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STRATEGIC ASSET ALLOCATION WITH LOW-RISK STOCKS: A BOOTSTRAP ANALYSIS

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Traditional asset allocations such as the 60/40 portfolio of stocks/bonds are not as well diversified as many investors believe since almost all the portfolio's returns are driven by the stock component. This paper examines a novel approach to strategic allocation by combining stocks with low betas and high dividend yield. Our "beta-yield" portfolio exploits the beta anomaly (low beta stocks have higher risk-adjusted returns than high-beta stocks) and the hedging property of high dividend yield stocks in declining markets. We use bootstrap simulations to analyze the long-term performance of the beta-yield portfolio and find that it outperforms 60/40, 70/30, and 80/20 stock/bond portfolios in terms of shortfall risk, Omega ratio, and Prospect Theory utility. The results hold even with relatively high loss-aversion, and when the beta-yield strategy is assumed to have counterfactually low average returns.

1 Introduction

The security market line for stocks is flat or even negative, implying that stocks with low betas have higher average returns than is predicted by theory. Although this anomaly is known since the time of Black *et al.* (1972), there was little follow-up research for the next 20 years until Fama and French's (1992) decisive finding that beta fails to explain the cross-section of stock expected returns. Fama and French's result was subsequently confirmed by Falkenstein (1994) and Haugen and Baker (1996) using longer sample periods.

Interest in the beta anomaly revived in recent years with the publication of several studies documenting an inverse relationship between average stock returns and risk. In a comprehensive study, Blitz and van Vliet (2007) find that low beta

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stocks have higher average returns in both the U.S. and global stock markets, and that this anomaly has persisted to recent times. In the same vein, Ang *et al.* (2006, 2009) find that stocks with low idiosyncratic volatility significantly outperform those with high idiosyncratic volatility in both the U.S. and international stock markets. More recently, Frazzini and Pedersen (2014) show that the beta anomaly extends to other asset classes such as bonds, commodities, and currencies. They also find that a "bet against beta" strategy which shorts high-beta assets and long leveraged low beta assets yield significantly positive risk-adjusted returns.

Two decades earlier, Black (1973) suggested another interesting way to exploit the low beta anomaly: using low beta stocks to substitute the market portfolio for a given stock/bond asset allocation. For example, instead of a 60/40 stock/bond portfolio, an investor might want to consider a portfolio with 80% in low beta stocks and the rest in bonds. Neither Black (1973) nor other researchers after him has pursued this idea in detail.

In this paper, we take up Black's suggestion by providing simulation evidence on the advantages of low beta stocks in the context of strategic asset allocation. Strategic asset allocation has traditionally focused on using diversified portfolios of stocks and bonds to generate high average returns while providing downside risk protection. The 60/40 stock/bond allocation epitomizes this "balanced" approach for many institutional investors such as pension and endowment funds, while a typical advice given to young investors saving for retirement is that they should allocate more than 70% of their savings to stocks. For example, according to Ibbotson *et al.* (2007), the typical stock allocation recommended by investment advisors range from 74% for a 20-year horizon to 86% for a 30-year horizon. The beta anomaly does raise questions about the optimality of this approach to long-term investing. In particular, due to compounding, even if low beta stocks have the same (arithmetic) mean return as riskier stocks, they may still outperform portfolios that are heavily weighted toward equities in terms of long-run cumulative returns. The dominance of stocks in traditional asset allocations also raises the question of whether they are riskier than investors are willing to bear. This issue is clearly more salient for loss-averse investors for whom losses loom larger than gains. The empirical evidence of loss-aversion is extensive (see Kahneman and Tversky, 1979; Tversky and Kahneman, 1992; Rabin and Thaler, 2001; Dimmock and Kouwenberg, 2010).

We show that an attractive solution for loss-averse investors with long investment horizons is to combine stocks with low betas and high dividend yield. This "beta-yield" strategy has the following attractive features. First, the two investment styles are complementary. High dividend yield stocks are only slightly more volatile than low beta stocks and both are much less volatile than the market portfolio. In addition, low beta stocks themselves have high dividend yields compared to high-beta stocks. Second, combining the two investment styles offer diversification benefits as high dividend yield stocks tend to outperform low yield stocks in declining markets (Fuller and Goldstein, 2011). Third, the beta-yield strategy may also have psychological benefits for myopic loss-averse investors (Benartzi and Thaler, 1995). Since dividend payments provide a cushion against capital losses, high dividend yield stocks may encourage investors to have a long-term focus.

Consistent with the above hypothesized protective features, we show that the beta-yield portfolio has a lower portfolio beta, lower maximum drawdown and higher Omega ratio (upside returns to downside risk) than either a pure low beta or pure high dividend yield portfolio. Yet, despite its lower beta, the beta-yield portfolio has nearly the same Sharp ratio as its constituents.

As mentioned earlier, the advantage of low-risk investing becomes greater when returns are compounded over a long period of time. Consistent with this, we show via bootstrap simulations that the beta-yield strategy outperforms traditional stock/bond portfolios such as the 60/40, 70/30, and 80/20 portfolios in having (a) higher cumulative returns across all percentiles of the return distribution, (b) lower shortfall risk for target returns of between 4% and 6% a year, (c) higher Omega ratios for the same range of target returns, (d) higher mean and median values for the Prospect Theory utility function of Tversky and Kahneman (1992), and (e) still outperforms the stock/bond portfolios even when its historical arithmetic average return is reduced to that of the market portfolio. We conclude that the beta-yield strategy is a compelling alternative to traditional stock/bond allocations for loss-averse investors.

The rest of this paper is organized as follows. Section 2 describes our data sources, beta estimation methodology, and gives details on how we form our beta-yield portfolio. Section 3 contains a preliminary analysis of the characteristics of beta-sorted and dividend yield-sorted portfolios. The common features of low beta and high dividend yield stocks are highlighted and evidence on the robustness of the beta anomaly provided. Section 4 describes our bootstrap simulation methodology and discusses the assumptions underlying our choice of target returns for assessing relative performance. Simulation results are discussed in Sections 5 and 6. Section 7 concludes with a summary of the key findings.

2 Data and methodology

2.1 *Stock universe*

Our raw data consist of daily and monthly returns for the sample period from June 1968 to December 2012. Data for stock returns come from the CRSP (Center for Research on Security Prices) database. We include all ordinary common stocks traded on the New York Stock Exchange (NYSE) and American Stock Exchange (Amex). Following standard practice, we restrict our sample to firms with share codes 10 and 11 in CRSP, thus excluding ADRs, foreign shares, REITs, financials, and closed-end funds. We use daily stock returns along with market returns from the CRSP daily index file to calculate betas. On each portfolio formation date, we remove stocks that are in the bottom quartile by market capitalization to exclude the most illiquid stocks that would be too costly to trade for any reasonable size trading volume. Stocks with prices less than $5 are also eliminated to mitigate microstructure effects associated with low-priced stocks such as bid–ask bounce (Jegadeesh, 1990).

2.2 *Beta estimation*

Every month, we form quintile portfolios based on each stock's estimated (ex-ante) market betas. The estimated beta for stock i can be written as

$$\beta_i^{TS} = \hat{\rho}\frac{\hat{\sigma}_i}{\hat{\sigma}_m} \qquad (1)$$

where $\hat{\sigma}_i$ and $\hat{\sigma}_m$ are estimates of the standard deviations of the stock and the market (the CRSP value-weighted stock index) and $\hat{\rho}$ is an estimate of their correlation. Following Frazzini and Pedersen (2014), we use 1-year rolling standard deviations for volatilities and a 5-year rolling period for correlations to account for the tendency for correlations to move more slowly than volatilities (De Santis and Gerard, 1997). Since the accuracy of covariance estimation improves with the data frequency (Merton 1980), we use daily data to estimate standard deviations and correlations. Specifically, we use daily log-returns to estimate volatilities and overlapping 3-day log-returns to estimate correlations, where the 3-day overlap is used to control for non-synchronous trading. The overlapping 3-day return for stock i is given by

$$r_{i,t}^{3d} = \sum_{k=0}^{2} \ln(1 + r_{t+k}^{i}). \quad (2)$$

We require at least 3 years of non-missing data to compute correlations and at least 6 months of non-missing data to estimate volatilities. To reduce the influence of outliers, we shrink the time-series estimate of beta (β^{TS}) toward the cross-sectional mean (β^{XS}):

$$\hat{\beta}_i = w_i \hat{\beta}_i^{TS} + (1 - w_i)\hat{\beta}^{XS}. \quad (3)$$

Instead of using stock-specific and time-varying shrinkage factors as in Vasicek (1973), we use simply set $w_i = 0.6$ and $\beta^{XS} = 1$ for all periods and all stocks. This method is similar to Frazzini and Pedersen (2014) who find that this method gives results that are very similar to those using the more complicated Vasicek method.

2.3 Portfolio formation

Each month, starting from June 1968, we rank stocks independently by their estimated betas and dividend yield to form 25 beta-dividend yield quintile portfolios. Betas are estimated using the method described above. Dividend yield is defined as the sum of cash dividends paid over the prior year divided by the stock price at the portfolio formation date. All portfolios are equally-weighted and rebalanced monthly. We denote Bl (B5) as the lowest (highest) beta quintile. Since zero-dividend firms tend to be concentrated among the smallest firms (Christie, 1990), we designate Dl as the zero-dividend yield quintile, D2 as the lowest nonzero dividend yield quintile and so on with D5 as the highest nonzero dividend yield quintile. Our beta-yield strategy focuses on B1D5, the portfolio comprising stocks with the lowest beta and the highest dividend yield.

3 Preliminary analysis

Table 1 reports summary statistics of firm characteristics within portfolios sorted by beta (Panel A) and dividend yield (Panel B). "Ex-ante Beta" refers to the time-series average of the pre-ranking betas for the typical firm in each quintile. "Ex-post Beta" is computed based on a time series regression of each portfolio's excess returns on the market's excess returns over the whole sample period. Frazzini and Pedersen (2014) find that on average, ex-post betas are very close to the ex-ante betas. This is also the case for our sample.

Panel A shows that beta sorts produce a fairly wide range of ex-ante betas from 0.69 (Bl) to 1.46 (B5). Price and beta are inversely related, but there appears to be no clear relationship for firm size. Low beta stocks tend to have higher dividend yield than high beta-stocks despite there being no strong connection between beta and the traditional measure of value, the book-to-market ratio.[1] Panel B shows that high dividend yield stocks share some common features with low-beta stocks. Specifically, the typical stock in D5 belongs to a large firm, has low beta and relatively high average excess returns compared to Dl. D5 has higher book-to-market ratios than Bl indicating that D5 has a more of a value tilt than Bl. In short, low beta and high dividend yield stocks embody complementary investment styles.

Table 2 reports the risk and return characteristics of B1D5, its constituents Bl and D5, and the 60/40 stock/bond portfolio. Panel A reports mean–variance statistics, including mean excess returns, standard deviation of returns and betas estimated using the CAPM and Fama–French three-factor (FF-3) model.

Table 1 Summary characteristics of beta and dividend yield portfolios.

A. Beta sorts	Bl (Low beta)	B2	B3	B4	B5 (High-beta)
Ex-ante beta	0.69	0.88	1.02	1.17	1.46
Ex-post beta	0.61	0.88	1.05	1.22	1.51
Mean Ex return (%)	0.66	0.76	0.69	0.66	0.43
Price ($)	135.9	92.8	50.4	34.6	27.5
Firm size ($ millions)	4,644	4,261	4,200	4,102	4,402
Dividend yield (%)	4.28	3.00	2.55	2.05	1.39
Book-to-market ratio	0.69	0.66	0.65	0.64	0.67
B. Dividend yield sorts	**Dl (Zero-yield)**	**D2**	**D3**	**D4**	**D5 (High-yield)**
Ex-ante beta	1.22	1.13	1.05	0.99	0.87
Ex-post beta	1.36	1.19	1.07	0.98	0.74
Mean Ex return	0.45	0.47	0.69	0.76	0.73
Price ($)	123.3	38.4	35.7	34.8	30.3
Firm size ($ millions)	1,492	4,320	4,766	6,051	5,233
Dividend yield (%)	0.00	0.93	2.14	3.38	5.97
Book-to-market ratio	0.65	0.53	0.62	0.70	0.80

This table reports descriptive statistics of portfolios formed by sorting stocks based on market betas (Panel A) or dividend yield (Panel B). The sample period is from January 1968 to December 2012. Each month from January 1968, we form portfolios by sorting stocks into equal-sized quintiles based on their estimated (ex-ante) betas or dividend yield as described in the text. Stocks are equally weighted within each quintile. The table reports time series means of the typical firm's ex-ante beta, ex-post beta, stock price, firm size (in $ millions), dividend yield (percent per year), and book-to-market ratio. Ex-post beta is the realized loading on the market portfolio in a full-sample CAPM regression. "Mean Ex Return" (percent per month) is a portfolio's mean excess return relative to the 1-month Treasury bill rate.

The 60/40 portfolio has a mean excess return of 0.37% per month, Sharpe ratio of 0.12, and beta of 0.65. Its mean excess return and volatility is 86% and 69% respectively of the stock index's, indicating that equities have a dominant influence in a "balanced" portfolio like the 60/40. In line with existing evidence on the beta anomaly, the low beta portfolio Bl outperforms the 60/40 in mean excess return (0.66%) and Sharpe ratio (0.19) despite having a relatively low beta of 0.61. D5 has the highest mean return (0.73%) among the portfolios which is not surprising given its high HML loadings, but its beta of 0.74 makes it less attractive from the viewpoint of low-risk investing.

As a long-term investment, B1D5 is the most interesting contender to traditional stock/bond portfolios. First, it has a lower beta (0.48) than either Bl or D5, consistent with the effects of diversification across the two investment styles. At the same time, B1D5 has comparable average returns and Sharpe ratios as the pure low beta portfolio, Bl. Thus, B1D5 best fits the description of a low-risk, high average return portfolio.

As a robustness check, we replicated Table 2 for January 1990 to December 2012 (roughly the second half of our sample period). The results (not tabulated) show that B1D5 continues to have a low market beta of 0.37 compared to the market

Table 2 Summary statistics of risk and returns.

	6040	Bl	D5	B1D5
A. Mean–variance statistics				
Avg. no. of stocks	—	225	246	103
Mean Ex return	0.37	0.66	0.73	0.64
Std. deviation	3.19	3.52	4.09	3.48
Sharpe ratio	0.12	0.19	0.18	0.18
MKT beta	0.65	0.61	0.74	0.48
SMB beta	−0.06	0.19	0.15	0.02
	(−2.95)	(4.10)	(3.73)	(0.56)
HML beta	0.01	0.43	0.61	0.49
	(0.50)	(6.69)	(12.77)	(7.60)
B. Downside risks				
Skewness	−0.21	−0.56	−0.24	−0.05
Kurtosis	4.02	7.08	5.68	5.78
Max. drawdown	−0.38	−0.41	−0.69	−0.35
Omega ratio	1.36	1.66	1.61	1.63

This table reports risk and returns statistics for the 60/40 stock/bond portfolio and portfolios formed by sorting stocks based on market beta, dividend yield and both of these characteristics. The sample period is from January 1968 to December 2012. The 60/40 portfolio has a weight of 60% in the CRSP value-weighted stock index and 40% in 20-year Treasury bonds. The other portfolios are formed monthly starting from January 1968 and are equally-weighted. Bl is the lowest beta quintile, D5 is the highest dividend yield quintile, and B1D5 is the intersection of Bl and D5. Panel A reports the average number of stocks in Bl, D5, and B1D5, mean–variance statistics for each portfolio including mean monthly excess return, standard deviation of excess returns, Sharpe ratio, market beta, and betas with respect to the other two Fame–French factors, SMB and HML. Returns and standard deviations are in percent per month. Panel B reports higher moment statistics to assess portfolio downside risk, including skewness and kurtosis coefficient, Omega ratio (with threshold return of zero), and maximum drawdown.

betas of Bl (0.43) and D5 (0.62), while its Sharpe ratio of 0.2 is comparable to that of Bl (0.21) but higher than that of the 60/40 portfolio and D5 (both 0.18).

One reason that may explain the excellent risk-adjusted performance of low beta stocks is the secular upward trend in institutional ownership of U.S. equities over the last 50 years. Cornell and Roll (2005) and Baker *et al.* (2011) argue that because most institutional investors are on fixed-benchmark mandates, they will aim for good performance on a relative basis (i.e., a high information ratio). From this relative-return perspective, low beta stocks are very risky, which explains why institutional investors pass up the superior risk-return opportunities provided by low beta stocks.

Another explanation for the beta anomaly is the strong preference for highly volatile, "lottery-type stocks" by individual investors (Kumar, 2009; Ang *et al.*, 2006, 2009; Barberis and Xiong, 2013). Investors' penchant for speculative stocks explains why low-risk stocks tend to be underpriced while high-risk stocks tend to be overpriced. As long as these structural factors are entrenched in financial markets, the beta anomaly is likely to persist.

Panel B reports various statistics that capture downside risks, including skewness, kurtosis, the Omega ratio, and maximum drawdown over the sample period. This panel shows that low volatility does not mean low downside risk. For example, Bl is more than twice as negatively skewed as the 60/40 portfolio. Its distribution is also more leptokurtic than the other portfolios. Investors of Bl therefore face more negative surprises than is predicted by the normal distribution. By contrast, B1D5 has the least negatively skewed return distribution.

A conservative measure of downside risk is maximum drawdown, which is the largest loss incurred by an investor who purchased the asset and exited from it at the worst time over a particular period. D5 has the largest maximum drawdown of –69%, substantially larger than that of 60/40 and Bl. B1D5 has the smallest maximum drawdown risk among the four portfolios, making it appealing to loss-averse investors.

Buy-and-hold investors may still be concerned with short-term downside risks if they are myopic loss-averse as argued by Benartzi and Thaler (1995). Panel B reports the Omega ratio (Shadwick and Keating, 2002) which is the average of exceedances divided by the average of shortfalls, where an exceedance is a return above a specified threshold and a shortfall is a return below the threshold. We use the Omega ratio as a simple way to capture the desire for high average returns and low downside risk. The Omega ratio also has the attractive property of being consistent with second-order stochastic dominance, i.e., if a risk-averse investor prefers prospect X to prospect Y, the Omega ratio for X will be greater than that of Y. This result applies to all risk-averse investors regardless of the precise shape of their utility functions.[2] Table 2 reports Omega ratios based on a threshold return of zero. The Omega ratio for the 60/40 strategy is 1.36. All other strategies have Omega ratios of at least 1.6. Results for the sample period 1990 to 2012 shows that in ascending order, the Omega ratios are 1.59 for the 60/40 portfolio, 1.63 for D5, 1.69 for B1D5, and 1.71 for Bl. Thus, B1D5 continues to do well in terms of average returns relative to downside risk in more recent times.

4 Long-term performance

For a long-term investor, achieving a high terminal wealth probably matters more than whether monthly alphas are positive or not. Since the geometric mean return is inversely related to volatility for a given value of the arithmetic mean, the benefit of low-risk investing is enhanced when returns are compounded over time. History confirms this fact, as shown in Figure 1 which tracks the cumulative returns from investing $1 in January 1968.

A dollar invested in the 60/40 portfolio grew to $58.50 over the 45-year period. As inflation

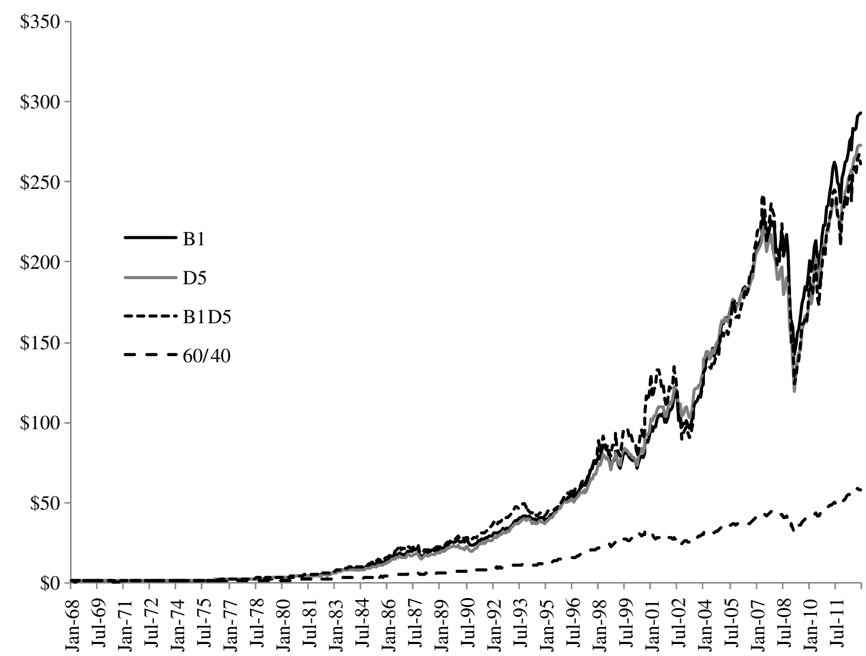

Figure 1 Value of $1 invested in 1968.

Sample period is January 1968 to December 2012. Bl: lowest beta quintile; D5: highest dividend yield quintile; B1D5: intersection of Bl and D5; 60/40: 60% in CRSP value-weighted stock index and 40% in 20-year treasury bonds.

eroded the real value of a dollar to about 15 cents, the 60/40 strategy produced a real gain of only \$8.77 or a geometric average return of 4.9% per annum. The other strategies outperformed the 60/40 strategy by a very wide margin. To provide a fair comparison with the 60/40 strategy, the returns of Bl, D5, and B1D5 were adjusted so that their full-sample betas are the same as the beta of the 60/40 strategy (0.65). We do so by levering the returns of Bl and B1D5 since these portfolios have lower historical betas than the 60/40 strategy and de-levering the returns of D5, whose beta is 0.74. In real terms, a dollar invested in Bl, D5, and B1D5 grew to \$43.93, \$41.26, and \$39.37, respectively, which translate to geometric average returns of between 8.5% and 8.8%. Geometric average returns without the beta adjustment are quite similar: 8.2% for B1D5, 8.5% for Bl, and 9% for D5. Thus, over the last half a century, low-risk stocks provided real returns that were far superior to that of the 60/40 strategy.

Although Figure 1 is consistent with theory, historical data is limiting because there are too few independent long-horizon returns for a given sample. For example, even with a century of returns, we have only ten independent 10-year returns, and many strategic allocation decisions stretch beyond that horizon. To address this small sample issue, we use bootstrap simulations to examine the distribution of long-horizon returns in a non-parametric way. We re-sample from blocks of data rather than individual data points, thus preserving the covariance structure within each block. Details of this block bootstrap approach are discussed below.

4.1 The block bootstrap

We use the moving block bootstrap method introduced by Knusch (1989) to generate long-horizon returns. The block bootstrap is an improvement over the classical bootstrap for i.i.d. data introduced by Efron (1979). The idea is to divide the data into overlapping blocks and randomly "stitch" the selected blocks to generate new data. An important advantage of block re-sampling is that it enables us to mimic the same dependence structure as the original data, whatever that structure may be. Large sample properties of the block bootstrap have been extensively studied in the statistics literature (see Lahiri 1991, 1992). Jing (1997) shows that the block bootstrap performs well both at the center and tails of distributions. Hansson and Persson (2000) use the block bootstrap (with a 5-year block length) to study time diversification.

We apply the block bootstrap to assess the performance of the B1D5 strategy vis-a-vis three stock/bond strategies that are commonly implemented in practice: 60/40, 70/30, and 80/20. We focus on the B1D5 portfolio since earlier results show that this portfolio has excellent mean returns and downside risk properties compared to Bl and D5. We assume a 20-year investment horizon in all our simulations.

The performance of the block bootstrap depends on the choice of the block length. When the block length is 1, the method reduces to the i.i.d. bootstrap and all correlations structure is lost. When the block length is very large, most of the dependence structure is maintained but there will be very few blocks to sample from. Similar to Hansson and Persson (2000), we choose a block length of 5 years, as this should be sufficiently long to capture most of the slow varying features of the data, while short enough to yield a large number of blocks to sample.

The simulation procedure is as follows. First, we form overlapping blocks of returns of length 60 months using the original data. Thus, the returns from month 1 through month 60 constitute the first block, the returns from month 2 through month 61 form the second block and so forth. Next, we randomly re-sample four overlapping blocks of 5 years each and "stitch" them in order to compute a long-horizon return of 20 years by compounding the monthly returns over that horizon. We repeat this process by sampling with replacement 1,000 times. Finally, we collect all the 1,000 simulated 20-year returns and use them to calculate the following performance measures: (a) percentiles of the cumulative return distribution (b) shortfall risk (the number of simulated cumulative returns that are below that implied by the target return), (c) Omega ratio, and (d) percentiles of the prospect theory utility function. We use three target rates of return to compute these performance measures: 4%, 5%, and 6%. In particular, we use these target returns to define reference point to quantify loss-aversion under prospect theory. The assumptions underlying these target returns are discussed in the next section.

4.2 Loss-aversion

Prospect theory (Tversky and Kahneman, 1979, 1992) posits that agents frame outcomes in terms of gains or losses relative to a reference point, and weight losses more than gains (i.e., they are loss-averse). There is substantial evidence that loss-aversion provides a better description of observed investor behavior than does risk aversion (e.g., Rabin and Thaler, 2001; Barberis *et al.*, 2006; Dimmock and Kouwenberg, 2010). Benartzi and Thaler (1995) argue that even long-term investors exhibit "myopic loss-aversion" because they evaluate their portfolios frequently.

Let W denote terminal wealth, R the reference point, and $X = W - R$ relative terminal wealth. We use the original form of the value or utility function as proposed by Tversky and Kahneman (1992) to quantify the effects of loss-aversion for long-term investors.

$$V(X) = X^{\alpha} \quad \text{if } X \geq 0$$

$$V(X) = -\lambda(-X)^{\alpha} \quad \text{if } X \geq 0$$

where V is the utility function defined over X, α measures the curvature of the utility function and λ is the loss-aversion parameter, $\alpha < 1$ implies that individuals are risk averse over gains and risk seeking over losses, and $\lambda > 1$ implies that individuals are loss averse. Tversky and Kahneman (1992) estimated α and λ to be 0.88 and 2.25, respectively. Using household survey data, Dimmock and Kouwenberg (2010) find that the best-fit median value for λ is 2.46, which is close to Tversky and Kahneman's estimate. However, the best-fit mean value for λ is 5.61, indicating that the distribution of the loss-aversion parameter is skewed to the right. For our simulations, we assume that $\alpha = 0.88$ and we calculate utility using two values for λ : 2.25 and 5.5.

The reference point is another key parameter in Prospect Theory. The reference point may be the status quo such as current wealth or a desired level of future wealth. Since we are interested in long-horizon returns, a reasonable reference point is a target level of future wealth. As we are running a horse race between B1D5 and the 60/40, 70/30, and 80/20 stock/bond portfolios, we will set as reference points, target returns for these portfolios based on reasonable long-term forecasts for inflation, equity risk premium, and bond returns. Table 3 sets out our assumptions for deriving target returns.

We use the consumer price index (CPI) to measure inflation and assume that CPI inflation rates vary between 2% and 4% a year. The lower forecast captures the low inflation environment of recent years, while the highest forecast is close to the average U.S. CPI inflation rate over our sample period. Recent yields on long-term Treasury bonds reflect investors' expectations of low inflation going forward.

Table 3 Bootstrap simulation assumptions.

Panel A. Forecast assumptions			
Forecast scenario:	1	2	3
Inflation rate	0.020	0.030	0.040
Real equity return	0.035	0.035	0.035
Nominal equity return	0.055	0.065	0.075
Long-term treasury Bonds	0.025	0.025	0.025
Panel B. Projected returns for stock/bond portfolios			
Stock/bond portfolio	1	2	3
60/40	0.043	0.049	0.055
70/30	0.046	0.053	0.060
80/20	0.049	0.057	0.065

This table presents long-term return forecasts for three stock/bond portfolios shown in Panel B. The underlying assumptions used to derive these forecasts are listed in Panel A. Details of the forecast derivations are described in the text.

Following Dimson *et al.* (2013), we use the 20-year Treasury bond yield at the end of 2012 (2.5%) as our forecast of the average real return for long-term Treasury bonds and add an inflation premium of 0.5% a year to this rate to derive forecasts for nominal bond returns.

Fama and French (2002) and Dimson *et al.* (2013) argue that a number of unique "good luck" factors favored equity and bond investors in the post-1950 period. Using a much longer time series of (global) equity returns from 1900 to 2012, and adjusting for non-repeatable good luck factors, Dimson *et al.* propose 3% to 3.5% as reasonable estimates of the real geometric average return for global stock markets. We use the upper forecast of 3.5% as our estimate future real equity returns for the U.S. Taken together, the above assumptions lead to nominal return forecasts of between 4.3% and 6.5% depending on the stock/bond mix. We therefore use target returns of 4% to 6% a year to assess the performance of the various investment strategies.

5 Simulation results

This section reports our bootstrap simulation results. Figure 2 shows cumulative returns averaged over 1,000 simulations for various combinations of B1D5 and the stock/bond portfolios. The weight for B1D5 is shown on the horizontal axis. The figure clearly shows that cumulative returns are monotonically increasing in the percentage invested in B1D5. In fact, a 100% allocation to B1D5 leads to the highest cumulative returns not only on average as shown in Figure 2, but also across all percentiles of the distribution (not shown for brevity). These results strongly suggest that B1D5 is likely to perform well on the other performance criteria. This is confirmed by Table 4 which reports statistics that capture downside risk and loss-aversion for various combinations of B1D5 and the 60/40 portfolio.

The top row of the table shows the weight for B1D5. A "0" indicates that all funds are invested in the 60/40 strategy throughout the 20-year horizon, while a "1" indicates that all funds are invested in the B1D5 strategy over the same period. For simplicity, we vary the weights vary in intervals of 0.1. The results in Panels A to C correspond to target returns (τ) of 4% to 6%. For each panel, we report the number of simulated cumulative returns that fall short of the target cumulative return, the Omega ratio, and the mean and median values of the prospect theory utility function with $\lambda = 2.25$.

Table 4 shows that the best-performing portfolio is B1D5 as a standalone strategy. B1D5 has the smallest shortfall risk, the highest Omega ratio, and the largest mean and median values for the prospect theory utility function. The worst performing strategy is the 60/40. The results further show that even a small allocation to B1D5, say from 0% to 20%, leads to sizeable reductions in shortfall probability and substantial

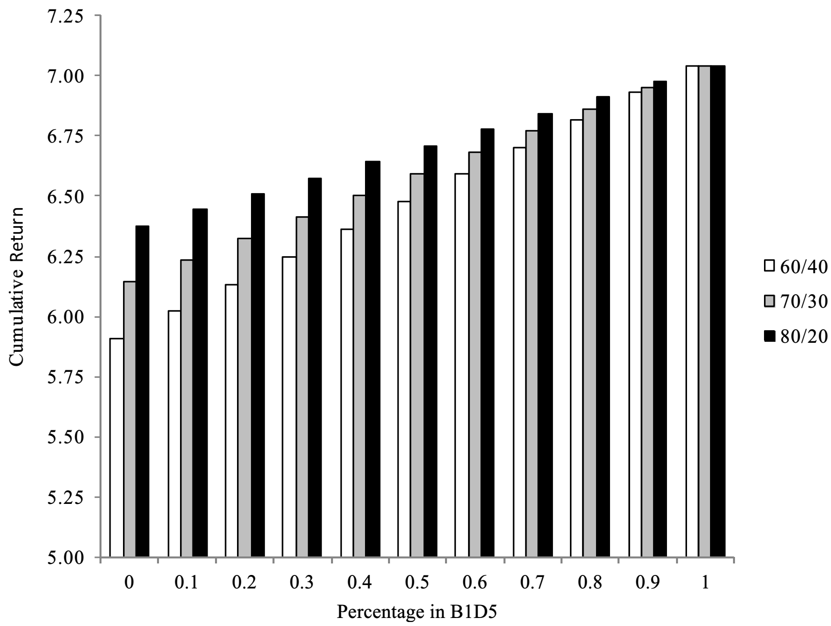

Figure 2 Bootstrap cumulative returns.

Cumulative returns from 1,000 bootstrap simulations for various combinations of the low-risk portfolio, B1D5 and three stock/bond portfolios (60/40, 70/30, and 80/20). The investment horizon is 20 years.

improvements in the other performance criteria. This result holds for all target returns.

Since the target return is a hurdle rate, on average, utility should decrease as τ increases. Moreover, since B1D5 has lower shortfall risk than the 60/40 strategy, the decrease in mean utility should be smaller for B1D5 than for the 60/40 strategy. Table 4 confirms these intuitions. For example, a two-percentage point increase in r from 4% to 6% leads to a 34% decrease in mean utility for the 60/40 strategy but only a 10% decrease for the B1D5 strategy. This result also holds for median utility.

Table 5 repeats the above results for λ of 5.5. Shortfall probabilities and Omega ratios are not shown as they are not a function of λ. The other results are qualitatively similar to those reported in Table 4. In particular, B1D5 and 60/40 are still the best and the worst performing strategies, respectively. Since a higher λ implies a larger penalty for losses, both mean and median utility are now generally lower for a given target return. Nonetheless, the B1D5 strategy is much less affected by the increase in λ than the 60/40 strategy. For example, holding the target return constant at 4%, increasing λ from 2.25 to 5.5 decreases the mean utility of the 60/40 strategy by 5% but leaves the mean utility for B1D5 virtually unchanged. The corresponding decrease for $\tau = 6\%$ is 33% for the 60/40 strategy and less than 1% for B1D5. Overall, these results indicate that the B1D5 is the optimal strategy even for a highly loss-averse investor.

We now examine the performance of B1D5 vis-à-vis the 70/30 and 80/20 portfolios (Table 6). To save space, we only report the results for the 6% target return (results for lower target returns are qualitatively similar). Because the weight for stocks is now larger, the 70/30 and 80/20 stock/bond portfolios raise the performance bar for the B1D5 strategy. However, B1D5 remains

Table 4 Bootstrap simulation results for various combinations of the 60/40 stock/bond portfolio and the low-risk B1D5 portfolio (λ = 2.25).

	B1D5 and 60/40					
% in BlD5	0%	20%	40%	60%	80%	100%
			4% target			
< Target	88	31	20	13	10	6
Omega ratio	0.77	2.50	5.31	10.50	17.89	27.42
Value function						
Mean	2.96	3.94	4.84	5.71	6.55	7.38
Median	2.60	3.42	4.17	4.80	5.48	6.14
			5% Target			
< Target	140	67	33	21	17	11
Omega ratio	0.34	1.05	2.42	4.50	7.88	12.47
Value function						
Mean	2.52	3.55	4.48	5.36	6.22	7.05
Median	2.23	3.08	3.83	4.47	5.15	5.82
			6% Target			
< Target	219	123	81	43	31	24
Omega ratio	0.15	0.42	1.00	1.95	3.31	5.26
Value function						
Mean	1.95	3.04	4.02	4.93	5.80	6.65
Median	1.79	2.65	3.42	4.07	4.76	5.43

This table reports the performance across 1,000 simulation runs for various combinations of the 60/40 stock/bond portfolio and the low-risk B1D5 portfolio. The 60/40 portfolio has a weight of 60% in the CRSP value-weighted stock index and 40% in 20-year Treasury bonds. B1D5 consists of stocks in the lowest beta and the highest dividend yield quintiles. The percentage allocated to B1D5 is indicated in row labeled "% in B1D5". The raw data for the simulations are monthly portfolio returns from January 1968 to December 2012. A moving block bootstrap with a block size of 5 years is used to simulate 20-year cumulative returns. The table reports three performance measures based on the simulated returns and target returns of 4%, 5%, and 6% a year. Shortfall risk is the number of simulated cumulative returns that fall below that implied by a target return. Omega ratio (Shadwick and Keating, 2002) is the average of returns above a target return divided by the average of returns below the target return. The last two rows in each panel show the mean and median of the Prospect Theory value function with $\alpha = 0.88$ and $\lambda = 2.25$ as in Tversky and Kahneman (1992).

as the optimal strategy based on all four performance criteria. For example, the mean utility of B1D5 is more than three times that of the 70/30 and 80/20 strategies. It also has appreciably lower shortfall risk and higher Omega ratio than any of the other asset combinations.

Figure 3 shows box plots of the 10th, 25th, 50th, 75th, and 90th percentiles of the prospect theory utility functions (based on $\lambda = 2.25$) for all the asset combinations discussed so far. Results for $\lambda = 5.5$ are very similar and are omitted to save space.

Figure 3 confirms that B1D5 stochastically dominates all other strategies across all target returns. In fact, Figure 3 shows that an investor need not invest all his funds in the B1D5 portfolio to

Table 5 Mean and median of the prospect theory utility function for various combinations of the 60/40 stock/bond portfolio and B1D5 ($\lambda = 5.5$).

% in BlD5	0%	20%	40%	60%	80%	100%
			4% Target			
Mean	2.80	3.87	4.80	5.68	6.54	7.37
Median	2.60	3.42	4.17	4.80	5.48	6.14
			5% Target			
Mean	2.20	3.40	4.40	5.31	6.18	7.03
Median	2.23	3.08	3.83	4.47	5.15	5.82
			6% Target			
Mean	1.31	2.73	3.85	4.82	5.72	6.59
Median	1.79	2.65	3.42	4.07	4.76	5.43

This table reports means and medians of the Prospect Theory utility function for various combinations of the 60/40 stock/bond portfolio and the low-risk B1D5 portfolio. All returns are simulated using a moving block bootstrap as described in Table 4 for an investment horizon of 20 years. The parameters of the utility function are $\alpha = 0.88$ and $\lambda = 5.5$. Results are shown for three target returns: 4%, 5%, and 6% a year.

Table 6 Bootstrap simulation results for various combinations of stock/bond portfolio and BlD5 ($\tau = 6\%$).

% in BlD5	0%	20%	40%	60%	80%	100%
			A. 70/30 and B1D5			
			$\lambda = 2.25$			
< Target	225	135	83	45	33	24
Omega ratio	0.14	0.37	0.88	1.80	3.16	5.26
Value function						
Mean	2.07	3.14	4.10	4.99	5.83	6.65
Median	1.82	2.79	3.50	4.15	4.82	5.43
			$\lambda = 5.5$			
< Target	225	135	83	45	33	24
Omega ratio	0.14	0.37	0.88	1.80	3.16	5.26
Value function						
Mean	1.34	2.78	3.90	4.87	5.75	6.59
Median	1.82	2.79	3.50	4.15	4.82	5.43
			B. 80/20 and B1D5			
			$\lambda = 2.25$			
< Target	252	146	94	50	33	24
Omega ratio	0.13	0.33	0.78	1.66	3.03	5.26

(*Continued*)

Table 6 (*Continued*)

B. 80/20 and B1D5						
$\lambda = 2.25$						
Value function						
Mean	2.17	3.23	4.17	5.04	5.86	6.65
Median	1.93	2.85	3.56	4.24	4.82	5.43
$\lambda = 5.5$						
< Target	252	146	94	50	33	24
Omega ratio	0.13	0.33	0.78	1.66	3.03	5.26
Value function						
Mean	1.32	2.80	3.94	4.91	5.77	6.59
Median	1.93	2.85	3.56	4.24	4.82	5.43

This table reports the performance across 1,000 simulations for various combinations of the 70/30 and 80/30 stock/bond portfolios and the B1D5 portfolio for a target return (τ) of 6% a year. The percentage allocated to B1D5 is indicated in row labeled "% in B1D5". The raw data for the simulations are monthly portfolio returns from January 1968 to December 2012. A moving block bootstrap with a block size of 5 years is used to simulate 20-year cumulative returns. The table reports three performance measures based on the simulated returns. Shortfall risk is percentage of simulated cumulative returns that fall below that implied by the target return of 6%. Omega ratio is the average of returns above a target return divided by the average of returns below the target return. The last two rows in each panel show the mean and median of the Prospect Theory utility function with $\alpha = 0.88$ and $\lambda = 2.25$ and 5.5.

benefit from low-volatility investing. For example, a 50–50 portfolio split between B1D5 and any of the stock/bond combinations also stochastically dominate the latter. Importantly, the utility function for this 50–50 strategy is always positive even in a "worst case" (e.g., 10th percentile) scenario.

What accounts for the superior long-run performance of B1D5? To answer this question, we repeat our simulations for portfolios that combine the 60/40 strategy with various weights for Bl and D5. We vary the weights for Bl from 0% to 100% (and thus the weights for D5 from 100% to 0%) in steps of 10%. As in previous tables, we vary the weight for the 60/40 strategy from 0% to 100% in steps of 20%. This gives a total of 66 portfolio combinations. At one extreme, we have a portfolio that invests only in the 60/40 strategy. At the other extreme, we have portfolios with only the low-risk strategies, specifically either Bl alone, D5 alone, or combinations of these two strategies. Table 7 displays the performance measures for all the portfolios based on a 20-year investment horizon and 1,000 bootstrap simulations. Specifically, we report (a) shortfall risk (the number of simulated cumulative returns below the target return (assuming a target return of 4% per annum)), (b) the mean prospect theory value function, (c) the value function Omega ratio, computed by dividing the average of value functions that exceed the target by the average of value functions falling short of the target, and (d) the Omega ratio for cumulative returns. These results are display in Panels A to D of the table.

One clear pattern that emerges from Table 7 is that for any combination of Bl and D5, the

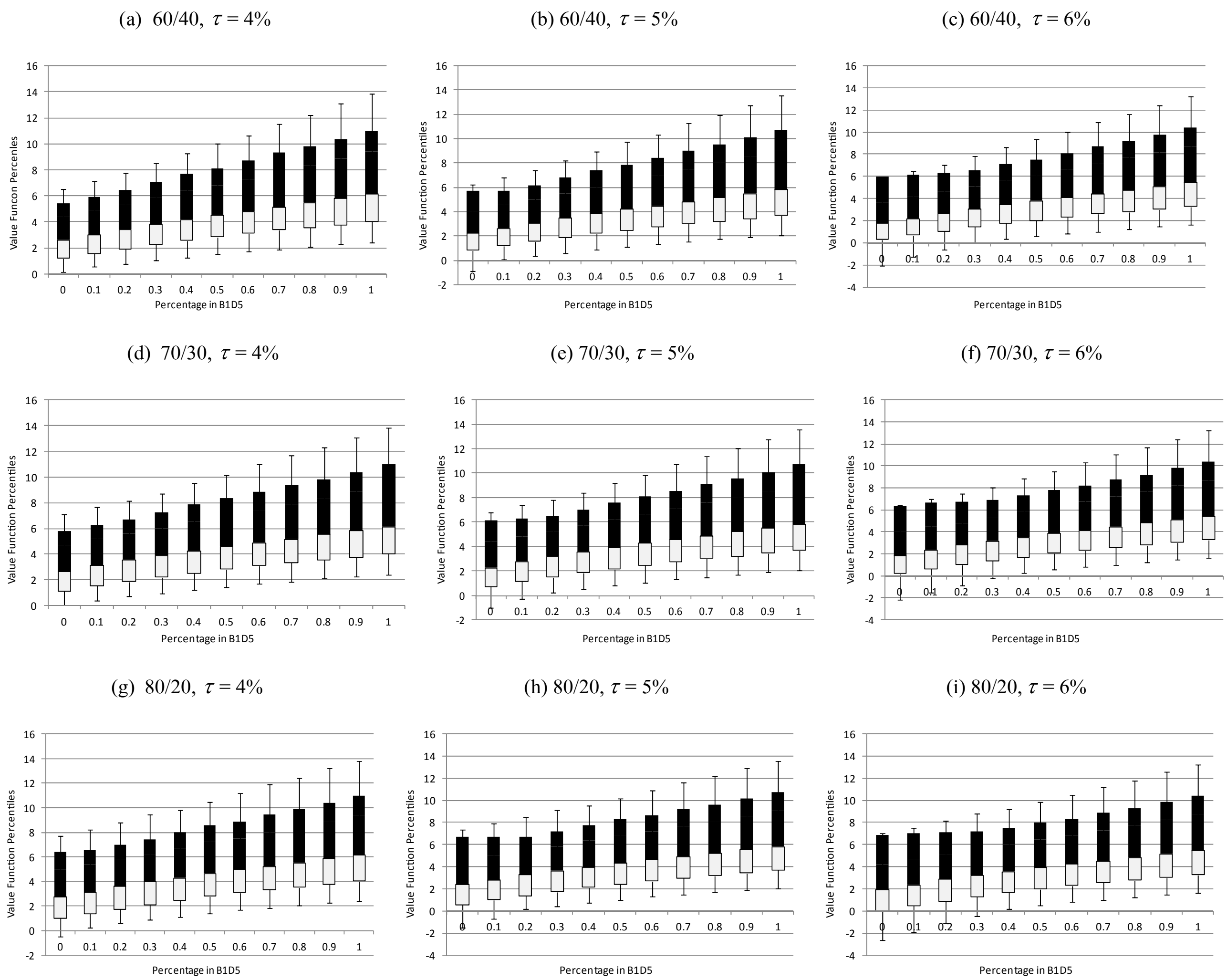

Figure 3 Percentiles of prospect theory utility function.

performance measure improves monotonically as we go from 100% in the 60/40 strategy to 100% in a low-risk strategy. That is, as the weight of low-risk stocks increases in the overall portfolio, shortfall risk decreases, while the average value function and Omega ratios for both the value function and cumulative returns increase. In this sense, both Bl and D5 play a role in improving long-run investment outcomes.

Reading the numbers across each row provides further insights into the relative contribution of Bl and D5 to performance improvements. The key results are as follows. First, Bl contributes more to reducing shortfall risk than D5, while D5 contributes more to increasing the mean value function. This could be linked to the fact that on average, D5 has higher monthly returns in "good" economic states than Bl, and good economic states outnumber "bad" economic states by 50% over the sample period.[3] Second, although D5 outperforms Bl in terms of mean monthly returns, D5 is more volatile and has a lower Sharpe ratio than Bl as shown earlier in Table 2. Higher volatility reduces some of the long-term benefits of strategies like D5 because it reduces the geometric mean return, a point we investigate further in the next section. Third, consistent

Table 7 Bootstrap simulation results: Contribution of Bl and D5.

% B1	% in Bl or D5 (60/40)						% B1	% in Bl or D5 (60/40)					
(D5)	0 (100)	20 (80)	40 (60)	60 (40)	80 (20)	100 (0)	(D5)	0 (100)	20 (80)	40 (60)	60 (40)	80 (20)	100 (0)
	Panel A. Number < Target							Panel C. Value function Omega ratio					
0 (100)	86	34	25	20	15	16	0 (100)	0.27	0.87	1.54	2.35	3.18	3.75
10 (90)	86	33	25	19	14	15	10 (90)	0.27	0.86	1.56	2.43	3.29	4.02
20 (80)	86	33	25	18	14	13	20 (80)	0.27	0.86	1.58	2.51	3.41	4.23
30 (70)	86	33	26	17	14	12	30 (70)	0.27	0.87	1.60	2.58	3.54	4.43
40 (60)	86	33	26	18	13	12	40 (60)	0.27	0.85	1.60	2.65	3.69	4.64
50 (50)	86	33	25	18	13	11	50 (50)	0.27	0.85	1.61	2.71	3.82	4.86
60 (40)	86	34	25	18	12	11	60 (40)	0.27	0.84	1.61	2.77	3.94	5.07
70 (30)	86	34	25	18	12	11	70 (30)	0.27	0.84	1.61	2.84	4.05	5.31
80 (20)	86	34	25	18	12	11	80 (20)	0.27	0.84	1.61	2.89	4.17	5.60
90 (10)	86	36	25	16	13	10	90 (10)	0.27	0.83	1.61	2.91	4.27	5.83
100 (0)	86	37	26	17	13	10	100 (0)	0.27	0.82	1.59	2.90	4.37	6.05
B1D5	86	31	20	13	9	6	B1D5	0.27	0.87	1.79	3.45	5.78	8.61
	Panel B. Mean value function							Panel D. Cumulative returns Omega ratio					
0 (100)	2.96	4.34	5.61	6.83	8.02	9.18	0 (100)	0.77	2.50	4.59	7.10	9.70	11.66
10 (90)	2.96	4.31	5.56	6.76	7.92	9.06	10 (90)	0.77	2.49	4.65	7.35	10.08	12.50
20 (80)	2.96	4.28	5.51	6.69	7.83	8.95	20 (80)	0.77	2.48	4.70	7.60	10.46	13.19
30 (70)	2.96	4.26	5.46	6.61	7.74	8.83	30 (70)	0.77	2.47	4.76	7.82	10.89	13.85
40 (60)	2.96	4.23	5.41	6.54	7.64	8.72	40 (60)	0.77	2.46	4.79	8.03	11.36	14.55
50 (50)	2.96	4.21	5.36	6.47	7.55	8.60	50 (50)	0.77	2.45	4.81	8.23	11.76	15.30
60 (40)	2.96	4.18	5.31	6.39	7.45	8.48	60 (40)	0.77	2.44	4.82	8.44	12.16	16.01
70 (30)	2.96	4.15	5.26	6.32	7.35	8.36	70 (30)	0.77	2.43	4.84	8.63	12.54	16.82
80 (20)	2.96	4.13	5.21	6.25	7.25	8.24	80 (20)	0.77	2.41	4.83	8.78	12.95	17.75
90 (10)	2.96	4.10	5.16	6.17	7.16	8.12	90 (10)	0.77	2.40	4.82	8.87	13.30	18.53
100 (0)	2.96	4.07	5.10	6.10	7.06	8.00	100 (0)	0.77	2.37	4.78	8.89	13.64	19.26
B1D5	2.96	3.94	4.84	5.71	6.55	7.38	B1D5	0.77	2.50	5.31	10.50	17.89	27.42

This table reports the performance across 1,000 simulations of portfolios formed by blending the 60/40 stock/bond strategy with various weights for Bl, and D5. The holding period is 20 years. The first column of each panel shows the weights for Bl (D5), whereas the first row of indicates the asset allocation between the low-risk strategies (Bl, D5 or their combinations thereof) and the 60/40 strategy. For example, the label 0 (100) denotes a portfolio that is 100% invested in the 60/40 strategy, the label 20 (80) denotes a portfolio that is 20% invested in the low risk strategies and 80% in the 60/40 strategy, and so on. Panel A reports the number of simulated cumulative returns that fall below that implied by the target return (set at 4% a year). Panel B reports the mean Prospect Theory value function with $\alpha = 0.88$ and $\lambda = 2.25$. Panel C reports the value function Omega ratio, computed by dividing the average value function for target exceedances by the average value function for target shortfalls. Panel D reports an analogous Omega ratio using cumulative returns.

with our first point, Bl's lower downside risk translates into higher Omega ratios for this strategy than for D5 as can be seen in Panels C and D of Table 7. Overall, the results indicate that Bl and D5 contribute differently to the risk-return tradeoff facing a long-term investor, and both are essential to the success of the blended strategy B1D5. Nonetheless, as the last row of each panel in Table 7 shows, the risk-adjusted performance of B1D5 exceeds any of the combination of Bl and D5, indicating that B1D5 provides considerable "synergies" for the long-term investor.

6 The benefits of low volatility

As mentioned, due to compounding, higher volatility implies a lower geometric average return, an effect that is more pronounced

Table 8 Bootstrap simulation results for various combinations of stock/bond portfolio and BlD5 ($\tau = 6\%$; $\lambda = 2.25$).

			A. 60/40 and modified B1D5			
% in BlD5	0%	20%	40%	60%	80%	100%
< Target	219	121	109	104	103	107
Omega ratio	0.15	0.18	0.22	0.25	0.27	0.27
Value function						
Mean	1.95	2.16	2.36	2.53	2.69	2.84
Median	1.79	1.90	2.06	2.17	2.17	2.25
			B. 70/30 and modified B1D5			
< Target	225	204	185	168	154	154
Omega ratio	0.14	0.17	0.21	0.24	0.26	0.27
Value function						
Mean	2.07	2.26	2.43	2.58	2.72	2.84
Median	1.82	2.03	2.17	2.21	2.20	2.25
			C. 80/20 and modified B1D5			
< Target	252	144	82	53	40	32
Omega ratio	0.13	0.16	0.19	0.23	0.26	0.27
Value function						
Mean	2.17	2.35	2.50	2.63	2.75	2.84
Median	1.93	2.09	2.22	2.26	2.25	2.25

This table reports the performance across 1,000 simulations for various combinations of three stock/bond portfolios and a modified B1D5 portfolio for a target return (τ) of 6% a year. The modified B1D5 portfolio has the same arithmetic average monthly return as the overall stock market but the same volatility as the original B1D5 portfolio. The percentage allocated to the modified B1D5 portfolio is indicated in row labeled "% in B1D5". The raw data for the simulations are monthly portfolio returns from January 1968 to December 2012. A moving block bootstrap with a block size of 5 years is used to simulate 20-year cumulative returns. The table reports three performance measures based on the simulated returns. Shortfall risk is percentage of simulations runs where the cumulative return is below that implied by the target return of 6%. Omega ratio is the average of returns above a target return divided by the average of returns below the target return. The last two rows in each panel show the mean and median of the Prospect Theory value function with $\alpha = 0.88$ and $\lambda = 2.25$.

for riskier securities. Conversely, long-term investors are well placed to benefit from low-risk securities even if these securities do not have higher average returns than riskier ones. To see the power of lower volatility over the long-term, we now put the B1D5 strategy to a more stringent test. Over our sample period, the arithmetic mean return for B1D5 is 12.9%, compared to 10.4% for the market. We carry out a counterfactual test by asking how B1D5 would perform if its arithmetic mean return is reduced to that of the stock market. We will call this counterfactual strategy, the modified B1D5 strategy. If the advantage of low-risk investing is mainly due to lower volatility, then the modified B1D5 strategy should still outperform the stock/bond portfolios. The results, shown in Table 8, confirm this hypothesis. To save space, we only show the results for a 6% target return.

As expected, lowering the mean return for B1D5 worsens the downside risk, as reflected by larger shortfall risk and lower Omega ratios than those reported in Tables 4 and 5. Similarly, the mean and median utility for all but the pure stock/bond strategies are significantly lower than before. The modified B1D5 strategy, however, is still the optimal strategy on all four performance criteria, confirming the beneficial effects of lower volatility for long-term investors.

7 Conclusions

This paper presents a novel approach to strategic asset allocation from the viewpoint of loss-averse investors. Using simulations, we compare the performance of traditional asset allocations such as the 60/40 stock/bond portfolio with an alternative strategy in the form of a beta-yield portfolio that combines stocks with low betas and high dividend yields. There is pervasive evidence that low beta stocks outperform high-beta stocks in risk-adjusted returns. The beta-yield strategy exploits this beta anomaly. Empirical evidence also shows that high dividend yield stocks decline less in down markets than low dividend yield stocks. The beta-yield strategy also exploits this hedging property of high dividend yield stocks. Using bootstrap simulations of 20-year buy-and-hold returns, we show that the beta-yield portfolio strongly outperforms the 60/40, 70/30, and 80/20 stock/bond portfolios on several criteria, including terminal wealth, shortfall risk, Omega ratio, and Prospect Theory utility. Indeed, this result holds even when arithmetic mean return of the beta-yield portfolio is reduced to that of the market portfolio, consistent with the intuition that lower volatility significantly boosts the geometric mean return of the least volatile stocks. We conclude that the beta-yield portfolio or similar low-risk portfolios offer a compelling alternative to stock/bond allocations for long-term investors.

Notes

[1] To compute a firm's book-to-market ratio, we use the stock price at the portfolio formation date and the most recent fiscal year's book value of equity with a 6-month lag to ensure that the data are available to investors at the time. We define book values in the same way as Asness *et al.* (2013).

[2] The proof can be deduced from proposition 2 of Ogryczak and Ruszczvynski (1999).

[3] This result holds whether we classify economic states using the overall stock market's returns (positive for good states and negative for bad states) or using a broad-based measure of business activity such as the Chicago Fed National Activity Indicator (CFNAI).

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