Journal Of Investment Management, Vol. 13, No. 2, (2015), pp. 6–24

JOIM
www.joim.com

ALTERNATIVE CURRENCY HEDGING STRATEGIES WITH KNOWN COVARIANCES*

Wei Chen[a], *Mark Kritzman*[b] *and David Turkington*[c]

Informed investors understand that they should hedge at least some of their portfolios' currency exposure, but the best strategy for doing so remains an open question. We investigate a variety of currency hedging strategies, including linear strategies, non-linear strategies, and combinations thereof, for the purpose of helping investors determine which strategies best meet their objectives.

Informed investors recognize that hedging at least some of a portfolio's currency exposure, in most cases, improves its quality, but the best approach for doing so is not often obvious. We investigate a variety of currency hedging strategies, including linear strategies, non-linear strategies, and combinations thereof, to help investors determine their most suitable strategy.

Currency hedging has been the subject of much debate over many years, and yet there remains a significant amount of confusion. We present a comprehensive analysis of the costs and benefits of a wide range of currency hedging strategies in order to facilitate ex-ante comparison and to help investors choose the strategy that best suits their goals. We consider a range of linear hedging strategies that hedge a constant fraction of a portfolio's explicit and implicit currency exposure. In addition, we examine strategies that employ options to hedge currency risk based on a variety of contingencies. Finally, we explore hedging strategies that combine conventional and more sophisticated methods to manage currency risk.

Our results are intended to provide an ex-ante comparison of currency hedging strategies. In other words, we focus on the task of selecting an appropriate strategy when the covariance structure of the assets is known. The benefit of ex-ante analysis is that we can consider the full distribution of hypothetical investment

*Views expressed in this article are those of the authors, and may not represent the views of State Street Corporation or its affiliates.

[a]State Street Associates, 140 Mt Auburn St, Cambridge, MA 02138, USA. E-mail: wchen3@statestreet.com
[b]MIT Sloan and Windham Capital Management, 200 Clarendon St, Boston, MA 02116, USA. E-mail: mkritzman@windhamcapital.com
[c]State Street Associates, 140 Mt Auburn St, Cambridge, MA 02138, USA. E-mail: dturkington@statestreet.com

outcomes that might occur over the investment horizon. In contrast, a strategy's ex-post performance will depend on one specific realization of returns. It is likely (in fact, a near certainty) that the optimal strategy ex-ante will not precisely equal the optimal strategy ex-post. However, this does not imply that strategies should be selected on the basis of their ex-post performance. While out-of-sample backtests can provide useful information, historical realizations of returns are period-specific and backward-looking, which is especially problematic for evaluating options strategies in which there may have been only a few instances where they expired in the money. To test the robustness of each optimized linear hedging strategy, we compare the full-sample solutions with those of three different data subsamples: pre-crisis, turbulent periods, and quiet periods.

This article focuses exclusively on risk management; hence we assume that the expected returns of currency forward contracts are zero. We also ignore transaction costs because in most cases these costs are negligible, and because they apply similarly to all strategies and, therefore, would not meaningfully alter our comparative results.[1]

1 Why hedge?

Investors should hedge currency exposure to remove uncompensated risks, but not everyone agrees with this view. Those who disagree typically cite two reasons to rationalize their opposition to hedging. The first is the claim that currency returns wash out over the long run. Schmittmann (2010) provides compelling evidence to contradict this claim. Schmittmann shows that currency hedging is typically beneficial for risk reduction over horizons as long as five years for both stocks and bonds and from multiple base currencies. But even if we ignore these facts and assume that currency returns wash out over the long run, is it prudent to accept uncompensated risks over any horizon? Not according to John Maynard Keynes (1923), who wrote:

"Economists set themselves too easy, too useless a task if during tempestuous seasons they can only tell us that when the storm is long past the ocean will be flat."[2]

Investors should question whether they will survive Keynes' metaphorical storms if they fail to dampen interim risks arising from currency exposure. We, therefore, present value-at-risk (VaR) statistics based on within-horizon return distributions, rather than the traditional approach to VaR which only considers the distribution of returns at the end of the investment horizon.[3] For example, a three-year within-horizon VaR represents the largest cumulative loss an investor can expect to experience *at any point* within the next three years with a given level of confidence.

The second rationalization for not hedging is the notion that currency exposure introduces diversification. We suspect that investors who invoke this argument have an inflated perception of the extent to which currency exposure diversifies a portfolio, because they focus on the correlation between local asset returns and currency returns, which may be low or even negative. The relevant correlation, however, is the correlation between the investor's base currency-denominated returns and currency returns. This correlation is much higher because a significant fraction of a foreign asset's return is the currency return. For example, in the 20 years prior to March 2013, monthly USD/GBP currency returns were -9% correlated to U.K. equity returns (based on the MSCI U.K. index) when measured in British pounds. From a U.S. investor's perspective, though, the unhedged U.K. asset returns were $+48\%$ correlated with the currency returns.[4]

2 Why not hedge everything?

Some investors view currency fluctuations purely as additional risk and choose to hedge 100% of their portfolios' currency exposures. Perold and Schulman (1988), for example, advocated full hedging for U.S.-based investors. They suggested that any exposure to currency risk should be viewed as an active decision, largely because it is hard to justify an expectation that currency hedging will produce non-zero returns over long horizons. Indeed, for each investor who assumes a short position in a foreign currency to hedge it, there must be another investor on the other side who assumes the opposite position. In the long run, it is implausible to assume substantial gains to passive hedging for all investors.[5]

We also focus on risk reduction and passive hedging, and we assume that the expected returns of currencies are zero. But we recognize that currencies are not uncorrelated with the portfolio's assets; thus, hedging away all of a portfolio's currency exposure is usually sub-optimal. Consider a portfolio with a 10% standard deviation that is exposed to a single currency with a 12% standard deviation. In order to minimize the portfolio's risk, the amount of the forward contract to sell as a fraction of the total portfolio equals the portfolio's beta with respect to the forward contract.[6] Suppose the portfolio's returns are 60% correlated with the currency's returns. Given our volatility assumptions, the portfolio's beta equals 0.50 (that is, $0.10/0.12 \times 0.60 = 0.50$). Therefore, in order to minimize portfolio risk, the investor should sell the currency forward contract in an amount equal to 50% of the portfolio's value.

Exhibit 1 shows how the portfolio's volatility varies with the currency hedge ratio for the particular example explained above. At a hedge ratio of 0% the portfolio's volatility is unchanged from 10%. As the hedge ratio increases from 0%, the portfolio's volatility falls until, at a hedge ratio of 50%, it reaches a minimum of 8%. As the hedge ratio rises above 50%, however, the portfolio's volatility rises. In this example a fully hedged portfolio is just as risky as an unhedged portfolio.

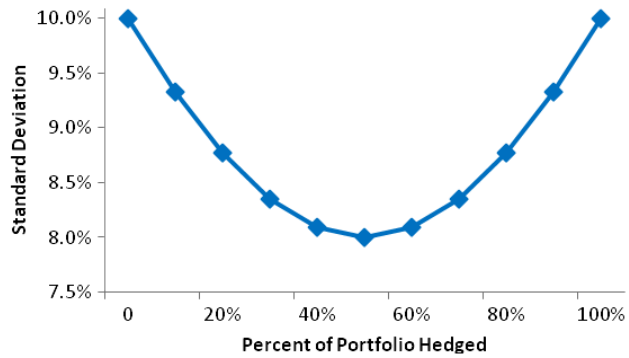


Exhibit 1 The minimum risk hedge ratio (an example).

Note: Figure assumes 10% asset volatility, 12% currency volatility, and a 0.6 correlation between the foreign asset and currency.

This type of hedging strategy, in which we hedge a constant fraction of currency exposure, is an example of a linear hedging strategy, so named because the portfolio's returns are a linear function of the hedged currencies' returns. We evaluate a variety of linear hedging strategies in the next section, which recognize that the optimal amount to hedge is specific to each currency. This idea is also highlighted by Campbell *et al.* (2010). They note that currency correlations with equity markets vary significantly across currencies, which implies that risk-minimizing investors should hedge some currencies more or less than others.

By their nature, linear hedging strategies remove currency gains as well as losses. This is not the only way to hedge currency risk. Investors who want to eliminate currency-related losses while also maintaining the potential for currency gains can employ non-linear hedging strategies, such as options.[7] In exchange for upside potential, investors who choose non-linear hedging strategies bear the cost of the option premium. We also evaluate a variety of non-linear hedging strategies and compare them with the linear strategies.

Black (1989) argued that there is a single "universal" hedge ratio which applies to all investors. However, Adler and Prasad (1992) noted that Black's finding relies on a number of strong assumptions about uniformity of risk tolerance, wealth levels, and the portfolio composition of investors, which do not hold in practice. To quote Yogi Berra: "In theory there is no difference between theory and practice, but in practice there is." Sound practice requires that investors determine the currency hedging approach that is best for them, taking into account their risk preferences, constraints, portfolio composition, and capital market expectations.

3 Linear hedging strategies

We evaluate four linear hedging strategies (in addition to no hedging and full hedging) with the objective of minimizing total portfolio risk. Each optimized hedging strategy described below imposes successively fewer hedging constraints. In each case, our objective is to solve the collection of short currency forward positions that jointly minimize total portfolio volatility.

No hedging: This strategy does not hedge any currency exposure. It simply leaves the portfolio unaltered.

Full hedging: This strategy hedges 100% of each foreign asset's currency exposure with its associated currency forward contract.

Currency-specific hedging with fixed asset weights: This strategy hedges some fraction of each foreign asset's currency exposure with its associated currency forward contract up to 100% but not less than 0%, taking into account the portfolio's volatility, each currency forward contract's volatility and correlation with the portfolio, as well as their correlations with each other, and holding constant the underlying asset weights, with the objective of minimizing total portfolio risk.

Cross-hedging: This strategy hedges some fraction of the total foreign assets' currency exposure collectively up to 100% rather than up to 100% of each foreign asset's currency exposure, but not less than 0% of any foreign asset's currency exposure. It is more flexible than currency-specific hedging with fixed asset weights because it allows one to hedge a foreign asset's currency exposure with the forward contract of another currency.

Over-hedging: This strategy hedges some fraction of the total portfolio's "effective" currency exposure, including its domestic component, up to 100% of total portfolio value, but not less than 0% of any foreign asset's currency exposure. It is more flexible than currency-specific hedging with fixed asset weights or cross-hedging because it allows one to hedge the currency exposure of the domestic component of the portfolio.[8]

Currency hedging with variable asset weights: This strategy seeks to reduce portfolio risk by simultaneously varying exposure to the portfolio's underlying assets as well as the currency forward contracts. This strategy is similar to one proposed by Jorion (1994), who argued that investors should determine their optimal asset allocation and currency exposure simultaneously rather than sequentially. We allow asset weights to vary individually from 0% to 100%, and the strategy is allowed to hedge up to 100% of the total portfolio value, but not less than 0% of any individual asset's currency exposure. To facilitate comparison with other strategies, we constrain the optimal portfolio to have the same expected return as the over-hedging case. We also impose a tracking error constraint with respect to the "over hedged" portfolio to ensure that the reallocated

Exhibit 2 Constraints for linear hedging strategies.

Linear hedging strategies	Asset allocation	Currency hedging constraints
Full hedging	Fixed	Each currency hedge amount = portfolio exposure to this currency
Currency-specific hedging	Fixed	Each currency hedge amount ≤ portfolio exposure to this currency
Allow cross-hedging	Fixed	Total currency hedging amount ≤ portfolio's total currency exposure
Allow over-hedging	Fixed	Total currency hedging amount ≤ 100%
Allow asset reallocation	Variable	Total currency hedging amount ≤ 100% with tracking error constraints for asset weights

portfolio's composition would appear reasonable to a prudent investor.[9]

The salient features of these hedging strategies are summarized in Exhibit 2.

We apply these hedging strategies to a global portfolio from the perspective of a U.S. investor. The portfolio is allocated as follows: 33% U.S. equity, 27% foreign equity, 12% U.S. bonds, and 28% foreign bonds.[10] Exhibit 3 shows the currency exposures for this portfolio as of March 2013.[11]

Exhibit 4 shows the expected returns, standard deviations, and correlations of the assets and associated currency forward contracts used to hedge this portfolio, based on monthly returns from February 1998 through March 2013. The expected returns of the currency forward contracts equal zero because, as mentioned previously, our focus is on risk management. The expected returns for assets are equilibrium returns based on the CAPM.[12] These returns are merely illustrative, and they are only relevant for the strategy in which we allow reallocation across assets.

With these assumptions, we are now prepared to evaluate a range of linear hedging strategies.

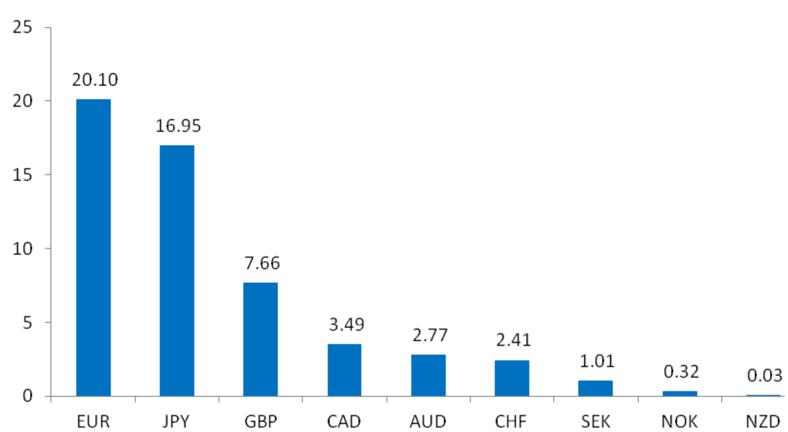


Exhibit 3 Currency exposure as % of total portfolio value.

Exhibit 4 Full-sample expected returns, standard deviations, and correlations.

		Expected return	Standard deviation	Correlation												
				A	B	C	D	E	F	G	H	I	J	K	L	M
A	US equity	8.90	16.32	1.00												
B	Foreign equity	9.85	17.96	0.87	1.00											
C	US bond	3.26	4.61	−0.33	−0.29	1.00										
D	Foreign bond	4.91	8.54	0.12	0.30	0.50	1.00									
E	AUD	0.00	13.10	0.58	0.73	−0.03	0.52	1.00								
F	CAD	0.00	8.84	0.57	0.67	−0.13	0.32	0.71	1.00							
G	CHF	0.00	11.14	0.09	0.33	0.22	0.78	0.54	0.33	1.00						
H	EUR	0.00	10.52	0.27	0.50	0.19	0.81	0.67	0.46	0.86	1.00					
I	GBP	0.00	8.57	0.22	0.41	−0.02	0.49	0.53	0.46	0.53	0.62	1.00				
J	JPY	0.00	11.01	0.00	0.05	0.30	0.65	0.12	0.00	0.33	0.22	0.09	1.00			
K	NOK	0.00	11.35	0.37	0.55	0.00	0.62	0.67	0.57	0.68	0.81	0.63	0.11	1.00		
L	NZD	0.00	13.65	0.47	0.63	0.04	0.54	0.84	0.55	0.60	0.66	0.51	0.14	0.58	1.00	
M	SEK	0.00	11.66	0.47	0.64	0.06	0.68	0.72	0.57	0.70	0.85	0.63	0.18	0.86	0.68	1.00

Again, these strategies hedge a constant fraction of an asset's currency exposure and are called linear strategies because the portfolio's return is a linear function of the hedged currencies' returns.

Results: Linear hedging strategies

Exhibit 5 shows the asset and currency forward contract exposures for each of these linear hedging strategies, as well as their expected volatility and within-horizon VaR based on the full sample of data (from February 1998 to March 2013).[13] The optimized linear hedging strategies (L-1 through L-4) represent the collection of short currency forward positions that jointly minimize total portfolio volatility, given the relevant constraints.[14]

To evaluate the regime sensitivity of these results, we compute similar optimal hedging results based on sub-samples for the pre-crisis period (February 1998 through December 2007), turbulent periods,

Exhibit 5 Optimal hedge ratios (%)—Full sample.

	No hedging	Full hedging	L-1 currency-specific hedging	L-2 allow cross-hedging	L-3 allow over-hedging	L-4 allow asset reallocation*
Hedgeable foreign exposure as % of total portfolio	54.74	54.74	54.74	54.74	54.74	92.00
Total hedge positions as % of total portfolio	0.00	54.74	49.53	54.74	93.20	78.78
US equity	33.30	33.30	33.30	33.30	33.30	8.00
Foreign equity	26.70	26.70	26.70	26.70	26.70	43.14
US bond	11.96	11.96	11.96	11.96	11.96	0.00
Foreign bond	28.04	28.04	28.04	28.04	28.04	48.85
Hedging positions						
	exposure	weight	weight	weight	weight	weight
AUD	2.77	−2.77	−2.77	−36.50	−26.97	−16.03
CAD	3.49	−3.49	−3.49	−1.01	−30.69	−16.89
CHF	2.41	−2.41	−2.41	0.00	0.00	0.00
EUR	20.10	−20.10	−20.10	0.00	0.00	−10.18
GBP	7.66	−7.66	−7.66	0.00	0.00	0.00
JPY	16.95	−16.95	−11.75	0.00	−9.06	−14.00
NOK	0.32	−0.32	−0.32	0.00	0.00	0.00
NZD	0.03	−0.03	−0.03	0.00	−2.64	−3.05
SEK	1.01	−1.01	−1.01	−17.23	−23.84	−18.62
Expected return	7.36	7.36	7.36	7.36	7.36	7.36
Portfolio volatility	10.57	8.76	8.74	6.88	6.34	5.99
Within horizon VaR**	−18.18	−13.40	−13.32	−8.65	−7.27	−6.44

*The tracking error aversion is 0.25; the tracking error of this portfolio is 3.53%.
**VaR is based on a 3-year investment horizon with 95% confidence.

and quiet periods. In each case, we re-estimate the variances and correlations for all assets and currencies based on the relevant sub-sample. For simplicity, we use the expected returns from Exhibit 4 in all cases. We define financial turbulence using multivariate outliers, following the methodology of Kritzman and Li (2010). The turbulent sample represents the 20% most unusual months in the data taking into account extreme returns, divergence of correlated assets, and convergence of uncorrelated assets. We define quiet periods as the 80% other months that are not classified as turbulent.

Exhibit 6 summarizes the volatility, value-at-risk, and total hedge positions across the four different data samples. Due to space constraints, we do not report the full set of optimal hedge positions corresponding to each data sub-sample.[15]

The results displayed in Exhibits 5 and 6 allow us to make several observations.

- The optimal hedging results vary only slightly across the four data samples. The results are generally consistent across the samples with respect to the optimal hedging positions and the risk reduction the hedges produce. The portfolio volatilities are ordinally identical across the four samples; they decrease from left to right in each case.
- Full hedging reduces portfolio risk, but not as much as other strategies that allow for more flexibility.
- The currency-specific hedging strategy is very similar to the full hedging strategy. For this portfolio, currency exposure does not provide much diversification benefit, and the currency-specific optimal solution is to

Exhibit 6 Optimal hedge ratios—summary (%).

	No hedging	Full hedging	L-1 currency-specific hedging	L-2 allow cross-hedging	L-3 allow over-hedging	L-4 allow asset reallocation*
Volatility						
Full sample	10.57	8.76	8.74	6.88	6.34	5.99
Pre-crisis	8.87	7.86	7.79	6.72	6.49	5.64
Turbulent periods	15.52	12.11	12.11	9.21	7.63	7.79
Quiet periods	8.75	7.58	7.50	6.07	5.74	5.26
Within horizon VaR						
Full sample	−18.18	−13.40	−13.32	−8.65	−7.27	−6.44
Pre-crisis	−13.49	−10.82	−10.67	−7.95	−7.34	−5.63
Turbulent periods	−30.23	−21.49	−21.49	−14.07	−9.89	−10.38
Quiet periods	−13.14	−10.12	−9.94	−6.52	−5.85	−4.82
Total hedge positions as % of total portfolio						
Full sample	0.00	54.74	49.53	54.74	93.20	78.78
Pre-crisis	0.00	54.74	44.67	54.74	83.02	71.10
Turbulent periods	0.00	54.74	54.74	54.74	100.00	83.39
Quiet periods	0.00	54.74	43.29	54.74	84.95	77.09

*Based on a 3-year horizon with 95% confidence.

hedge the full amount of most currencies when over-hedging and cross-hedging are not allowed.

- Cross-hedging tends to use the AUD, CAD, and SEK to hedge the portfolio's overall currency exposure. As shown in Exhibit 4, these currencies have the three highest correlations with foreign equities, which make them effective proxy hedges. One possible explanation for these high correlations is that both the Australian dollar and Canadian dollar are "commodity currencies" which tend to move in-line with global growth, and by extension equity prices. Taking short positions in these currencies provides a hedge for other risky currency exposures and also for inherent risk factors in the portfolio's equity allocation.[16]
- Over-hedging, which allows one to hedge the currency exposure of domestic assets, increases the total hedge ratio significantly by hedging the portfolio's implicit AUD, CAD, and SEK (and in some cases JPY and NZD) exposure. The implicit exposure of domestic assets to various currencies is apparent from the relatively high correlations between domestic equities and these currencies, as shown in Exhibit 4.[17]
- Allowing the asset weights to vary along with exposure to currency forward contracts yields the minimum risk portfolio. However, it should be noted that to the extent investors have less confidence in their currency views than their asset views, they may not want currency hedging considerations to dictate asset allocation.

4 Non-linear hedging strategies

We now consider using options to manage currency risk. We refer to these hedging strategies as non-linear strategies because the performance of the portfolio is a non-linear function of the hedged currencies' returns. The main benefit of using put options to manage currency risk is that they can reduce downside exposure while also maintaining the potential for upside gains. We evaluate the following four non-linear hedging strategies. In all cases, we assume that the investor purchases quarterly options that are held to expiration.

Currency-specific options: This strategy employs a portfolio of put options—one option for each currency exposure. We set the strike price to equal a given currency's starting value, the time to maturity to equal three months, and the notional value to equal the portfolio's explicit exposure to that currency. We use the Black–Scholes–Merton option pricing model to value these options.[18]

Basket option: This strategy employs a single basket option to hedge the collective currency exposure in accordance with their relative weights. We set the strike price to equal the currency basket's starting value, the time to maturity to equal three months, and the notional value to equal the portfolio's total exposure to foreign currencies. We use Monte Carlo simulation to price each basket option at its inception and to re-price it at the end of each month prior to its expiration.[19] Each time we price the option, we simulate 10,000 hypothetical return paths for each currency and calculate the basket option payoffs at the end of each path. We then take the average of these payoffs and discount it back to its present value based on the domestic risk-free rate.

Foreign asset contingent option: This strategy employs a basket option whose payoff is contingent on the simultaneous occurrence of two outcomes: (1) the depreciation of the collective currency value below its value at the inception of the option and (2) the depreciation of the *collective foreign asset value* below its value at the inception of the option. We again assume that we roll over the options every quarter and apply them to the portfolio's total foreign currency exposure.

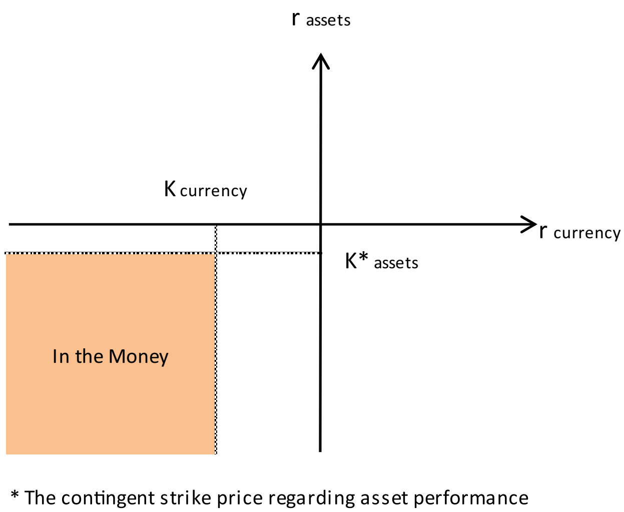


Exhibit 7 Contingent option payout structure.

Total portfolio contingent option: This strategy is identical to the previous contingent option, except that the second contingency pertains to the total portfolio value rather than only the foreign asset value. Exhibit 7 illustrates the contingencies that generate a payoff, where k equals strike price and r equals return.

We conduct simulations to evaluate the performance of the non-linear strategies. By basing these simulations on the same expected return and covariance estimates used in the linear hedging analysis and shown in Exhibit 4, we facilitate the comparison of linear and non-linear strategies. The simulations provide a more complete assessment of the costs and benefits of the options strategy than an out-of-sample backtest, because backtest results are highly period specific when there may have been only a few instances when options expired in the money. We do not show options results for the other sub-samples considered earlier, purely due to space constraints.

We perform the simulations as follows:

(1) We simulate three-year paths for both asset and currency prices based on the expected return, standard deviation, and correlation assumptions from Exhibit 4.
(2) For each quarter along these paths, we price the relevant option at the beginning of the quarter, one month later, and two months later, and calculate the payoffs at the end of the quarter.
(3) For each month within a quarter, we calculate the change in the option price for the month and divide by the total portfolio value to generate a time series of returns for the options.
(4) Finally, we combine the option returns with the unhedged portfolio returns to generate a time series of returns for each hedging strategy.
(5) We repeat this process 10,000 times to generate return distributions for these strategies.

5 Results: Non-linear hedging strategies

Exhibit 8 shows the performance statistics for each option strategy. The basket option is less expensive than multiple options because diversification among the currencies reduces the volatility of the currency basket, and thereby also reduces the average option payout. The contingent options are less expensive than the basket option because they provide less downside protection overall. However, under certain circumstances, when the portfolio's value falls or the foreign assets' value falls, the contingent options provide adequate protection, as indicated by the conditional payoff probabilities and conditional average payoff value.[20]

For non-linear hedges, standard deviation does not provide a useful description of risk. We look instead at the full distribution of annual return outcomes. Specifically, we isolate the impact of currencies on portfolio returns by subtracting the fully hedged portfolio's returns from those of each strategy along each return path in our simulations.

Exhibit 8 Option performance statistics (%)

	NL-1, multiple options	NL-2, basket option	NL-3, contingent option on foreign asset and currency	NL-4, contingent option on total asset and currency
Option premium as % of portfolio	−1.05	−0.75	−0.67	−0.60
Mean payoff as % of portfolio	1.05	0.76	0.55	0.48
Probability of payoff > 0	90.49%	50.65%	29.47%	26.07%
Probability of payoff > 0 when portfolio is down	99.30%	**75.71%**	72.86%	**75.71%**
Mean payoff when portfolio is down	1.677	1.831	1.884	1.837
Probability of payoff > 0 when foreign asset is down	99.84%	**84.63%**	**84.63%**	72.05%
Mean payoff when foreign asset is down	1.853	1.872	1.878	1.885

Exhibit 9 shows the distribution of currency return impact for each hedging strategy. The dotted line represents the stylized normal distribution curve fitted to the unhedged portfolio. The fully hedged portfolio has no currency impact, and serves merely as a point of reference. The currency exposures of the unhedged portfolio introduce both upside and downside returns. Compared to the unhedged portfolio, the four options provide visible downside protection, as indicated by the left side of their distributions. As one would expect, the less expensive options provide less downside protection. The four option strategies provide less downside protection than full hedging; however, they also allow for substantially higher upside potential as indicated by the right side of their distributions.

Exhibit 10 shows the within-horizon VaR for the total portfolio for each of the non-linear hedging strategies. These statistics can be compared with the within-horizon VaR numbers shown in Exhibit 5 for linear hedging strategies.

5.1 *Combining linear and non-linear strategies*

In the previous section, we set the notional value of the non-linear hedging strategies to equal the portfolio's foreign exposure. It may, however, be optimal to hedge more or less than this amount, or to combine both linear and non-linear hedging strategies. We use full-scale optimization rather than mean–variance optimization to solve for the optimal combination of hedging positions to account for the non-normality of option strategy returns. As explained by Cremers *et al.* (2005), full-scale optimization maximizes expected utility based on the complete distribution of asset returns. It also allows for more flexible definitions of investor utility. We assume a kinked utility function that strongly penalizes returns below a threshold, θ. Exhibit 11 illustrates a sample utility function.

In applying full-scale optimization, we allow linear hedge positions to vary between 0% and 100% of the total portfolio value, and we also allow the allocation to the individual put options to range from 0% to 100% of the total portfolio value. Exhibit 12 shows the hedge positions that result from full-scale optimization with kinked utility thresholds of 0% and −5%, respectively. In both cases, the optimal solution uses five forward contracts to hedge. These are the same five currencies hedged in the optimal linear hedging strategy in Exhibit 5 which allows over-hedging.

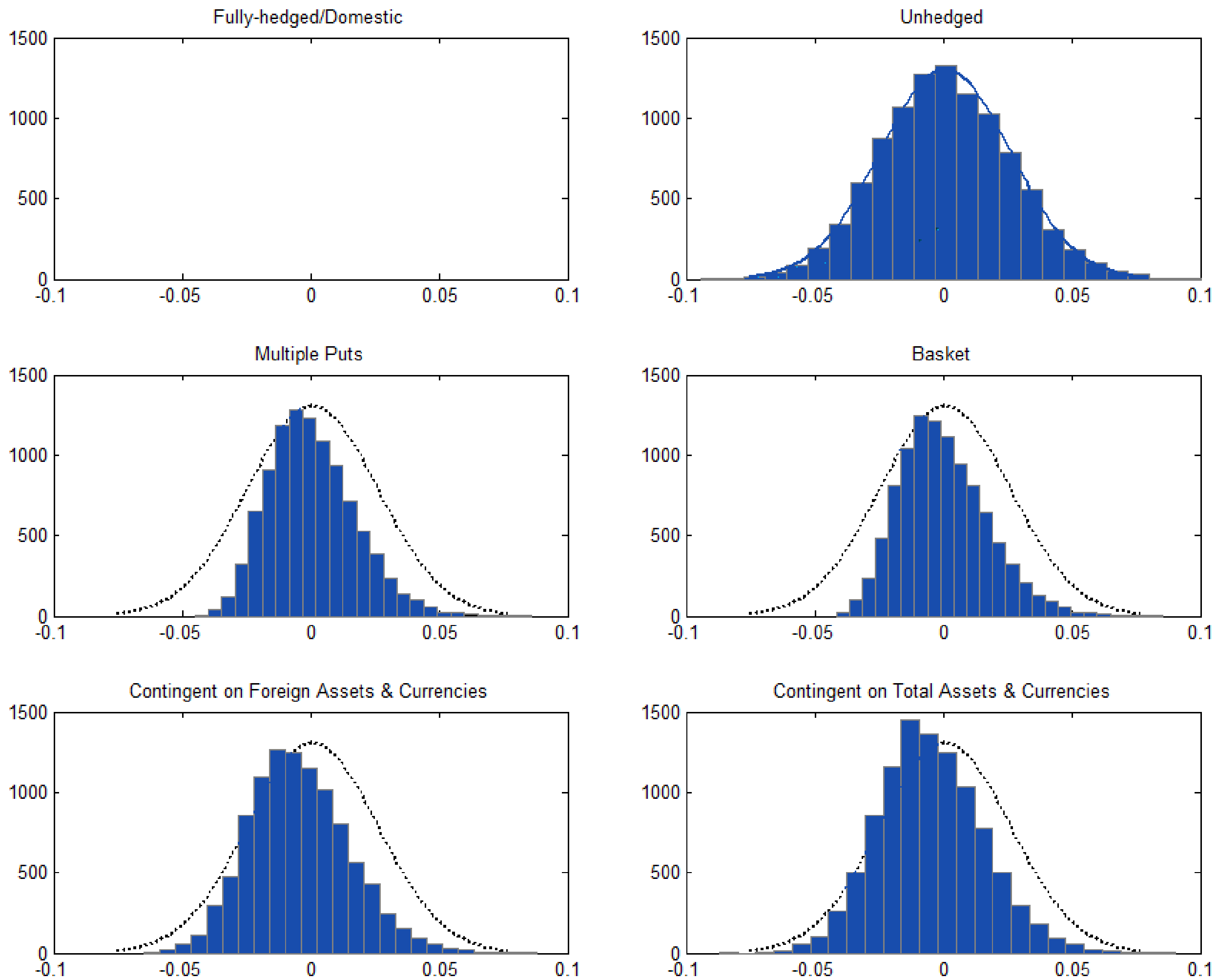


Exhibit 9 Distribution of currency impact on portfolio returns.

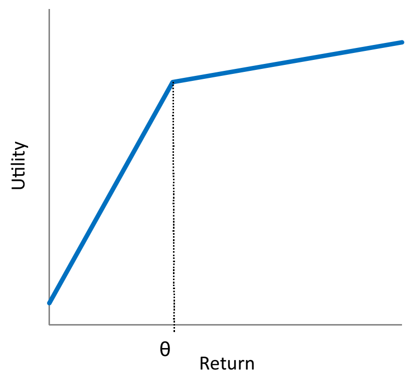
Exhibit 10 Within-horizon VaR for non-linear hedging strategies (%).

	No hedging	NL-1, multiple options	NL-2, basket option	NL-3, contingent option on foreign asset and currency	NL-4, contingent option on total asset and currency
Within-horizon VaR	−18.18	−14.16	−14.10	−13.97	−13.98

*Based on a 3-year horizon with 95% confidence.

In Exhibit 12, the optimal hedging solution also allows for hedging with put options. With the loss threshold set to 0%, it is optimal to hedge a small portion of the AUD, JPY, and NZD with an option in addition to the short forward position. With the loss threshold set to −5%, options were not selected. These non-linear strategies seek to avoid large losses while preserving upside potential at the portfolio level.

Exhibit 13 summarizes our results by highlighting the trade-offs associated with each of the hedging strategies we have considered.[21]



$$U(x) \begin{cases} = \ln(1+x), & \text{for } x \geq \theta \\ = 10 * (x-\theta) + \ln(1+\theta), & \text{for } x < \theta \end{cases}$$

Exhibit 11 Kinked utility function (an example).

Exhibit 12 Full-scale optimal hedging results including currency forward and options (%).

	No hedging weight	Full-scale optimal with threshold 0% weight	Full-scale optimal with threshold −5% weight
Unhedged portfolio	100.00	100.00	100.00
Forward positions			
AUD	0.00	−27.29	−25.89
CAD	0.00	−31.27	−34.56
CHF	0.00	0.00	0.00
EUR	0.00	0.00	0.00
GBP	0.00	0.00	0.00
JPY	0.00	−8.46	−7.77
NOK	0.00	0.00	0.00
NZD	0.00	−2.47	−3.63
SEK	0.00	−23.37	−23.73
Put option positions			
AUD	0.00	0.38	0.00
CAD	0.00	0.00	0.00
CHF	0.00	0.00	0.00
EUR	0.00	0.00	0.00
GBP	0.00	0.00	0.00
JPY	0.00	0.97	0.00
NOK	0.00	0.00	0.00
NZD	0.00	0.03	0.00
SEK	0.00	0.00	0.00
Within-horizon VaR*	−18.18	−6.72	−6.78

*Based on a 3-year horizon with 95% confidence.

Exhibit 13 Summary of trade-offs across hedging strategies (%)

	No hedging	Full hedging	Linear hedging				Non-linear hedging				Full scale optimiza-tion
			L-1, Currency specific hedging	L-2, Allow cross hedging	L-3, Allow over hedging	L-4, Allow asset reallocation	NL-1, Multiple options	NL-2, Basket option	NL-3, Contingent option on foreign asset & currency	NL-4, Contingent option on total asset & currency	
Volatility	10.57	8.76	8.74	6.88	6.34	5.99	na	na	na	na	na
Hedging Cost*	0.00	0.00	0.00	0.00	0.00	0.00	-1.05	-0.75	-0.67	-0.60	0.00
Downside Loss**	-10.11	-4.53	-4.38	1.35	3.13	4.18	-6.53	-6.61	-7.85	-7.86	3.03
Upside Loss***	0.00	-1.70	-1.72	-3.60	-4.09	-4.40	0.24	1.93	1.54	3.38	na

smallest
largest

*Hedging cost refers to premium of the options.

**Downside loss represents the end-of-horizon VaR for the full sample.

***Upside loss represents the 95th percentile of the return distribution minus the 95th percentile of the unheded portfolio's return distribution.

The results we have presented assume that historical covariances will prevail in the future which, of course, will not hold true precisely. But we have little cause to doubt that these results will prevail qualitatively out-of-sample. In fact, Kinlaw and Kritzman (2009) provided evidence that a collection of optimized linear hedging strategies performs well out-of-sample.

6 Summary

We evaluated a variety of linear and non-linear currency hedging strategies, as well as combinations of both. Although there is not a unique hedging strategy that is universally superior, we are able to draw several general conclusions.

- In most cases, full hedging reduces risk relative to an unhedged strategy, but not by as much as optimal hedging using linear strategies.
- More flexible linear hedging strategies reduce risk more and offer greater downside protection than more constrained linear hedging strategies.
- A linear hedging strategy that simultaneously optimizes currency and asset class exposure provides the best outcome of linear hedging strategies. However, if investors have more confidence in their views about the expected return and risk of asset classes than they do about currencies, they may not want to allow currency exposures to influence asset class exposures. A reasonable compromise may be to employ mean–variance-tracking error optimization to anchor the asset class exposures to a predefined set of weights.
- Non-linear hedging strategies offer clear trade-offs between the degree of protection and the cost of the strategies.
- Hedging with multiple currency options is more protective than using basket options, because the former strategy pays off if any of the options are in the money, whereas the latter strategy pays off only if the average currency return places the option in the money. However, the basket option is less expensive than multiple options because the volatility of the portfolio of currencies is reduced by diversification.
- Options that are contingent on both currency performance and foreign asset or portfolio performance are less expensive than options that are contingent only on currency performance, because assets and currencies are less likely to

breach their strike prices simultaneously than currencies alone.

- Full-scale optimization allows investors to identify optimal combinations of linear and non-linear hedges for the purpose of addressing more elaborate descriptions of investor utility including utility functions that account for higher moments. It is therefore difficult to compare this approach to other strategies based only on limited observations from return distributions. We do know from first principles, though, that full-scale optimization will always yield the best in-sample outcome. Whether this superiority persists out-of-sample is an empirical issue.
- Most of these observations are qualitatively apparent from first principles. Our contribution is to quantify these trade-offs given reasonable assumptions about asset class and currency covariances.

Appendix A: Monte Carlo simulation to price options

Monte Carlo simulation is a procedure to generate random numbers from a given distribution with predefined parameters. For option pricing, Monte Carlo uses the risk-neutral valuation method.

We simulate paths to obtain the expected payoff of the option in the risk-neutral world, and then discount the cash flow at a risk-free rate r:

(1) Sample a random path for S_t in a risk-neutral world. If there are multiple assets to consider, we use multi-variable simulation which takes into account the correlations among assets.
(2) Calculate the payoff of the option given the simulated price.
(3) Repeat the previous steps to generate many sample payoffs of the option in a risk-neutral world.
(4) Calculate the mean of the sample payoffs to get an estimated expected payoff in a risk-neutral world.
(5) Discount the expected payoff using the risk-free rate to estimate the present value of the option.

Appendix B: Pricing basket options

The basket option is a European put option on the total currency exposure of the portfolio. The underlying "asset" for the basket option is defined as:

$$B_t = B_{t-1} * \left(1 + \sum_{k=0}^{n} \omega_k * r_{k,t}\right)$$

where ω is weight of the currency exposure within the portfolio and r is the return of each currency exposure at time t.

The payoff of the basket option at maturity is:

$$\max\{0, (1-k) * B_0 - B_T\}$$

where k is the strike price.

Assume the distribution of currency exchange rate S is lognormal. Then in a risk-neutral world, for each exposed currency,

$$\frac{dS_{i,t}}{S_{i,t}} = (r_d - r_f)dt + \sigma_i dZ_i^Q$$

where $r_d(r_f)$ is the risk-free rate of the domestic (foreign) currency, and σ_i is the expected volatility of the underlying asset. Here, we use a simplified historical volatility to represent the expected volatility. dZ_i^Q denotes the Brownian motion under the Q measure—the risk-neutral measure.

From Ito's lemma, the process followed by $ln S_{i,t}$ is:

$$dln S_{i,t} = \left(r_d - r_f - \frac{\sigma_i^2}{2}\right) dt + \sigma_i dZ_i^Q$$

Table A1 Short-term interest rate (%).

USD	AUD	CAD	CHF	EUR	GBP	ILS	JPY	NOK	NZD	SEK
0.44	4.00	1.29	0.05	0.18	0.70	2.12	0.20	2.30	2.86	2.19

If there is no correlation among currencies, then for each currency, we can show that:

$$lnS_{i,T} - lnS_{i,0} = \left(r_d - r_f - \frac{\sigma^2}{2}\right) T + \sigma_i \epsilon \sqrt{T}$$

where $S_{i,T}$ is price of currency i at maturity, S_0 is the current price, T is the expiration time, and ε is a random variable which is normally distributed with 0 as the mean and 1 as the standard deviation.

However, the correlation of the currencies within the basket cannot be omitted. For each Brownian motion, we have:

$$dZ_i^Q * dZ_j^Q = \rho_{i,j} dt \quad (i \neq j)$$

We use Monte Carlo simulation to price the option, which takes into consideration the currency correlations and simulates multiple variables (all currency forward prices) at the same time. With the forward price of the currencies, we calculate B_T, estimate the payoff at maturity, and then discount the payoff to present time using domestic risk-free rate r_d.

During the Monte Carlo simulation process, we make the following assumptions about the basket option:

- Option type: European put
- Strike Price: B_0 (at the money option)
- Annualized risk-free rates as shown in Table A1
- T: Three months

Appendix C: Pricing contingent options

The currency contingent option aims to hedge currency risk when both assets and currencies suffer from loss.[22] It pays off just like a plain vanilla basket option only when both the assets and the currency exposures breach specific thresholds (K) at the same time. The payoff formula is shown below:

$$\begin{cases} \text{Max}\{0, (1 - K_1) * B_{T-1} - B_T\}, \\ \qquad \text{if } A_T < (1 - K_2) * A_{T-1} \\ 0, \quad \text{if } A_T \geq (1 - K_2) * A_{T-1} \end{cases}$$

where A_T is the asset value; K_1, K_2 equal 0 in our analysis (at the money option).

We can use Monte Carlo simulation to simulate the asset prices and the currency prices at the time of option expiration (three months in this case):

$$\frac{dS_{i,t}}{S_{i,t}} = (r_d - r_f)dt + \sigma_i dZ_i^Q$$

At the same time, Brownian motions are correlated among assets and currencies:

$$dZ_i^Q * dZ_j^Q = \rho_{i,j} dt \quad (i \neq j)$$

We use Monte Carlo simulation to price the option. At the beginning of each quarter, we simulate the prices of both assets and currencies based on the above assumptions. Then we calculate the payoff based on the payoff rule in each path. Finally, we average all the payoffs, and discount the result back to the beginning of the period using risk-free rate r_d.

$$E_{\text{payoff at expiration}} = \frac{1}{n} * \sum CF_{\text{payoff of } i\text{th path}}$$

$$\text{Contingent option price} = e^{-r_d * (\frac{3}{12})} * E_{\text{payoff at expiration}}.$$

In the case of non-linear hedging with various kinds of options, we assume that options are bought quarterly and held to expiration, which does not require an excessive amount of turnover. Investors should consider the transaction costs they face in the context of their own hedging policy.

Notes

[1] In the case of non-linear hedging with various kinds of options, we assume options are bought quarterly and held to expiration, which does not require an excessive amount of turnover. Investors should consider the transaction costs they face in the context of their own hedging policy.

[2] Keynes, J. M. *A Tract on Monetary Reform* (1923). This sentence followed Keynes' famous aphorism, "In the long run we are all dead."

[3] See Kritzman and Rich (2002) for further details on within-horizon risk measurement. End-of-horizon risk statistics may also be useful, but in the interest of brevity we do not report them in this article.

[4] The following equations from Kritzman (2000) show how to map the correlation between the local asset returns and the currency returns onto the correlation between the investor's base currency-denominated asset returns and the currency returns.

$$\rho_{FA,CC} = \frac{(\rho_{L,FX}\sigma_L + \sigma_{FX})}{\sigma_{FA}}$$

$$\rho_{FA,CC} = \frac{(\rho_{L,FX}\sigma_L + \sigma_{FX})}{(\sigma_L^2 + \sigma_{FX}^2 + 2\rho_{L,FX}\sigma_L\sigma_{FX})^{1/2}}$$

[5] Black (1989) offers a contrary view. He points out that the expected return from foreign currencies is positive for all investors in the world due to Siegel's paradox. However, this effect is small compared to the volatility that currencies introduce. For example, based on data from September 1990 through March 2013, the annualized expected return of an equally-weighted portfolio of G10 foreign currencies averaged across all ten investors' perspectives equals 66 basis points. This compares with an average annualized volatility of 11% for these currencies over that period. Even more importantly, Siegel's paradox relates to expected return arithmetic, and when it comes to actual purchasing power, it is impossible for all investors to experience gains for foreign currencies over a given period. For example, Kritzman points out that if purchasing power parity holds, then Siegel's paradox does not permit economic gains (see Kritzman's book: *Puzzles of Finance*, 2000, John Willey & sons).

[6] This notion is no different than hedging the market exposure of a U.S. equity portfolio by selling S&P 500 futures contracts. The amount to sell in order to minimize risk is determined by the equity portfolio's beta with respect to the S&P 500 index.

[7] Dynamic hedging rules, such as those based on constant proportion portfolio insurance, are other examples of non-linear strategies which present similar trade-offs. For simplicity, we focus on options in this article.

[8] The portfolio's domestic assets have implicit currency exposure to the extent they have a non-zero correlation with any currency.

[9] We introduce an additional tracking error term into the utility function, as described by Chow (1995). In our example, we solve for the portfolio that has the same expected return as the previous hedging scenarios. We set tracking error aversion equal to 0.25, which in this case produces an optimal portfolio with 3.4% tracking error with respect to the next-best hedging strategy.

[10] We assume a 60/40 allocation into equities and bonds. The equity allocation for each country is based on the MSCI Developed World Equity Index. The bond allocation for each country is based on the Citigroup World Government Bond Index.

[11] We use the Euro to proxy for the Danish kroner and the U.S. dollar to proxy for both the Singapore dollar and Hong Kong dollar, as these currencies trade within tight bands around the currencies to which they are pegged. We ignore the Israeli Shekel because it comprises a very small portion of the portfolio and is relatively more costly to hedge than the other developed market currencies.

[12] Using monthly historical returns from February 1998 through March 2013, we estimate each asset's beta to be a broad market portfolio consisting of 60% MSCI World Equity and 40% Citigroup World Government Bonds. Each asset's equilibrium return equals a risk-free rate of 3.5% plus the product of its beta and an assumed market risk premium of 4%.

[13] We also report the hedged portfolio's expected return, but because we assume that currency forward contract expected returns are 0%, the portfolio's expected return remains constant at 7.36%. We estimate within-horizon VaR values by simulating 10,000 three-year paths based on the relevant covariance matrix of the portfolio assets and currency forward contracts.

14 The hedging results are also robust to changes in the U.S. versus foreign portfolio allocation. We repeated the analysis from Exhibit 5 using upper and lower limits on the U.S. versus foreign allocation based on the time series country weights of the MSCI Developed World Equity Index and the Citigroup World Government Bond Index from February 1998 through March 2013. Using the largest U.S. equity and U.S. bond weights throughout this period to calculate the U.S. versus foreign allocation results in weights of 35% U.S. equity, 25% foreign equity, 14% U.S. bonds, and 26% foreign bonds. Using the minimum historical U.S. allocations results in weights of 29% U.S. equity, 31% foreign equity, 8% U.S. bonds, and 32% foreign bonds. Portfolio risk reduction is consistent with Exhibit 5 in each scenario, and the optimal hedge ratios match closely. For example, each currency-specific hedging solution hedges all currencies fully except Japan, which is partially hedged. Each over-hedging solution hedges the same five currencies as in Exhibit 5, with the same ranking across currencies for the amount hedged.

15 We find that the specific hedge positions are very consistent across data samples. For example, the cross-hedging and over-hedging scenarios consistently hedge substantial exposure to some combination of AUD, CAD, JPY, NZD, and SEK, but never hedge any of the CHF, EUR, GBP, or NOK. The asset reallocation scenario consistently results in large overweight positions to foreign assets, and never includes hedge positions to the CHF, GBP, or NOK (with the exception of a −3.41% hedge position to NOK for the turbulent sub-sample).

16 In addition, Kinlaw and Kritzman (2009) discuss currencies such as AUD and CAD where the domestic economy is heavily influenced by commodities. Because these currencies tend to be positively correlated with domestic equities, foreign currency exposure is often a welcome addition to Australian and Canadian investors' portfolios, whereas foreigners often find it desirable to hedge the AUD and CAD.

17 It is interesting to note that the rolling three-year monthly correlation between a 60/40 U.S. stock/bond portfolio and an equity-market-weighted basket of G10 foreign currencies is above 0.7 as of March 2013. In contrast, this rolling correlation remained below 0.5 from 1988 through 2008 and was often negative during this 20-year period. This high correlation suggests that even domestic assets have an implicit exposure to foreign currencies which can be hedged to reduce risk.

18 See Black and Scholes (1973).

19 We assume that individual currency returns are log-normally distributed. Because the sum of log-normally distributed random variables is not log-normal, we do not use the Black–Scholes–Merton formula to price a basket option.

20 Note that sometimes the conditional payoff probability between the basket option and the contingent option is the same (highlighted black bold numbers). The reason is because the basket option provides the same protection as the contingent option in the given circumstances.

21 The results presented in this article pertain to a U.S.-based investor. Schmittmann (2010) appropriately highlights the fact that an investor's base currency matters when determining a currency hedging policy. The techniques we present can be applied to investors who have other base currencies and who hold different asset portfolios. While the optimal hedging weights and performance details differ, we have replicated the analysis in Exhibit 12 for a Euro-based investor, for instance, and found qualitatively similar results.

22 For more information see Kritzman and Rich (1998).

References

Adler, M. and Prasad, B. (1992). "On Universal Currency Hedges," *Journal of Financial and Quantitative Analysis* **27**(1), 19–38.

Adler, T. and Kritzman, M. (2007). "Mean-Variance Versus Full-Scale Optimization: In and Out of Sample," *Journal of Asset Management* **7**, 302–311.

Black, F. (1989). "Universal Hedging: Optimizing Currency Risk and Reward in International Equity Portfolios," *Financial Analysts Journal* (July/August), 16–22.

Black, F. and Scholes, M. (1973). "The Pricing of Options and Corporate Liabilities," *The Journal of Political Economy* **81**(3) (May–June), 637–654.

Campbell, J. Y., Serfaty de Medeiros, K., and Viceira, L. M. (2010). "Global Currency Hedging," *The Journal of Finance* **65**(1) (February), 87–121.

Chow, G. (1995). "Portfolio Selection Based on Return, Risk, and Relative Performance," *Financial Analysts Journal* **50**(2) (March/April), 54–60.

Chow, G., Jacquier, E. Kritzman, M., and Lowry, K. (1999). "Optimal Portfolios in Good Times and Bad," *Financial Analysts Journal* **55**(3) (May/June), 65–73.

Cremers, J., Kritzman, M., and Page, S. (2005). "Optimal Hedge Fund Allocations," *Journal of Portfolio Management* **31**(3) (Spring), 70–81.

Jorion, P. (1994). "Mean Variance Analysis of Currency Overlays," *Financial Analysts Journal* **50**(3), 48–56.

Karlin, S. and Taylor, H. (1975). *A First Course in Stochastic Processes*, 2nd edition (Academic Press).

Keynes, J. M. (1923). *A Tract on Monetary Reform* (Macmillan and Co., limited).

Kinlaw, W. and Kritzman, M. (2009). "Optimal Currency Hedging In and Out of Sample," *Journal of Asset Management* **10** (April), 22–36.

Kritzman, M. (2000). "Currency Hedging and the Risk of Loss," *The Journal of Alternative Investments* **3**(3) (Winter), 27–32.

Kritzman, M. (2000). *Puzzles of Finance*: *Six Practical Problems and Their Remarkable Solutions*, Paperback edition (John Wiley & Sons).

Kritzman, M. and Rich, D. (1998). "Risk Containment for Investors with Multivariate Utility Functions," *The Journal of Derivatives* **5**(3) (Spring), 28–44.

Kritzman, M. and Li, Y. (2010). "Skulls, Financial Turbulence, and Risk Management," *Financial Analysts Journal* **66**(5) (September/October).

Kritzman, M. and Rich, D. (2002). "The Mismeasurement of Risk," *Financial Analysts Journal* **58**(3), (May/June).

Markowitz, H. (1952). "Portfolio Selection," *The Journal of Finance* **7**(1), 77–91.

Perold, A. and Schulman, E. (1988). "The Free Lunch in Currency Hedging: Implications for Investment Policy and Performance Standards," *Financial Analysts Journal* **44**(3) (May/June), 45–50.

Schmittmann, J. M. (2010). "Currency Hedging for International Portfolios," IMF Working Paper (June).

Keywords: Currency hedging; risk management; optimal hedge ratio; currency forwards; currency options