is a function of final wealth. It results in the counterintuitive recommendation that investors should not become more cautious as they approach retirement. This is discussed viz. "glide path" and "target-date" investment strategies. A discussion of Markowitz' views versus Samuelson's views on maximizing utility over the long run presents the pros and cons of repeatedly rebalancing to a M-V efficient portfolio that approximates the maximization of expected log wealth.

Chapter 10) "*Portfolio Selection as a Social Choice*", delves into normative aspects of portfolio choice—rather than attempting to describe how decisions are actually made (human decision makers), this chapter focuses on how decisions should be made (rational decision makers). It presents value judgements. Arithmetic mean versus geometric mean utility is reviewed, and Markowitz then corrects an error in the Nash solution to the two-person bargaining problem.

Chapter 11) "*Judgement and Approximation*", covers maximization of expected utility for dynamic practical games, along the lines of chapter 13 of Markowitz (1959), which uses implicit maximization of expected utility. Next, Markowitz turns to explicit maximization of expected utility, relying on, and explaining, the heuristic approaches of Markowitz and van Dijk (2003) and Kritzman, Myrgren and Page (2009) optimization techniques. Regarding the superiority of the two heuristic approaches, Markowitz concludes "time will tell". Finally, chapter 11 covers taxable accounts, utilizing the Blay-Markowitz Net Present Value methodology to motivate the "Tax Cognizant Portfolio Analysis" (TCPA). This process transforms pre-tax return sequences into vectors of present values and estimates the inputs required by the critical line algorithm from the present values. Because this procedure is overly sensitive to outliers in the data, the vector of means, and the matrix of variances and covariances are inferred from a sample-parameterized normal distribution rather than from the sample present values directly. Efficient frontiers and resampled frontiers are discussed, and the chapter addresses Merrill Lynch Wealth Management's asset allocation approach. Bucketing generally is discussed, including as a viable way to approach partially convex utility functions. Markowitz suggests such cases, not well served by mean-variance analysis, may be accommodated with mixed-integer linear programming.

Chapter 12) "*The Future*", proposes that even more robust financial simulators and decision support systems will be created as computers evolve into massively parallel architectures. Essentially this chapter is an in-the weeds continuation of chapter 7. It makes a detailed case for the continued development of SIMSCRIPT as a programming language well suited to future development of simulators and DSS. While SAP is acknowledged to be the most commonly-used current enterprise DSS generator, Markowitz lays out his vision for possible future development of the SIMSCRIPT language, with a detailed discussion of how SIMSCRIPT II could evolve into a future SIMSCRIPT M ("Modernized"). This chapter also reviews Moore's law, and points out that current computer architecture was invented by John von Neumann—the same genius who was a principal developer of game theory described earlier in the book. Markowitz concludes this volume with comments on the analogy of computers as brains, learning, emulation, processes, and parallelized processes. I especially appreciated the Brahms "would have driven me mad" quotation about J.S. Bach's "Chaconne", a complicated (up to 4 parts) piece for solo unaccompanied violin, and looked up the video of Perlman playing this beautiful piece on YouTube. I agree with Harry, it is extraordinary—a musical metaphor for Markowitz' own Portfolio Selection, in my humble view.

Volume 2 of Risk-Return Analysis is a bigger tome than Volume 1. To me it was more difficult to work through, and I ascribe this to the fact that I am not an advanced programmer. I personally use VBA, Python and R and could follow the discussion just fine, but I felt like chapters 7 and 12 in particular were overly specific to a language, SIMSCRIPT, that while clearly dear to Markowitz' heart, is essentially a proprietary RAND Corporation project which has since been commercially developed by defense contractor CACI, again, in a proprietary mode, while IBM left its own version to wither. I am grateful that Markowitz, who co-developed the SIMSCRIPT language in the early 1960s, has provided his detailed history and vision of the language. However I feel like the free, open source languages such as Python [1991 origin with 2016 current stable release], R [1993 origin with 2016 current stable release], and (at least at Jane Street) OCaml [1996 origin with 2016 current stable release], all of which have broad-based, self-organized, thriving volunteer development initiatives, are quants' preferred languages of choice today. That said, I did learn a lot from the SIMSCRIPT chapters.

The classic Markowitz (1959), "Portfolio Selection", runs 384 pages in total. Its Chapter 10, "The Expected Utility Maxim", is 37 pages long; the exposition and extension of Chapter 10 as Volume 1 of "Risk-Return Analysis" is 230 pages. Chapter 11 of Markowitz (1959), "Utility Analysis Over Time", is 13 pages long; the exposition and extension of Chapter 11 as Volume 2 of Risk Return Analysis is 376 pages (nearly 100 of which are devoted to SIMSCRIPT). Markowitz has put in an incredible effort to bring us up-to-date on these topics. Personally I think it would be great if the open-source developer community reignited the SIMSCRIPT cause.

I am eagerly anticipating Volume 3 of Risk-Return Analysis. You might, too.