JOURNAL OF INVESTMENT MANAGEMENT, Vol. 13, No. 3, (2015), pp. 5–23

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CONSUMPTION, INVESTMENT AND INSURANCE in the Game of Life*

Harry M. Markowitz

Markowitz (1991) proposed the development of a "Game of Life" simulator in which portfolio selection was just one type of move in the financial actions of a subject household. Sherri Grabot's invitation to Markowitz in the late 1990s to form and join the design committee of GuidedChoice (originally a 401k advisory service) permitted Markowitz and team to take the first steps in applying the theory in practice, with favorable results for participants.

1 Introduction

Markowitz (1991) argues that financial decisions for the individual or family should be considered as part of the "Game of Life" that the individual or family plays out. Even reducing this game to its essentials, it is much too complex to solve analytically, therefore requires computer simulation to think through. The object to be analyzed is the nuclear family, consisting of an unattached individual, a couple, or a family with children and perhaps a residing elder. Typically, in the course of events the residing elder (if any) dies; the children leave home to set up their own nuclear families; and the subject family then consists of husband and wife. One spouse dies; one survives. When the remaining spouse dies the family's wealth is distributed to heirs and charity, and the Game of Life is over for the subject family. The proposed Game-of-Life simulator would try to emulate a Rational Decision Making (RDM) family for guidance for recommended behavior rather than describing a real (perhaps dysfunctional) family.

Late in the 1990s Sherrie Grabot, CEO of GuidedChoice (GC) explained to me that GC was a 401(k) advisory service with a different business model than used then by others in the industry, such as Bill Sharpe's Financial Engines. I expressed interest in Grabot's enterprise, gave her a copy of Markowitz (1991), agreed to form

*The recollections recounted, and the views expressed, here are those of the author and do not necessarily reflect those of GuidedChoice, Inc. or any individual therein besides myself.

a San Diego-based design and production team, and to consult for GC. GC would not try to build a complete Game-of-Life model, but to set it as an ideal—a North Star—toward which we would direct our model-building, starting with the immediate objective of helping investors save for their retirement.

As it turned out, the GC design team—consisting of Ming Yee Wang, Gan Lin Xu, and me, under Sherrie Grabot's guidance—designed and supervised the building of a set of interrelated programs and procedures that constitute GuidedChoice's Data Support System (DSS). The GC simulator is embedded in this DSS. Basically, the DSS includes programs and procedures to

(1) Make forward-looking estimates of asset-class expected returns, variances and covariances;
(2) Generate an MV-efficient frontier at the asset-class level;
(3) Elicit from a new client (namely, a corporate 401k plan sponsor) plan specifics such as the securities in which plan participants can invest and the plan's provisions for matching participant's contributions;
(4) Assign a portfolio of plan-permitted investments to each MV-efficient asset-class portfolio;
(5) Elicit from a plan participant, and from the plan's record keeper on behalf of the participant, information needed by the GC simulator;
(6) Inform the participant of results of a simulation analysis concerning the probability distribution of possible post-retirement consumption levels, taking into account the participant's savings rate, the company's matching policy, the portfolio selected, Social Security payments, the participant's spouse's income (if the client so specifies), etc.
(7) Instruct the plan's record keeper to execute GC's advice if the participant so elects; and
(8) Report account status to the participant and the plan sponsor.

GC's first product, GuidedSavings, evaluates participant's savings and investment plans up to the time of their retirement. A subsequent second product, GuidedSpending, evaluates savings, investment, and consumption strategies perhaps starting before retirement and continuing through retirement.

At various points in this paper, I will make use of the EAS-E (Entity, Attribute, Set and Event) view of dynamic system description. The remainder of this paper starts by describing the EAS-E view. I next summarize the Markowitz (1991) sketch of a possible Game-of-Life simulator. The latter "sketch" is in fact quite sketchy, since it was an off-the-top-of-the-head rumination on a possible direction for research into financial planning for the so-called "individual" investor. I then present specifics concerning the GC "first step" toward a Game-of-Life advisory system. These include an extract from the EAS (Entity, Attribute, Set) table used in the design of the GC DSS database; the utility functions we used to score the outcomes of simulation runs; some observations on the probability distribution of the "principal components" used to generate the simulated history of asset-class returns; and the model GC is currently using concerning the possible evolution of interest rates. The discussion of the GuidedSavings utility function includes a comparison of the "glide path" implications of this utility function as compared to the glide paths typically recommended by life cycle funds—and why we consider ours to be the superior one. I then present some performance statistics and conclude with some thoughts about the future of financial models and systems.

2 The EAS-E worldview

The EAS-E (Entities, Attributes, Sets and Events) view of dynamic systems modeling was first used in the original SIMSCRIPT programming language (Markowitz *et al.*, 1963), now called SIMSCRIPT I to distinguish it from SIMSCRIPTs II and III that followed. SIMSCRIPT I was conceived as a simulation language, hence its name. In contrast, SIMSCRIPT II was conceived as a general purpose programming language including *database* entities, attributes, and sets. The idea here was that the basic SIMSCRIPT premise—that the world consists of Entities, Attributes and Sets and changes with Events—applies to "the world" as represented by a database as well as that in a simulation model. (See Markowitz, 1979.)

In an EAS-E simulation model: As of any instant of time, the simulated world is viewed as having a "*Status*" that is described by how many "*Entities*" (or individuals) of a given "*Entity-Type*" exist, what are the values of their "*Attributes,*" what "*Sets*" do they "*Belong*" to, and who are the "*Members*" of the sets they "*own*?" "*Events*" change status and/or cause future events. Since status consists of entities, attributes and sets, an event changes status by one or more actions that create or destroy an entity, read in or calculate an attribute value, or file an entity into, or remove an entity from, a set. An event may cause future event-occurrences or cancel ones already scheduled.

For example, in the JLMSim stock market simulator (Jacobs *et al.*, 2004) entity-types include Investor, Trader, Portfolio_Analyst, Security and Order Slip. A security "*owns*" (i.e., has associated with it) a set called Buy_orders and a set called Sell_orders, each of which has order slips as their members. Each is sorted by the bid (or asked) price attribute of the order slip.

The use of the word "owns" to describe a set associated with entities of a given type works well unless the system-to-be-described contains "owners" in a legal sense. An extreme example would be a system that contained an entity-type called Partnership that owned (in the EAS sense) a set called Owners (in the legal sense). The sense in which an A owns a B is usually clear from context.

It is not necessary to program in SIMSCRIPT to make use of the EAS-E worldview. For example, JLMSim was conceived in EAS-E terms, and then programmed in C++.

3 The Markowitz (1991) "Game of Life" proposal

Professor Mandell, editor of Financial Services Review, invited me to contribute an article related to financial research for the individual investor for the first issue of his journal. Since the subject is not my specialty, it was uncharacteristically risky of me to have accepted the invitation. But an evening of reflection convinced me that there were clear differences between the needs of individual and institutional investors. These differences suggest differences in desirable research methodology; and that a note on these differences might be of value.

I proposed that a Game-of-Life simulator could be developed with sufficient realism to help guide financial planning in fact. For example, some economic theories find it convenient to assume that the individual is immortal, or that death is a Poisson process independent of the age of the individual. For actual financial planning, however, aging and mortality are salient facts that must be included in the model. The model should also include the probability of an accident or disease that will keep the individual from work for an extended period, the probability distribution of time to recover or die, the costs of treatment and probability of relapse, since these possibilities should be major factors in financial planning, including the purchase of medical and disability insurance as well as the keeping of reserves. Other major life-decisions (or accidents) that affect an individual's or family's balance sheet and cash flows include the pursuit of an education; the buying of a house, car, or other consumer durables; the having of children; sending children to college; the decision to retire and perhaps to move to a warm climate.

In EAS-E terms, the proposed Game of Life model would distinguish between the entity-types *Person* and *Family*. Typically, at some stage a family "owns" (in the EAS-E sense) a set of individuals whose roles are husband, wife, children, and perhaps residing elder. Assets would be thought of as belonging to the family rather than the individual, to be used by husband and wife and (at husband and wife's discretion) by and for the children including for their education. Attributes of individuals include those needed to characterize health, the employment or employability of husband and wife, and the educational objectives of each child.

Among other assets, the family may own (in the EAS-E sense) one or more residences, which may be owned (in the legal sense) or rented. If owned (in the legal sense) the residence is characterized by its cost basis and a current market value. Whether owned or rented, a home includes furniture and appliances. A home and its furnishings are clearly illiquid assets, partly because of the time and cost to sell, but also the time and cost to move, and the mismatch between the furniture needs of the old and new residences.

Events in the Game of Life include periodic events such as receiving a salary check, having a birthday, and the time when an income tax payment is due; plus randomly occurring events such as becoming sick, becoming well, finding a job, losing a job, financing a house to buy, finding a buyer for a house to be sold, and replacing consumer durables. Changes in price levels, interest rates, and stock and real estate values could be computed periodically.

The model should also include—as essential to evaluating family investment practice in fact—things such as IRAs, Keoghs, Social Security, pension plans, various kinds of insurance, their costs, and the kinds of events they insure against. The simulated family must make decisions at various points in time, such as the level of this week's nondurable consumption, transfers from cash to other liquid assets, the decision to search for and then buy a new house, and the decision of a member to retire. A major purpose of the model would be to evaluate alternative decision rules in the making of such decisions.

As noted in Introduction, in the course of events the residing elder (if any) dies or is placed in a nursing facility; the children leave home to set up their own nuclear families; the original family (the subject of the model) then consists of husband and wife. When one dies the subject family consists of the survivor only. When the latter dies, the assets of the subject family are distributed to heirs, and the Game of Life is over for the subject family. Some kind of utility function would be needed to score each play of this game. Random repetitions of the game would be used to estimate the expected utility to the family of the game-as-a-whole when played with particular decision rules.

As with most simulation programs, a Game-of-Life simulator would have adjustable parameters allowing its user to control the level of detail included. For example, it may be sufficient for some purposes to characterize financial assets as stocks, bonds, and cash, whereas for other purposes it may be essential to distinguish various asset-classes such as Large Cap and Small Cap stocks, Non-U.S. developed and emerging market stocks, and various classes of bonds.

4 The GC DSS database

As noted in Introduction, GC's initial step toward a Game-of-Life simulator is imbedded in GC's Decision Support System. The DSS's memory of itself and its obligations are stored in its database. An examination of the kinds of information in this database can serve to illustrate what is involved in trying to support good financial decision-making in the real world.

Table 1 presents an extract from the EAS (Entity, Attribute and Set) table used to plan, implement, and document the GC DSS database. The first column of the table indicates "Entity-type," such as Person, Dependent, and Planned_disbursement. The second column contains the names of attributes of entities of the just listed entity-type. Examples of attributes include Birth-date, Soc_sec_num, and Marital_status of Person. The third column of the exhibit lists sets "Owned By"—in the EAS-E sense, that is, sets *associated with*—individuals of the particular entity-type. For example, each Person owns sets called Dependents, Portfolios, and Planned_disbursements. The fourth column of the exhibit indicates the data-type of an attribute or the entity-type of the members of a set. For example, the Birth_date of Person is in Date format, and the members of the set called Dependents are of entity-type Dependent.

The fifth column is labeled precision. It indicates, for example, that 32 characters are allotted for a State's name, which is also assigned a two-character State code. The final field of the exhibit includes comments and cross-references. The list of entity-types under the comments/cross references heading across from entity-type Authorization_memo, for example, indicates that individuals of this entity-type are referenced by The_SYSTEM and entities of entity-type Person. Such cross-references are not a standard part of an EAS table, but proved useful in implementing this complex system relatively quickly with minimal resources.

An EAS description invariably has an entity-type called The System representing the "system-as-a-whole." Table 1 shows that the GC System owns various sets of "top level" entity-types. Entities of other entity-types are accessed through these top level entities. Top level entity-types include everything from Sponsors (of 401k plans), Record_keepers, Securities, and so on down to entries in the Transaction_log_book, Event_log_book, and Error_log_book. Attributes of The_System include Federal limits on the dollars (D) or percent (P) that may be contributed to a 401(k) plan.

This extract from the full GC EAS table includes, as its only "compound entity" a Person_X_Comp_type combination. (Comp_type is short for "compensation type" such as full-time as opposed to overtime salary.) The full GC EAS documentation has many more examples of what SIMSCRIPT texts call "compound entities" and mathematicians would call "Cartesian product" spaces.

For the most part the contents of Table 1 should be self-explanatory. In part they reflect the context of the portfolio selection decision including income tax rates for various income brackets, both Federal and State, the latter varying from state to state; regulations and company policies with respect to how much income the participant can shelter in his or her 401(k) plan; the company's matching policy; the participant's birthdate and intended retirement age, and perhaps that of his or her spouse; the participant's dependents and planned disbursements to them; and the participant's other accounts such as, for example, 401(k) plans with prior employers.

Other data in the database are oriented toward reporting status information to the plan sponsor,

Table 1 GuidedChoice EAS table.

COM	ENT	ATTR	OWNS	Data/Memb	PREC	COMMENT/Cross references
COM	TYPE			TYPE	(*)	(*) All "Integer"s 32 bit All "Number"s 12,3
ENT	The_SYSTEM					
SET			Sponsors	Sponsor		
SET			Record_keepers	Record_keeper		
SET			GC_AC_frontiers	GC_AC_frontier		
SET			Trust_families	Trust_family		
SET			Security_types	Security_type		
SET			Securities	Security		
SET			Asset_classes	Asset_class		
SET			Partcpnt_auth_memos	Authorization_memo		Authorizastion memos of participants
SET			DB_users	DB_user		Other than participants
SET			Stdd_comp_types	Compensation_type		
SET			Stdd_plan_texts	Plan_text		
SET			Transaction_log_book	Transaction_log_entry		
SET			Event_log_book	Event_log_entry		
SET			Error_log_book	Error_log_entry		
SET			States	State		
ATT		Fed_limit_pretax_dlrs		Number		Currently $10,500
ATT		Fed_limit_pretax_pct		Number		Currently 25%
ATT		Fed_limit_total_dlrs		Number		Currently $30,000
ATT		Fed_limit_total_pct		Number		Currently 25%
SET			Fed_tax_brackets	Inc_tax_bracket		
ENT	State					SYSTEM
ATT		State_id		ID		

Table 1 (*Continued*)

COM	ENT	ATTR	OWNS	Data/Memb	PREC	COMMENT/Cross references
ATT		State_name		Text	32	
ATT		State_code		Char	2	
SET			Inc_tax_brackets	Inc_tax_bracket		
ENT	Inc_tax_bracket					SYSTEM, State
ATT		Inc_tax_bracket_id		ID		
ATT		Taxing_government		ID		State_id or -1 for federal govt
ATT		From_income		Number		
ATT		To_income		Number		
ATT		Marginal_rate_pct		Number		
ENT	Authorization_memo					SYSTEM, Person
ATT		Authorization_memo_id		ID		
ATT		Participant		Person_id		Person
ATT		Access_pin		Char	12	
ATT		Personal_question		Text	128	
ATT		Secret_answer		Text	128	
ATT		Pin_change_allowed		Char	1	
ATT		Pin_change_date		Date		<=Today
ENT	Person					||Person, Authorization_memo, GC_case, Session,
ATT		Person_id		ID		>Transaction_log_entry, Event_log_entry
ATT		First_name		Text	32	>Error_log_entry, Expenses_worksheet

(*Continued*)

Table 1 (*Continued*)

COM	ENT	ATTR	OWNS	Data/Memb	PREC	COMMENT/Cross references
ATT		Last_name		Text	32	
ATT		Middle_initial		Char	1	
ATT		Birth_date		Date		<Today
ATT		Gender		Char	1	.=MF
ATT		Contact_info		Contact_info_id		
ATT		Country_of_legal_residence		Text	32	
ATT		Marital_status		Char	1	YN
ATT		Why_in_DB		Char	1	.=PSB(Partcpnt, Spouse, both)
ATT		Other_income		Number		Not in Account's Person_comps
ATT		Total_income		Number		
ATT		Retirement_age		Integer		
ATT		Ret_income_goal_DorP		Char	1	D|P
ATT		AT_ret_income_goal		Number		
ATT		Spouse		person_id		Person
ATT		Expenses_worksheet		Expenses_worksheet_id		
ATT		Nr_dependents		Integer	2	
SET			Sign_ons	Authorization_memo		>=1 source doc says this is set?
SET			Dependents	Dependant		?should this be hooked onto marriage?
SET			GC_accounts	Account		
SET			Portfolios	Portfolio		
SET			Positions	Position		.= positions in all of Person's portfolios

Table 1 (*Continued*)

COM	ENT	ATTR	OWNS	Data/Memb	PREC	COMMENT/Cross references
SET			Pensions	Pension		
SET			Planned_disbursements	Planned_disbursement		
SET			Non_GC_plans	Non_GC_plan		Plans of Person rprsntng spouse go here
ENT	Dependent					‖Planned_ disbursement
ATT		Dependent_id		ID		
ATT		Participant		Person_id		
ATT		Dependent_name		Text	32	
ATT		Dependent_birthdate		Date		>=Today
ATT		Dependent_gender		Char	1	.=M|F
ENT	Planned_ disbursement					Person
ATT		Planned_disbursement_id		ID		
ATT		Participant		Person_id		
ATT		Dependent		Dependent_id		.=0 if not for college
ATT		Disbursement_period		Char		.=M|Q|S|Y
ATT		Disbursement_amt		Number		
ATT		Inflation_adjust_amt		Char	1	YN
ATT		Start_date		Date		
ATT		End_date		Date		>=Start_date
ATT		Disbursement_type		Integer		
ATT		Disbursement_name		Text	128	
ENT	Account					Plan, Person‖Pension, Session

(*Continued*)

Table 1 (*Continued*)

COM	ENT	ATTR	OWNS	Data/Memb	PREC	COMMENT/Cross references
ATT		Account_id		ID		>Transaction_ log_entry,
ATT		Participant		Person_id		
ATT		Plan		Plan_id		
ATT		Sponsor		Sponsor_id		
ATT		Contact_info		Contact_info_id		
ATT		Tax_state		Text	32	1 of 2 !=""
ATT		Spouse		Spouse_id		
ATT		Employee_status		Enum		1 Active 2 Hardship 3 Terminated
ATT		Eligibility_date		Date		
ATT		Eligibility_match		Char	1	YN
ATT		Eligibility_pension		Char	1	YN
ATT		Years_service		Integer		
ATT		Highly_compensated		Char	1	YN
ATT		Hire_date		Date		<=Today
ATT		GC_advice_accepted		Char	1	YN
ATT		Start_advice_date		Date		
ATT		End_advice_date		Date		
ATT		Eligibility_profit_share		Char	1	YN
ATT		Phone_access		Char	1	YN
ATT		Annual_salary		Number		
ATT		Pretax_earnings		Number		
ATT		Posttax_earnings		Number		
ATT		Last_use		Date		<=Today
ATT		RK_update_date		Date		
ATT		Accepted_case		GC_case_id		
ATT		Date_case_accepted		Date		
ATT		Base_case		GC_case_id		

Table 1 (*Continued*)

<table>
<tr><th>COM</th><th>ENT</th><th>ATTR</th><th>OWNS</th><th>Data/Memb</th><th>PREC</th><th>COMMENT/Cross references</th></tr>
<tr><td>ATT</td><td></td><td>Initial_advice</td><td></td><td>GC_case_id</td><td></td><td></td></tr>
<tr><td>ATT</td><td></td><td>Modified_advice</td><td></td><td>GC_case_id</td><td></td><td></td></tr>
<tr><td>ATT</td><td></td><td>Next_case</td><td></td><td>GC_case_id</td><td></td><td></td></tr>
<tr><td>ATT</td><td></td><td>Last_session</td><td></td><td>Session_id</td><td></td><td></td></tr>
<tr><td>ATT</td><td></td><td>OK_rcvd_prime_bnf_NE_spouse</td><td></td><td>Char</td><td>1</td><td>YN OK received for prime beneficiary not spouse</td></tr>
<tr><td>SET</td><td></td><td></td><td>Beneficiaries</td><td>Beneficiary</td><td></td><td></td></tr>
<tr><td>SET</td><td></td><td></td><td>Portfolios</td><td>Portfolio</td><td></td><td></td></tr>
<tr><td>SET</td><td></td><td></td><td>Person_Comp_types</td><td>Person_Comp_type</td><td></td><td></td></tr>
<tr><td>ATT</td><td></td><td>Contrib_spec_PorD</td><td></td><td>Char</td><td>1</td><td>P|D If Plan permits either P (%) or D ($)</td></tr>
<tr><td>SET</td><td></td><td></td><td>Current_contribs</td><td>Contrib_instruction</td><td></td><td></td></tr>
<tr><td>SET</td><td></td><td></td><td>BT_contrib_allocs</td><td>Contrib_allocation</td><td></td><td></td></tr>
<tr><td>SET</td><td></td><td></td><td>AT_contrib_allocs</td><td>Contrib_allocation</td><td></td><td></td></tr>
<tr><td>SET</td><td></td><td></td><td>PS_contrib_allocs</td><td>Contrib_allocation</td><td></td><td></td></tr>
<tr><td>SET</td><td></td><td></td><td>Archived_cases</td><td>GC_case</td><td></td><td></td></tr>
<tr><td>ENT</td><td>Portfolio</td><td></td><td></td><td></td><td></td><td>Person, Account|| Contrib_instruction</td></tr>
<tr><td>ATT</td><td></td><td>Portfolio_id</td><td></td><td>ID</td><td></td><td></td></tr>
<tr><td>ATT</td><td></td><td>Participant</td><td></td><td>Person_id</td><td></td><td></td></tr>
<tr><td>ATT</td><td></td><td>Account</td><td></td><td>Account_id</td><td></td><td>NULL if not owned by GC_Accouont</td></tr>
<tr><td>ATT</td><td></td><td>Tax_type</td><td></td><td>Investment_tax_type_id</td><td></td><td>Enum in EJB; Entity in Administrator</td></tr>
</table>

(*Continued*)

Table 1 (*Continued*)

COM	ENT	ATTR	OWNS	Data/Memb	PREC	COMMENT/Cross references
ATT		Portfolio _name		Text	128	
ATT		Accum_AT_contrib		Number		
ATT		Monthly_planned_ contrib_dlrs		Number		
ATT		Inflation_adjust_contrib		Char	1	YN
SET			Positions	Position		
SET			AC_exposures	Exposure		
ENT	Position					Person, Portfolio‖ Kept_investment
ATT		Position_id		ID		
ATT		Security		Security_id		
ATT		Portfolio		Portfolio_id		
ATT		Person		Person_id		
ATT		Security_type		Security_type_id	32	Must be on security type list
ATT		Tax_type		Enum		
ATT		Quantity		Number		Shares or face value
ATT		Valuation_method		Enum		Mkt price, user price, user total
ATT		User_supplied_price		Number		
ATT		Date_of_user_info		Date		
ATT		Total_value		Number		
ATT		Restricted_for_participant		Char	1	YN Company requirement
ATT		Date_unrestricted		Date		
ATT		Total_cost_basis		Number		

Table 1 (*Continued*)

COM	ENT	ATTR	OWNS	Data/Memb	PREC	COMMENT/Cross references
ENT	Person_Comp_X_type					Person, Compensation type Combo
ATT		Person_Comp_type_id		ID		
ATT		Account		Account_id		
ATT		Compensation_type		Compensation_type_id		
ATT		Compensation_type_name		Text	128	
ATT		Pay_periods_per_year		Integer		
ATT		Amount_per_pay_period		Number		
ENT	Compensation_type					SYSTEM, Plan, Eligible_comp_type‖
ATT		Compensation_type_id		ID		>Person_Comp_type, Contrib_instruction
ATT		Plan		Plan_id		
ATT		Compensation_type_name		Text	32	
ENT	Eligible_comp_type					Plan(2)‖PCH_ECT, Contrib_instruction
ATT		Eligible_comp_type_id		ID		>Savings_rate_spec
ATT		Plan		Plan_id		
ATT		Eligible_comp_type_name		Text	32	
SET			Compensation_types	Compensation_type		Overlapping sets

and the running of the DSS itself. The database must remember, for example, the set of accounts to be reviewed "now" for possible rebalancing transactions, and for the collection of GC's fee.

The events of a Decision Support System are those points in time at which system status is updated, reported, or called upon to take actions. Thus a complete EAS-E summary of the GC DSS would spell out more about Items (1) through (8) listed in Introduction.

GuidedChoice provided me an ideal proof-of-concept opportunity with which to demonstrate: (a) the EAS-E view's ability to neatly document complex systems, and (b) a step toward a Game-of-Life simulator and its use within a practical Decision Support System. The procedure for building the GC DSS was for Sherrie Grabot to initially explain to the design team what she wanted from her DSS. The design team then spelled out implementation details, frequently checking with Grabot about fine points. The design team documented the database structure—GC's own model of itself and its commitments—with an ever-growing EAS table.

Although the system was put through many tests before release, much was learned after the system encountered real client data—often because the client's portfolio selection problem had a greater variety of angles and wrinkles than the GC design team had anticipated. As noted above, overall the complex GC DSS was implemented with minimal resources and rolled out relatively painlessly.

The designs of financial decision support systems are rarely if ever part of financial education, but they are a large part of financial practice. Often such DSSs are not identified as such, and have no system-wide documentation. When a person or department needs something they tell the guy or gal from IT. Often it is found afterward—sometimes a long time afterward—that what was wanted was not said or written, what was said or written was not understood, what was understood was not programmed, and what was actually programmed was never documented. I contend that ideally a DSS should be planned from the ground up with the help of the EAS-E view. But no matter how an existing DSS came into being, it can be provided an EAS-E documentation. This may take some digging, but the result will be a system that is more easily understood—by management and by technical newcomers, for example—and thus more easily maintained.

5 Utility functions

A practical Game-of-Life simulator imbedded in a DSS needs to do more than just characterize status and events to "*do no harm*" and perhaps some good for its users. It needs, in particular, (1) a "good enough" model of how the modeled system evolves through time, including asset-class returns; (2) how investor decisions (such as savings rate and portfolio choice) affect desirables such as retirement wealth and (3) a utility function with which to evaluate alternative probability distributions of consumption-streams or retirement wealth scenarios. The present section discusses the utility functions used in the two GC products. The following sections discuss some aspects of the GC return generation model.

Both GuidedSavings and GuidedSpending offer initial advice based on the Monte Carlo evaluation of various proposals. (The participant is encouraged to vary the parameters of this initial advice and review the resulting Monte Carlo analysis as often as they wish.) To form its initial advice, a utility function was needed for each of GC's two products. These are used to assign a utility number to each run of the Monte Carlo analysis and, from these, an estimate of the expected utility of a proposal. In the case of GuidedSavings, utility had to be a function of wealth at retirement time:

$$U = U(W_T) \tag{1a}$$

For GuidedSpending utility had to be a function of the participant's simulated consumption stream plus bequest amount

$$U = U(C_1, C_2, \ldots, C_T, W_T) \tag{2}$$

We will discuss each of these in turn.

(1) *GuidedSavings*. Mossin (1968) and Samuelson (1969) have shown that if $U(W_T)$ in Equation (1a) is logarithmic or a power function:

$$U = \log(W_T) \tag{1b}$$

$$U = W_T^{\alpha} \tag{1c}$$

and certain other conditions are true, then the utility function would be myopic. Specifically, if investment opportunities did not change, the optimal asset allocation would not change as the participant approaches retirement. Since it seems plausible that the participant should typically invest more cautiously as retirement approaches, alternatives are frequently sought.

Certain life-cycle models assume that the present value of one's future labor is like a bond. As this implicit bond shrinks in value the participant is advised to shift from stocks to bonds, thus effectively maintaining a fixed stockbond ratio. The problem with this argument is that many investors probably should consider their employment incomes more like equity returns than fixed income. Examples of such include everyone who was concerned about their job, or at least their take-home pay, during the great recession. About the only individuals who might consider their salary to be "fixed income" are tenured (associate or full) professors. Thus the life cycle plans in question may be thought of as "of tenured professors, by tenured professors, for tenured professors."

A more plausible explanation for our intuition that the investor should become more cautious as retirement approaches has to do with the opportunity for growth in the long run when much time remains versus the seriousness of substantial losses if little time remains. The GC system designers found that, with suitable choice of parameters, this motivation and its effect can be adequately represented by the simple device of having some aspiration level $\tilde{W}$ for retirement real wealth, and letting

$$U(W_T) = \begin{cases} (W_T/\tilde{W})^{\alpha}/\alpha) & \text{for } W_T \leq \tilde{W} \\ \log(W_T/\tilde{W}) + \alpha^{-1} & \text{for } W_T > \tilde{W} \end{cases} \tag{3}$$

for some $\alpha < 0$. For sufficiently large retirement wealth, the investor seeks return in the long run (in the maximum growth rate sense). At lower wealth levels, the investor is more cautious.

$U(W_T)$ in Equation (3) is continuous, but has a "kink" (i.e., a discontinuous first derivative) at $W_T = \tilde{W}$. On the other hand, the derived utility function $U(W_{T-1})$ produced by one step in the dynamic programming process typically does *not* have a discontinuous first derivative. Rather $U'(W_{T-1})$ is continuous, but the second derivative $U''(W_{T-1})$ has a discontinuity. If we repeat the dynamic programming process we obtain $U(W_{T-2})$ whose second derivative is continuous but its third derivative has a discontinuity, and so on.[1] Thus, the single-period utility functions derived from Equation (3) by the dynamic programming process for $t = T - \tau$ are (for $\tau \geq 2$ or 3) smooth functions that are approximately logarithmic for high $W_t/\tilde{W}$ and approximately a more cautious power function for low $W_t/\tilde{W}$.

Specific results with the utility function in Equation (3) depend on specific assumptions, of course, but the general nature of the results can be seen if we make some commonly made simplifying assumptions concerning the return generating process. In particular, we will consider some glide path characteristics of Equation (3) utility function assuming that the returns on all available portfolios are log normally distributed (conveniently overlooking the fact that a portfolio consisting of securities each of whose return distribution is log normal will typically not itself have a log normal return distribution). We will also assume here that the investor chooses the same distribution repeatedly. Thus the conditional distribution of retirement wealth W_T at an earlier time t is also log normally distributed. We emphasize that these are not an assumption of GC's GuidedSavings model, but of our illustrative example.

If $W_T/\tilde{W}$ is log normally distributed then

$$(W_T/\tilde{W}) = e^y \tag{4a}$$

where y is normally distributed. In particular, $y \geq 0$ is equivalent to $W_T \geq \tilde{W}$. Rather than view an investor's utility as a function of W_T as in Equation (1a), we may view it as a function of y. Specifically, for Equations (1b) and (1c), respectively

$$\begin{aligned} V(y) &= \log_e(e^y) \\ &= y + \alpha^{-1} \end{aligned} \tag{4b}$$

and

$$\begin{aligned} V(y) &= (e^y)^\alpha/\alpha \\ &= e^{\alpha y}/\alpha \end{aligned} \tag{4c}$$

Thus the utility function in Equation (3) becomes

$$V(y) = \begin{cases} e^{\alpha y}/\alpha & \text{for } y < 0 \\ y + \alpha^{-1} & \text{for } y \geq 0 \end{cases} \tag{5}$$

Let m be the mean and σ be the standard deviation of the y distribution. Then the mean and standard deviation of the sum of T i.i.d. draws from this distribution are

$$m_T = Tm \tag{6a}$$

$$\sigma_T = \sqrt{T}\sigma \tag{6b}$$

Letting $\hat{w} = \log(\hat{W})$ and $\tilde{w} = \log(\tilde{W})$, if there are T periods (say, years) until retirement, then the wealth level $\hat{W}$ which is k standard deviations above $\tilde{w}$ satisfies

$$\hat{w} = \tilde{w} + \sqrt{T}k\sigma - Tm \tag{7}$$

It is shown in an endnote[2] that if we let

$$K = \frac{k\sigma}{m} \tag{8}$$

Then $\tilde{w} = \hat{w}$ at $T = 0$ and $T = K^2$. Also, $\hat{w} - \tilde{w}$ is greatest at

$$T = K^2/4 \tag{9}$$

at which time

$$\hat{w} - \tilde{w} = k^2\sigma^2/(4m) \tag{10}$$

For example, if $m = 0.1$, $\sigma = 0.2$ and $k = 3$, then $K = 6$ and $\hat{w} = \tilde{w}$ at $T = 0$ and 36 years. In other words, with these parameter values one may reasonably expect maximizing expected log to be near optimal—from now until retirement—if $w_{36} = \tilde{w}$. The greatest ratio $W_T/\tilde{W}$ is required at

$$\begin{aligned} T &= K^2/4 \\ &= 9 \text{ years} \end{aligned}$$

At that time the ratio required to "safely" ignore the prospect of ending with W_T below $\tilde{W}$ is

$$\begin{aligned} \hat{W}/\tilde{W} &= \exp(k^2\alpha^2/4m) \\ &= 2.46 \end{aligned} \tag{11}$$

Thus, in this respect at least, the glide-path associated with the utility function defined in Equation (3) is quite different than the one implied by the human capital explanation. In the latter, bond holdings increase monotonically in a manner unrelated to the investor's wealth. In the case of the utility function in Equation (3), for a given ratio of current wealth to target wealth, choice is a non-monotonic function of time.

(2) *GuidedSpending*. The GC utility function of form Equation (2) uses two numbers available in GuidedSpending but not in GuidedSavings. GC asks the user for two levels of consumption, C_U and C_L. At any point in simulated time in a Monte Carlo run, a proposed current consumption C is determined by an actuarial calculation that allows for the participant living somewhat longer than expected. If C exceeds C_U the difference, $C - C_U$, is saved. If C is less than C_L then $C_L - C$ is dissaved—if available. As described below, C_U and C_L are used in the calculation of U, as is an aspiration level of bequest B. If the participant declines to supply either B or C_L and C_U, then default values are supplied at levels dependent on the user's likely retirement wealth.

To compute the U attached to a particular consumption history and bequest, we form a score S by combining the history's average consumption level A and its maximum year-to-year decline in consumption D:

$$S = A - \alpha D$$

The idea here is that it is better to start poor and end rich than vice versa. We then normalize S forming a "normalized score," NS, such that if $A = C_U$ and $D = 0$ then $U = 1$ whereas if $A = C_L$ and $D = 0$ then $U = 0$. A term reflecting W_T versus B is added for a final score FS. Finally, utility is computed as a function of FS

$$U = f(\text{FS})$$

where f is a smooth curve with $U = 1$ as an asymptotic upper bound and with U dropping off steeply as FS drops below zero.

6 The GC model of return evolution

An investment/consumption simulator requires a model of how returns evolve. Here we will sketch two features of the GC returns generating process, to illustrate the considerations involved.

(1) GC performed a Principal Components Analysis (PCA) to model the covariance structure of asset-class returns. This is plausible for its own sake and is especially important because our asset-class data series are of differing length. It is frequently assumed that PCs are i.i.d. log normal. We found that our first few principal components were autocorrelated and most likely generated by a Pearson Family Type IV distribution. Thus, i.i.d. draws from a normal or log normal distribution for successive values of PCs would be quite unrealistic.

(2) Because of the current historically low interest rates, and the probability of these rising over time, we distinguished between near-term expected total returns to fixed income securities as compared to long-term estimates, and considered the probable paths from the one to the other. After exploring alternative models the GC design team decided that certain parameter settings for the CIR (Cox *et al.*, 1985) model, best described the range of interest rate sequences we have seen in the past and might well happen from this point forward.

7 Have we helped?

A frequent consequence of a participant's use of GuidedChoice's interactive Monte Carlo-based system is for the participant to increase his or her savings rate. Remarkably, it was found that participants who used GuidedSavings increased their savings rates by an average of 110%.

From time to time the technical team also does special analyses for the marketing support folk. For example, Figure 1 is based on an analysis by Tom Anachini of the technical team, at the request of one of GuidedChoice's larger clients as conveyed by the client support team. The figure

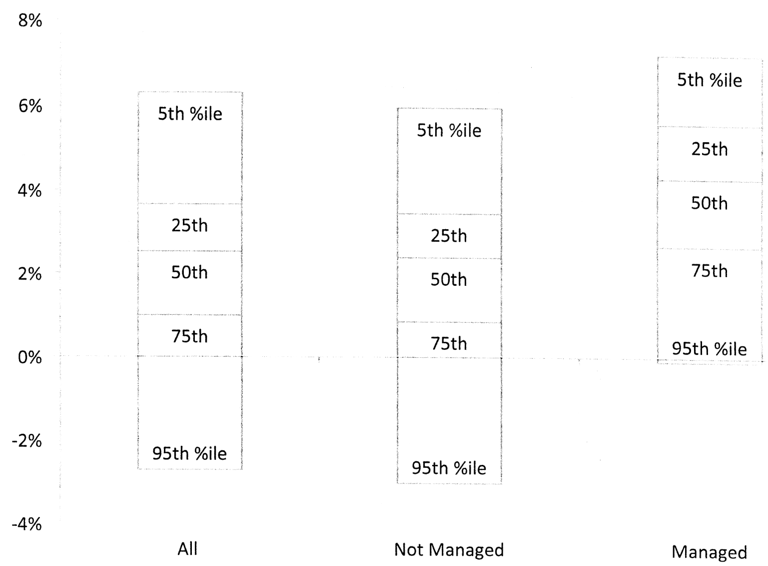

Figure 1 GuidedChoice plan client retirement savings plan return distributions 5.5 year returns to 6/30/2013, annualized active participants.

shows the annualized return (since inception by the client) to: (a) all 401(k) participants, (b) those who did not use GuidedChoices service, and (c) those who did. The difference in results is spectacular. The median level of annualized return to accounts managed by GuidedChoice was roughly two percentage points (200 bps) greater than those we did not manage. (Variations within each category are due in part to differences in start and end times for participant's participation). More generally, by collecting participant individual rate of return data from several record keepers and plan clients, GuidedChoice found that their managed accounts outperformed non-managed accounts by 1% to 3%. There was also a sizable reduction in performance variability.

8 The future

A DSS design must partition the world into those aspects which the DSS will take into account and those which will be ignored or left to human judgment. The Markowitz (1991) Game-of-Life proposal seeks to move more aspects of the financial planning process from the human set to the DSS set in this partition. As noted previously, to do so the DSS designers must model (in their simulator and DSS database):

(1) How the expanded state-space evolves;
(2) How decisions affect payoffs (such as the C_t and W_T); and
(3) How to assign utilities to payoff trajectories.

Of these, the last seems to me to be the most difficult. For example, in addition to "mortality tables," the existence of disability insurance has induced insurance companies to tabulate "morbidity tables" that show the probabilities of transition from health to illness and back; and the Bureau of Labor Statistics has data that would help a DSS design team model the transition from employed to unemployed—and perhaps back again.

But assigning utilities to enhanced payoff trajectories is more challenging. Specifically, rather than characterizing consumption at time t by a single number, c_t, the simulated family's enjoyment for the period would depend on the size of the family, whether it lives in a large house or small apartment, whether it now has to move because someone has a new job elsewhere, etc. The approach required here is both "behavioral" and "rational." It should be behavioral in that it reflects plausible human choices. It should be rational, for example, in that the RDM family understands the consequences of high-interest-rate credit-card debt.

Granted that it may take a considerable effort by many disciplines, and much debate back and forth, to produce a plausible algorithm for assigning utilities to such trajectories, the process should produce substantial light as well as heat concerning actual and ideal financial planning objectives. As von Neumann said.[3]

"If you think mathematics is hard, it is because you think life is easy."

Notes

[1] If the probability distribution of single-period return R has an absolutely continuous density function, then the assertion in the text follows from the fundamental theorem of calculus.

[2] Let

$$s = \sqrt{T} \tag{N1}$$

then

$$\begin{aligned} \hat{w} - \tilde{w} &= k\sigma s - ms^2 \\ &= s(k\sigma - ms) \end{aligned} \tag{N2}$$

We see immediately that $\hat{w} = \tilde{w}$ at $s = 0$ and at

$$s = \frac{k\sigma}{m} = K \tag{N3}$$

Setting its first derivative to zero we see that $\hat{w} - \tilde{w}$ is greatest at

$$\begin{aligned} \hat{s} &= \frac{k\sigma}{2m} \\ &= K/2 \end{aligned} \tag{N4}$$

At this point it equals

$$\begin{aligned} \hat{w} - \tilde{w} &= \frac{k^2\sigma^2}{2m} - \frac{mk^2\sigma^2}{4m^2} \\ &= \frac{k^2\sigma^2}{4m} \end{aligned} \tag{N5}$$

Since $T = s^2$, we have $\hat{w} - \tilde{w} = 0$ at $T = 0$ and $T = K^2$. The difference is greatest at

$$T = \frac{k^2\sigma^2}{4m^2} = K^2/4 \tag{N6}$$

The max difference is still as in Equation (N5), since s and T do not enter the latter result.

[3] Quoted in "Archaeology of computers: Reminiscences, 1945–1947", Communications of the ACM, volume 15, issue 7, July 1972, special issue: Twenty-fifth anniversary of the Association for Computing Machinery, p. 694."

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Keywords: Financial planning; 401k