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IMPACT OF CREDIT MARKETS ON DYNAMIC STOCHASTIC REAL AGGREGATE PRODUCTION

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This paper provides a dynamic stochastic macro-financial model that describes the impact of the credit market on real production risk and provides some empirical evidence of the reasonableness of the model. Our model shows that the uncertain real sector output affects the performance of the credit market, which in turn, impacts the real production of an economy, resulting in a positive feedback effect. Our model shows that an increase in financial sector leverage and household sector leverage would induce a stronger feedback effect and increasing marginal production of financial leverage. Our model identifies the key risk drivers in measuring the performance of an economy that can be used to attribute quarterly gross domestic product (GDP) growth rate over the sample period 2000 Q1 to 2013 Q3. The empirical results can be used to interpret the underlying causes of economic boom–bust cycles and provide insights into a sustainable GDP growth pattern.

This macro-finance model has many applications. For example, the risk drivers of the GDP growth rates can be used to study equity broad-based market returns (Ho and Lee, 2014).The model can also be used to specify a structural macro-finance model that can be used to evaluate efficacy of some financial regulations (Ho and Lee, 2015b).

1 Introduction

Gross domestic product (GDP) is arguably the most important measure of economic performance of an economy. An important approach to analyze stochastic GDP growth rate is to determine its attribution by specifying the components that constitute its value. Based on the attributions, market participants can better interpret the underlying economics of the reported GDP growth rate. Recently, the size of the credit market as an attribute to the uncertainty of GDP growth rate is of particular importance. From mid-2007 to mid-2013, the US debt market has grown $30 trillion and the size of China's shadow banking industry has grown to $7.5 trillion.

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Financial economists are concerned with the impact of the growth of credit market on the real output of an economy. Yet, to date, few macro-financial models have empirical relevance that captures the dynamic stochastic characteristics of the GDP growth rate, taking the credit market into account.

For example, one attribution model is based on the aggregate expenditure constructed on the accounting identity stating that GDP is the sum of consumption, investment, government expenditure, and trade imbalance. This accounting identity is static and does not provide us insights into forecasting or time series estimation. Production functions such as the Cobb–Douglas function in macroeconomics are used. These production functions assert that the next period output depends on current capital and labor inputs. These models are dynamic, providing a model to be estimated using time series, but they pay no attention to uncertainties, which are considered as "noise", without modeling the stochastic terms.

Risk structure of GDP growth rate is important in characterizing the performance of an economy. This is because GDP quarterly growth rates are volatile and such "risk" can only be well defined if the market has already established the "expected growth" value. If GDP has grown in the multiple consecutive cycles, then economists have to decide if the economy has entered a secular growth cycle with higher expected growth rate or simply some transient positive production outcome which is called "noise". Therefore, modeling the risk structure of GDP growth rate and explaining the temporal changes are necessary to understand the growth rate movements.

A growing literature of dynamic stochastic equilibrium models seeks to fill this void. However, to date, these models typically have not incorporated the role of the financial sector as an integral part of the economic system and have failed to be empirically relevant.

Using a macro-financial model to specify the risk structure of GDP growth rate can answer the following questions: Is the financial sector destabilizing economic growth? How do household leverage and financial leverage affect the stochastic economic growth? Which drivers of economy provide sustainable secular growth? Which drivers form the components of economic growth trends? These questions are important to designers of the financial system, financial regulators, and corporate managers.

We formulate a dynamic stochastic model that describes the impact of real production risk on the financial sector performance, which in turn impacts on production. Using the flow of risk analysis, our model shows that financial leverage and household leverage induce a positive serial correlation on production. This feedback effect increases (or decreases) in an accelerated rate with an increase (or decrease) in financial sector and household sector leverages. This non-linear relationship can be destabilizing to economic growth as market participants tend to increase both household leverage and financial leverage when the economy grows steadily, and the accelerated increase in the feedback effect may lead to market fragility.

Intuitively, our model can be explained as follows. We have shown that an economy's production based on equity financing, including internal financing, is most important to identify the performance of an economy. This production output increases the total aggregate asset, and this additional asset can be used as collateral for credit, funding new projects at a lower cost of capital by lowering the informational cost of transacting. If the economy can consistently generate increased equity-financed outputs, then the credit market would provide an additional positive net present value production. But the converse is also true. In an economic downturn, the credit market would further add to the loss in production in a "de-leveraging" cycle. We define this uncertain output from equity funding as production risk. We find both the production risk and the feedback to have significant explanatory power to the GDP stochastic growth rate.

The production risk and the feedback effect enable us to formulate a seven macroeconomic factor stochastic dynamic GDP growth model. The purpose of this empirical study is not to determine the fundamental factors of the stochastic GDP growth rate as in empirical factor asset valuation models, but to study how some of the macro-factors may attribute to the changes in the GDP growth rate. Our empirical evidence tends to support this theoretical model. Our GDP growth rate attribution results for the sample period from 2000 to 2013 enable us to interpret the underlying factors that explained the GDP growth through the Great Moderation period (2002–2007), the Great Recession (2007–2010), and Slow Recovery (2010–2013). The results find that the production risk and the feedback effect from financial sector are both significant. Their impact on the boom and bust cycles of the economy provides insights into a measure of a sustainable GDP growth rate pattern.

Our model differs from macro-finance models that deal with likelihood of sovereign defaults (Cornelius, 2000; Merton and Broglie, 2005; Gray *et al.*, 2007). These models analyze the impact of the credit market on the sovereign default risk by using option such as pricing models. By way of contrast, our model is not concerned with sovereign defaults but on the impact of the financial sector on the performance of an economy.

This paper proceeds as follows: There are two main sections in this paper. First, we describe the real economy as a network tied to the financial sector and use the flow of risk methodology to formulate the aggregate asset dynamic stochastic model which is then analyzed empirically. Second, we empirically examine a GDP quarter growth rate attribution model using a Cobb–Douglas production function framework by extending from our macro-financial model.

2 Dynamic stochastic aggregate asset model

This section derives a model of total aggregate asset's dynamic stochastic process. This paper extends from the Ho *et al.* (HPS, 2012, 2013) framework in relating the financial sector to the real outputs by providing an explicit model of the lagged production risk structure. For exposition clarity, we summarize HPS model assumptions in this section. HPS assumes the agents in the economy invest in the aggregate asset for future consumption, with a design of the economy that maximizes real outputs. The productive capacity depends on an exogenous production risk and the expected net return of the aggregate asset of the economy. HPS notes that the outstanding credit borrowed must be equal to the credit lent in the financial sector. The agents' aggregate borrowing is constrained by the size of aggregate real asset used as collaterals. An increase in the size of the financial sector enhances the allocation of resources resulting in higher real outputs, which increases the aggregate asset in the economy. This in turn would lead to a larger financial sector, until the financial sector size is at an optimal level in balancing an increase in bankruptcy cost.

2.1 *Technology of the economy: Aggregate asset and production risk*

HPS first considers the real sector without the financial sector. In this case, the dynamics of the aggregate (real) asset K in the economy depend on a number of factors: the outputs of these assets, the proportion of this production which is reinvested, and the production shocks on the economy. The aggregate real asset K evolves over time according to production, investment, and consumption in the economy. Using a multi-period discrete time model, we assume that the aggregate real asset K is a linear stochastic process at time $t + 1$:

$$K_{n+1} = K_n + (h - c)K_n + K_n\varepsilon_n \qquad (1)$$

where, at time n, h is the output per unit of aggregate real asset; c is the combined effect of the depreciation rate net of investments, and organic growth of the real sector independent of the financial sector; and ε_n the idiosyncratic outputs which we assume to be independent and identical normal distribution with a constant standard deviation of σ, where $n = 0, 1, \ldots$.

This paper deals with the risk of the real sector growth and therefore the production risk ε_n is particularly important in our discussion. Within the context of our model, the production risk is the uncertain real sector output that results in the change in the aggregate asset value, which generates future outputs as explained in HPS. This idiosyncratic output, "production risk" ε_n, requires further explanation. These are idiosyncratic proportional changes in the aggregate real assets generated by exogenous factors, for example natural and man-made disasters that deplete real asset value or breakthroughs in technological innovations that enhance real asset value. In this economy, the household, which includes all agents in the economy including individuals, corporations, and government, owns all the aggregate assets K_n and therefore, the household net worth is also K_n.

In this stylized economy, we do not model the process by which agents in the economy choose to invest or consume the real assets as it is not essential for understanding the financial system. But instead we assume that the agents consume and invest in the economy which provides a constant rate of return h. We assume that c is a positive constant, and therefore the value deducted from the growth of real sector is proportional to the size of the current real sector size.

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2.2 *Institutional framework of the economy: Financial system and market frictions*

HPS assumes that the financial sector consists of financial agreements among the agents of the economy. For simplicity, the model assumes that the agreements are one-period bonds. To make implicit an interaction between the real economy and the financial sector, we assume that the financial sector has the potential for improving the production of real assets, but can also destroy assets in the case of bankruptcy.

HPS model assumes frictions in the economy with search cost and transaction cost, and considers a financial system that improves the allocation of real resources and enhances the performance of the production economy. But these benefits are offset in part by financial distresses and their associated deadweight loss resulting from bankruptcy costs.

Specifically, we note that the aggregate household asset A^H must be equal to the total debt outstanding, the aggregate household liability, L, since every dollar amount borrowed must equal the amount lent. The aggregate household liability is supported (collateralized) by the aggregate real asset, K. Therefore, the total asset of the household is sum of the financial asset and the aggregate real asset, $(L + K)$, and the total liability is L. So, by accounting identity, the net worth is K. Since the net worth is K, we can define the household leverage l_H to be the ratio of the total liabilities to net worth:

$$l_H = L/K \quad (2)$$

The financial sector can be modeled as an aggregate bank, which HPS calls the Tier 1 financial system. (Tier 2 incorporates the financial market, an extension that does not affect our conclusions of this paper.) The bank has asset A^B, liability L^B, and capital C. By definition of capital, which is asset net the liability,

$$A^B = L^B + C \quad (3)$$

But the flow of funds from the household liability to the household asset must pass through this aggregate bank. And therefore, each household debt (liability) is the bank's loan (asset). That is:

$$L = A^B \quad (4)$$

Substituting Equation (3) into Equation (4), we get:

$$L = L^B + C$$

Since the household aggregate asset equals the household aggregate liability, we can then conclude that the aggregate household assets are separated into two classes: capital C and investments A, which is the aggregate bank's liability L^B. Capital is the total asset net of the total liabilities of all the financial institutions in the financial system. And therefore,

$$L = A + C \quad (5)$$

The bankruptcy cost in the household sector has to pass from the aggregate household liability side of the household balance sheet to the aggregate household asset side of the balance sheet via the financial sector. The capital can be viewed as a junior tranche of the aggregate household asset that absorbs the default costs first. Therefore, C is a buffer to credit losses. For this reason, we can define the financial leverage l_F to be the aggregate bank's total asset (equaling the aggregate household liability) to its capital,

$$l_F = L/C \quad (6)$$

2.3 *Flow of risk: Risk structure of the aggregate asset stochastic process*

This subsection uses the financial sector framework discussed above and the flow of risk method to derive the aggregate asset dynamic stochastic process, particularly, its risk structure showing how the financial leverage, household leverage, and the time series of production risks are related. The flow of risk describes how the sequence of idiosyncratic outputs ε_n flow through the financial system and generate the uncertainties of investments in the aggregate real asset K. According to Equation (1), the risk ε_n is measured per unit K_n value, and the risk ε_n is a normal distribution with mean 0 with constant standard deviation of σ.

According to Equation (1), the production risk leads to stochastic changes in the aggregate real asset value K. We assume for the time being that the financial sector size L and the capital C adjust at the end of each period such that the household leverage l_H and the financial leverage l_F remain constant at the beginning of each period. The idiosyncratic real output induces uncertainty to the size of the aggregate real asset, which is collateralizing the household liability. Therefore, the idiosyncratic outputs induce uncertain bankruptcy cost that flows from the household sector to the financial sector, and then back to the real sector.

To model this flow of risk, we first assume that the idiosyncratic output is realized at the beginning of a period. As a result of costly re-contracting in a market with friction, the aggregate household liability L would remain unchanged for a period. Therefore, the leverages l_H and l_F would be affected by the change in K. At the end of the period, re-contracting occurs and the leverages adjust back to l_H and l_F . This lag is important to the model. When there is a failure in production, our financial capital size and household debt level cannot be adjusted downward immediately. For example, when the real sector value falls 10%, the debt level does not fall 10% immediately. The debt level would still be at the ratio to the previous real sector size. The lag would lead to high debt level for a period and hence higher bankruptcy than it would be. Therefore, the lag creates a flow of risk of bankruptcy cost from the household liability to household asset via the financial sector.

2.4 *The pathway of the flow of risk via household sector*

The production risk triggers the flow of risk, starting from the household aggregate liabilities, passing through the financial sector to the household assets, raising or lowering the financial risk capital of the household sector. The production risk is assumed to affect the credit market size linearly, proportional to the household leverage. For this paper, we interpret this relationship via expected bankruptcy cost. The impact of the credit market on the real sector can also be interpreted as the marginal effect of the credit market on the opportunity of positive net present value projects.

There are two pathways in the flows of risk. The first pathway starts with the production risk that triggers household defaults, and the dead weight loss passes to the aggregate real asset. The second pathway is the dead weight loss that passes from the household liability to the financial sector, resulting in dead weight loss in the financial sector. And that dead weight loss then passes from the financial sector to the aggregate real asset K. The first path relates the household liability to the real sector and; the second path relates the household liability and the financial sector to the real sector. For example, a home owner's default adds a dead weight loss to the real sector. Also, the homeowner's default lowers the aggregate bank's capital that may trigger financial institutions' default. That will also add the dead weight loss to the real economy. The two paths are described as follows:

For simplicity, we assume that the default rate is proportional to household leverage. We denote household default rate by δ_H. Using the superscript "–" denotes the value just prior to the realization of the production risk and default events, we have

$$\delta_H^- = \beta^H l_H. \tag{7}$$

Immediately after the realization of the production risk, the real asset value is $K(1+\varepsilon_n)$ and the household leverage becomes

$$l'_H = \frac{L}{K(1+\varepsilon_n)}. \tag{8}$$

By assuming that the standard deviation of ε_n is small, we can ignore the second order terms, then we have $1/(1+\varepsilon_n) = 1 - \varepsilon_n$ and then the household leverage just before bankruptcies occur is given by:

$$l'_H = l_H - l_H\varepsilon_n \tag{9}$$

Therefore, Equation (7) shows that if the real sector stochastic output is positive, the household leverage falls, and conversely, if the production stochastic output is negative, the household leverage would increase.

After the production risk has prevailed, at the beginning of the period $n+1$, and given that the default rate δ_H is proportional to the prevailing leverage, we have Equations (10) and (11) from Equations (7) and (9).

$$\delta_H = \beta^H l'_H = \beta^H l_H - \beta^H l_H \varepsilon_n \quad (10)$$

$$\delta_H = \beta^H l_H (1 - \varepsilon_n) \quad (11)$$

Then the following proposition describes the flow of risk via the household financial sector by identifying the stochastic term of Equation (11) and the results are summarized below.

Proposition 1. *Household sector defaults*

The default risk that flows to the real sector is given by $\beta^H l_H \varepsilon_n$, where β^H is the impact of household leverage on the default rate. The flow of funds and the flow of risk are both linearly related to the household leverage. The stochastic default rate would flow back to the real sector and it also flows to the financial sector.

2.5 *The pathway of the flow of risk via household and financial sectors*

The second pathway flows from the household liabilities to the financial sector before passing back to the household assets. In the process, dead weight losses are generated from the financial sector affecting the aggregate real asset value. Again, we assume that the financial sector default rate is proportional to the financial leverage.

Proposition 2. *Financial sector default*

The default rate from the financial sector is given by

$$\delta_F = \beta^F \beta^H l_F l_H (1 - \beta^H l_F \varepsilon_n - \varepsilon_n). \quad (12)$$

β^F is the impact of the financial leverage on the financial sector default rate.

Proof. Provided in Appendix A.

The result shows that the flow of funds is proportional to the product of the financial leverage and the household leverage. However, the flow of risk is a quadratic form of the financial leverage with a multiplicative factor of the household leverage.

This quadratic term can be explained intuitively. As expected, the financial sector defaults are affected by the household default. Therefore, we expect to see that the financial sector default contains the household leverage as a factor. When household leverage is low, the financial sector default rate should also be low. But the flow of risk is no longer linear to the financial leverage. This is because the household default is absorbed by the risk capital, which then increases the financial leverage. As explained above, capital in the aggregate on the household asset is the "junior tranche" in absorbing the default, and that in turn increases the financial default rate, resulting in a compounding effect as specified in the model.

This compounding effect can further be illustrated by a numerical example. Suppose that the US financial sector aggregated outstanding debt is estimated to be \$66 trillion. If the capital ratio is 10%, then the household asset has devoted \$6.6 trillion to support the functioning of the financial sector risks, and the financial leverage is 10. When there is a failure of idiosyncratic production at the beginning of the period the result is \$1 trillion default costs. That costs will be absorbed by the capital of the aggregate bank, resulting in a capital of \$5.6 trillion or a financial leverage of 11.6 (= \$65/\$5.6). In an alternative scenario, suppose that the capital ratio is 5%, and then the financial leverage will be 20. Given the same failure of outputs, the financial leverage will increase to 50, and hence the proportional increase is much higher when the initial leverage is higher, resulting in a proportionally higher bankruptcy costs from the financial sector.

We assume the bankruptcy cost k^H and k^F per unit default rate of the household sector and financial, sector, respectively to be constant. Let k be the combined bankruptcy cost.

Proposition 3. *Bankruptcy costs*

The total bankruptcy cost that flows to the real sector is given by:

$$k = \alpha l_H + \kappa l_F l_H - (\alpha + \kappa l_F + \gamma l_F^2) l_H \varepsilon_n \tag{13}$$

where

$$\alpha = k^H \beta^H, \quad \kappa = k^H \beta^H \beta^F \quad \text{and}$$
$$\gamma = k^H \beta^{H^2} \beta^F$$

with β and k representing default rates and default cost per unit default, respectively, and the superscripts H and F denote the household and the financial sectors, respectively.

That is, the default cost flow of funds (as opposed to the flow of risk) is linear to the financial leverage and directly proportional to the household leverage, bilinear to both leverages. However, as noted before, the flow of risk is not bilinear to the leverages.

Proof. The total default cost is given by the sum of household default and financial sector defaults:

$$k = k^H \delta^H + k^F \delta^F \tag{14}$$

Substituting for δ_H and δ_F, we have

$$k = k^H(\beta^H l_H - \beta^H l_H \varepsilon_n) + k^F(\beta^H \beta^F l_F l_H (1 - \beta^H l_F \varepsilon_n - \varepsilon_n))$$

Simplifying the above equation,

$$k = \alpha l_H + \kappa l_F l_H - (\alpha + \kappa l_F + \gamma l_F^2) l_H \varepsilon_n$$

where

$$\alpha = k^H \beta^H, \quad \kappa = k^H \beta^H \beta^F \quad \text{and}$$
$$\gamma = k^H \beta^{H^2} \beta^F$$

□

Now we can proceed to specify the dynamic model of the aggregate real asset.

Theorem: *Aggregate asset dynamic stochastic model*

Let K_n be the aggregate real asset value at time n. The stochastic movement of K_n is derived as a linear stochastic process with a drift term and two stochastic terms. The dynamics stochastic aggregate asset model is given by

$$K_{n+1} = K_n + (h - c + (b - \alpha) l_H - \kappa l_F l_H) K_n + K_n \varepsilon_n + K_{n-1}(\alpha + \kappa l_F + \gamma l_F^2) l_H \varepsilon_{n-1} \tag{15}$$

where:

$h =$ output per unit of the aggregate real asset
$c =$ consumption and depreciation rate net of investments
$b =$ positive effect of the household leverage
$\alpha = k^H \beta^H$ bankruptcy cost rate of household leverage
$\kappa = k^H \beta^H \beta^F$ combined financial and household bankruptcy cost
$\gamma = k^H (\beta^H)^2 \beta^F$ compounding dead weight loss of bankruptcy feedback effect
$\varepsilon_n =$ idiosyncratic output

Proof. According to Equation (1), without the financial sector, K follows a stochastic dynamic equation

$$K_{n+1} = K_n + (h - c) K_n + K_n \varepsilon_n \tag{16}$$

From Proposition 3, the total bankruptcy cost per unit value K that flows to the real sector is given by

$$k = \alpha l_H + \kappa l_F l_H - (\alpha + \kappa l_F + \gamma l_F^2) l_H \varepsilon_n \tag{17}$$

And the real output increases by bl_H.

Incorporating the financial sector impact of Equation (17) into Equation (16), we get the dynamics of the aggregate real asset given by

$$K_{n+1} = K_n + (h - c + bl_H - \alpha l_H - \kappa l_F l_H) K_n + K_n \varepsilon_n + K_{n-1}(\alpha + \kappa l_F + \gamma l_F^2) l_H \varepsilon_{n-1} \tag{18}$$

□

This result specifies the stochastic production risk ε_n without the credit market, when l_H and l_F have no value. The lagged stochastic term of the aggregate asset $(K_{n-1}(\alpha + \kappa l_F + \gamma l_F^2) l_H \varepsilon_{n-1})$ will be noted as the "feedback effect".

The model shows that the flow of risk through the financial sector leads to positive serial correlation of the change in the real asset value. This positive serial correlation results in a positive feedback effect on the real sector. That is, when there is a drop in real output, the dead weight bankruptcy cost would affect the real output the following period, as the financial crisis of 2008 clearly demonstrates that both the household sector defaults and the financial sector defaults affect the following period economic production.

Note that default risk is induced by the idiosyncratic outputs. When the real output production exceeds market's expectation, the default rate would also fall. The feedback effect should also be observed when the economy outperforms expectation.

This result has direct implications on macro risk management as it shows that an increase in the credit market can enhance the productivity of the economy, but at the same time, induces a higher volatility of the aggregate real asset value, resulting in higher production risk. And therefore, macro risk management has to balance these two effects on the real output in managing the size of the credit market.

2.6 *Aggregate asset model: Empirical evidence*

For clarity of exposition, we keep our model parsimonious. We assume that the equilibrium leverages are determined by the structural parameters of the market and they are constant over the sample period. The idiosyncratic output ε_n with constant standard deviation is the only factor explaining the stochastic variations in the real asset value K_n. For this reason, the *aggregate asset equation* can be re-written as Equation (19), with g and g^F the constant growth rate and constant feedback effect, respectively,

$$K_n = (1+g)K_{n-1} + K_{n-1}\varepsilon_{n-1} + g^F K_{n-2}\varepsilon_{n-2} \tag{19}$$

with

$$g = h - c + bl_H - \alpha l_H - \kappa l_F l_H \tag{19a}$$

and

$$g^F = (\alpha + \kappa l_F + \gamma l_F^2) l_H \tag{19b}$$

2.7 *Data description*

All economic data are in real terms based on 2009 price. The sample period covers from 2000 Q2 to 2013 Q4, based on quarterly data. The GDP deflator (implicit price deflator for GDP) is a measure of the level of prices of all new domestically produced final goods and services in an economy. Like the consumer price index (CPI), the GDP deflator is a measure of price inflation/deflation with respect to a specific base year; the GDP deflator of the base year itself 2009 is equal to 100. The quarterly time series of the GDP are obtained from the Federal Reserve Board.

We use the household net worth as proxy to the aggregate real asset. Household net worth is the sum of the market value of assets owned by every member of the household minus liabilities owed by household members. Wealth in the US is commonly measured in terms of this household net worth. Here we use only household net worth rather than the sum of the corporate net worth and the household net worth to avoid the double counting. The labor data come from the quarterly civilian employment from US Department of Labor. The household net worth, investment, export, import, and the government expenditure data are collected from St. Louis Federal bank.

3 Empirical results

Time series data of household net worth are used to estimate Equation (19). The constants g and g^F are used as control variables to minimize the root mean squared of the errors ε_n. For our purpose, we calculate the inverse of the negative Hessian matrix, because maximizing the likelihood function is equivalent to minimizing the sum of residual squared which we have used to estimate the parameters and the asymptotic covariance matrix of the maximum likelihood estimator is the inverse of the negative Hessian matrix.[1]

The results are reported below.

$$K_n = \underset{(1.647)}{1.00519K_{n-1}} + K_{n-1}\varepsilon_{n-1} + \underset{(3.90^*)}{0.333K_{n-2}\varepsilon_{n-2}} \tag{20}$$

The result shows that the expected quarterly growth rate of the aggregate real asset is relatively low, 0.52%. Note that these growth rate estimates are deflated to real terms and therefore the low rate of return is reasonable. The feedback effect is high, 0.333, and significant with t-statistics of 3.90. The results show that approximately a third of the idiosyncratic product output is fed back to the real economy the following period, since K_{n-2}/K_{n-1} is close to 1.

The standard deviation of the quarterly production risk (ε_n) is 2.47% or annualized 4.95%. The standard deviation of the proportional change of K_n is 2.68%, or annualized 5.35%. This result shows that the risk of aggregate asset value is magnified by the feedback effect, with an increase of 0.4%, in comparing the standard deviation of the aggregate real asset with that of the production risk.

4 Discussion

This feedback effect is important in the literature in discussing financial fragility. Kindleberger (1978) describes how a financial crisis is triggered by a default event that would lead to a fall in prices across the market as agents adjust their expectations of returns, and that in turn would create business failures. Allen and Gale (1998) argue that the low asset returns would lead to banks unable to meet their obligations, resulting in fire sale of asset, depressing asset prices further. The feedback effect can also be observed when an economy is growing. Minsky (1986) argues that when the economy is growing, risk premium would tighten, resulting in higher leverage ratio, which in turn leads to economic expansion. This feedback effect results in further tightening of the risk premium and raising the leverage ratios. Dalio (2013) provides a detailed explanation of the feedback effect. He describes the mechanics of leveraging and de-leveraging of an economy based on monetary policy, resulting in changes in real outputs.

Ho *et al.* (2013) describe that both the financial leverage and the household leverage would increase when the economy expand reaching a partial equilibrium. Ho and Lee (2015b) extends HPS model to derive those optimal household and financial leverage levels. Our model provides a model to this feedback effect that has been discussed in this literature.

The contribution of our estimate of the feedback effect based on our model has two folds: (1) we have quantified the feedback effect, which is shown to be significant; (2) we have shown that observed change in aggregate asset is, in a significant way, an accumulation of changes of the aggregate of previous cycles and not just the innovation of the current cycle. Therefore, our results show that the change in the aggregate asset value per se does not only update new information, only portion of the value change in current information. The aggregate asset value dynamic stochastic process is not martingale. Therefore, the results suggest that we should isolate the real production factor from feedback effect to determine trends of economic growth.

5 GDP growth rate attribution

The previous section shows that the credit market induces a positive feedback effect to the change in aggregate asset. This section uses this result to extend a parsimonious empirical GDP growth rate model, which is then used to specify an attribution model of GDP growth. The purpose of this empirical model is to provide a relatively simple tool to analyze reported growth rate of GDP, enabling us to identify some of the attributes of the GDP growth rate in the boom and bust cycles, such as the role of government expenditure, employment rate, and other attributes that may relate to the underlying causes of economic cycles.

5.1 A GDP Growth Rate Model

We assume that the real asset K can be measured by the household net worth. Consistent with macro equilibrium models, the economic output can be viewed as a constant rate of return of the aggregate real asset, as discussed in HPS. This model assumes labor to be held constant as input to the GDP. To incorporate labor input into the production function, we use a dynamic form of a Cobb–Douglas function. That is, we consider the following *real output equation* of the real output GDP, denoted by Y_n:

$$\frac{\Delta Y_n}{Y_n} = a + \eta \frac{\Delta K_n}{K_n} + \eta^L \frac{\Delta L_n}{L_n} + \varepsilon_n^P \qquad (21)$$

where L denotes the civilian employment in the economy, η and η^L are elasticities of the aggregate asset and labor to the production respectively, and ε_n^P is the residual term. Equation (21) assumes that the economy is closed and that all outputs are consumed immediately such that outputs and expenditure are reported in the same period. These assumptions lead to estimation error of ε^P. This error term can be explained by the expenditure side of the real output identity. In particular, we assume that

$$\varepsilon_n^P = \beta_1 \frac{\Delta G_n}{G_n} + \beta_2 \frac{\Delta I_n}{I_n} + \beta_3 \frac{\Delta T_n}{T_n} + \delta_n \qquad (22)$$

where G, I and T are the real government consumption expenditure and gross investment, the real gross private domestic investment, and the difference between real export and real import of goods and services (trade balance) respectively.

Since these independent variables are constructed to explain the residual of Equation (21), they are constructed with no expected values with the time series data de-trended. The purpose of Equation (22) is not to identify how much these components can explain the GDP growth rate. The purpose of the estimate is to identify the factors not explained by the production function. Note that we have not considered using consumption as an explanatory variable in Equation (22) because our test has shown that the consumption independent variable has little explanatory power, even though consumption is an important component of GDP.

The system of Equations (19), (21), and (22) provides us an empirically relevant model whereby each equation can be estimated. Linear regressions are applied and the results are provided below.

Estimation results for Equation (21),

$$\frac{\Delta Y_n}{Y_n} = 0.00339 + 0.11042\frac{\Delta K_n}{K_n} + 0.43179\frac{\Delta L_n}{L_n} + \varepsilon_n^P \tag{23}$$
$$(4.68^*) \quad (3.95^*) \quad (3.02^*)$$

The adjusted R^2 is 37.64.

Equation (22) seeks to determine the factors that can explain the residuals of Equation (23), based on the expenditure perspective of the GDP measure. Estimation of Equation (21),

$$\varepsilon_n^P = 0.26230\frac{\Delta G_n}{G_n} + 0.13306\frac{\Delta I_n}{I_n} - 0.03127\frac{\Delta T_n}{T_n} + \delta_n \tag{24}$$
$$(5.61^*) \quad (10.72^*) \quad (-4.49^*)$$

The adjusted R^2 is 68.24.

5.2 Risk contribution by attributes

The results can be rewritten as a model of the quarterly changes of GDP. If we substitute Equation (20) into Equations (23) and (24), then the GDP growth rate model can be described below

$$\begin{aligned}\frac{\Delta Y_n}{Y_n} &= 0.00401 + 0.11042\varepsilon_{n-1} \\ &\quad + 0.03663(K_{n-2}/K_{n-1})\varepsilon_{n-2} \\ &\quad + 0.43179\frac{\Delta L_n}{L_n} + 0.26230\frac{\Delta G_n}{G_n} \\ &\quad + 0.13306\frac{\Delta I_n}{I_n} - 0.03127\frac{\Delta T_n}{T_n} + \delta_n\end{aligned} \tag{25}$$

Each term of Equation (25) is called an "attribute" of the GDP growth rate. Equation (25) shows that the GDP quarter growth rate is the sum of the constant term A_0 and risk attributes denoted by A_i ($i = 1, 2, \ldots, 7$). And the *Attribution Equation* is given below:

$$\frac{\Delta Y_n}{Y_n} = \sum A_i \quad \text{for } i = 0, \ldots, 7. \tag{26}$$

Each attribute is described as follows: *Productivity* (0.00401) : This attribute is derived from productivity (a) and the growth rate of the household net worth (g), which has been estimated to be almost negligible. For this reason, intuitively, we think of the intercept to be productivity in the Cobb–Douglas function sense. *Production Risk* ($0.11042\varepsilon_{n-1}$): This attribute is an important determinant of the GDP growth rate uncertainty, since the production risk attribute is the risk driver of the real outputs of the economy. *Feedback Effect* ($0.03663(K_{n-2}/K_{n-1})\varepsilon_{n-2}$): This attribute should be as important as the production risk by comparing the magnitude of the coefficients for the production risk and the feedback attributes. This result is consistent with the view that the reported GDP reflects not just the current outputs but also the outputs of previous periods; and this lagged effect is induced by the credit market. *Labor Attribute* ($0.43179\Delta L_n/L_n$): This attribute is derived from the employed labor, which has minimal growth over the sample period. Equation (25) shows the significant demographic effect on the GDP growth rate. *Government Attribute* ($0.26230\Delta G_n/G_n$): This attribute is derived from the observed government expenditure that has not been explained by the production function. The result to be discussed later shows the counter-cyclical action of the government expenditure to the performance of the real outputs. *Investment Attribute* ($0.13306\Delta I_n/I_n$): This attribute is derived from the observed investment expenditure. The investment attribute is an important component of the GDP. *Trade balance Attribute* ($-0.03127\Delta T_n/T_n$): This attribute is derived from the observed trade imbalance. Since the US is not export-oriented country compared with Japan, this attribute is relatively small. *Unexplained Attribute* (δ_n): This is the idiosyncratic term unexplained by the model.

We first study the correlations of the seven risk attributes, where the correlation matrix is denoted by Ω_{ij}. The results are provided below:

	Production	Feedback	Labor	Investment	Trade	Government	Unexplained
Production	1.00	0.05	0.17	0.31	−0.10	−0.07	−0.42
Feedback	0.05	1.00	0.44	0.50	−0.13	−0.36	−0.04
Labor	0.17	0.44	1.00	0.51	−0.30	−0.25	−0.25
Investment	0.31	0.50	0.51	1.00	−0.50	−0.24	−0.09
Trade	−0.10	−0.13	−0.30	−0.50	1.00	0.07	−0.01
Government	−0.07	−0.36	−0.25	−0.24	0.07	1.00	0.08
Unexplained	−0.42	−0.04	−0.25	−0.09	−0.01	0.08	1.00

The results can be explained intuitively. Trade imbalance and government expenditure tend to be anti-cyclical to the production. On the other hand, labor and investment are positively related to the production. Interestingly, investment is strongly correlated to the feedback effect, suggesting that investment level depends on the production of the previous cycle. Investment decisions are affected by the observed production as well as the projected future production. Investment is related strongly to labor, as one might expect.

We can now determine the risk contribution of each risk attribute to the volatility of the GDP quarterly growth rates. Let σ_i be the ith attribute's standard deviation estimated over the sample period. Since the attributions' variations explain the variations in the GDP growth rates, the following equation must hold:

$$\sigma = \frac{\Sigma\Sigma\Omega_{ij}\sigma_i\sigma_j}{\sigma} \tag{27}$$

In re-arranging, we get

$$\sigma = \frac{\Sigma\sigma_i\Sigma\Omega_{ij}\sigma_j}{\sigma} \tag{28}$$

Let

$$\beta_i = \frac{\Sigma\Omega_{ij}\sigma_j}{\sigma} \tag{29}$$

and

$$C_i = \frac{\sigma_i\beta_i}{\sigma} \tag{30}$$

Then we have derived the following *risk contribution equation*

$$\Sigma C_i = 1 \tag{31}$$

where C_i is interpreted as the contribution of risk by the ith attribute. That is, we can decompose the standard deviation of the GDP growth rate into seven components, C_i for $i = 1, \ldots, 7$. The quarterly GDP growth rate standard deviation decomposition (%) is presented in the table below:

Descriptions	Production	Feedback	Labor	Investment	Trade balance	Government	Unexplained
Standard deviation (%)	0.273	0.092	0.204	0.458	0.188	0.212	0.259
Contribution (%)	20.993	7.048	17.641	58.898	–5.965	–0.463	1.842

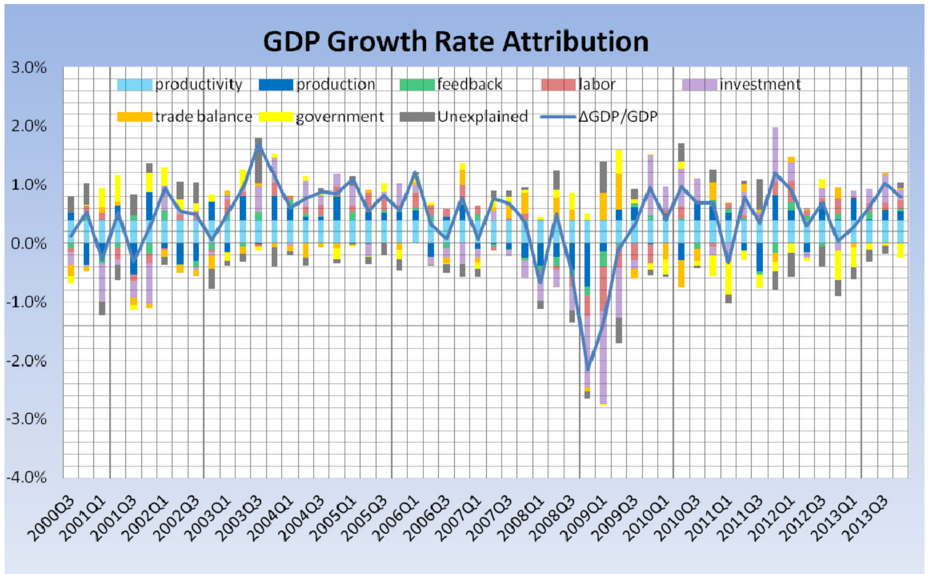
Note the value β_i adjusts for the correlations of the attributes. For example, if one attribute is negatively correlated with another, then the contributions of these two attributes would be lowered when their combined effect is considered. This measure of β_i is applied to calculate cross hedging strategies in investment literature in determining imperfect hedging strategies. The application of this methodology is the same for our purpose here.

The results show that the significant contributors are the investment (57%), production risk (21%), and labor (17.63%). The results can be explained intuitively. The quarterly standard deviation of investment is significant. 0.44% adds to the GDP growth on one standard deviation probability. Furthermore, investment is strongly positively correlated to the feedback and labor attributions, whereas the production does not have such positive correlation. Since GDP growth rate is driven by these attributes, investment becomes an important explanatory factor in the GDP growth uncertainty. Likewise the unexplained attribute is relatively small (3.66%), even though its standard deviation is not negligible. That is because the unexplained parameter has minimal correlations with other attributes. Also, both government expenditure and trade have negative contribution to risk because they tend to be counter-cyclical, providing a "hedge" to the risk of economic growth rate. The result suggests that investment and production risk together explain almost 80% of GDP growth rate variations and are important to identify GDP growth trends.

5.3 GDP quarterly growth rate attribution analysis

This risk contribution result can be extended further to study the inter-temporal behavior of these attributes using Equation (25). We can determine the attributes of the growth rate every quarter over the sample period. The attribution results are presented below graphically. The numerical results are presented in Appendix B. The sample period spans from 2000 to 2013, quarterly observations. The blue line is the quarterly growth rate in GDP. The bar chart depicts the attributions to the GDP growth rate for each quarter. For example, in the case of second quarter in 2006, the growth rate of GDP from the first quarter in 2006 is 0.3113%, to which the productivity contributes 0.4005%; production contributes –0.2402%; feedback effect contributes 0.0538%; labor contributes 0.1864%; investment contributes –0.1293%; trade balance contributes 0.0141; government contributes 0.0286% and unexplained contributes –0.0032%. The sum of the contribution is equal to the GDP growth rate.

The model enables us to interpret the underlying drivers of the GDP growth through the boom and bust cycles over the past 13 years. After the internet bubble and the 9/11 tragedy, and prior to the Great Moderation period (2002–2007), there was insignificant positive production; investment was lowered as the government expenditure increased. During the Great Moderation period, production, labor, and investment all increased. Note that investment was an important component in the GDP growth and that may



suggest that the problem of over investment in housing had become a major contributing cause for the Great Recession.

The over investment in housing deserves further elaboration. If the housing boom takes away from investment in plant that creates jobs, improves the trade balance, then the productivity as measured in our model would not increase. Since productivity growth is a main driver of GDP growth, over investment in housing can have a detrimental effect on the economy, resulting in one of the causes of the Great Recession.

The first downturn of production is in 2006 Q2, four cycles ahead of the Great Recession. The Great Recession (2007–2010) saw all the attributes fall, except for trade and government expenditure to support that market. Slow Recovery (2010–2013) period has two phases. For the first six cycles, global economy was going through the Euro-crisis. The recovery was led by investments and not by production. Starting from 2011 Q4, the results show that the production remained positive in consecutive cycles, similar in pattern to that of the Great Moderation period.

Ideally, the GDP growth should be stable, with no excess investment or government expenditure impact. In terms of the attribution graph, such sustainable GDP growth would be depicted with fairly positive constant production. Hence feedback attributes with also a positive labor attribute. If this growth can be realized over a stretch of time period, then these changes would be captured by the productivity attribute. That is, the economy would grow due to a significant accumulation of total aggregate asset and higher production efficiency (an increase of productivity). The GDP growth rate attribution for 2013 Q1, Q2, Q3, as depicted in the figure, seems to be consistent with this description, but of course, such consistency over three quarters does not imply that the US economy is actually on a sustainable growth path. (When the GDP growth is expected to be stable, the returns of the stock market should also capture such an expectation. Ho and Lee (2015b) also show that the S&P index returns were consistent with this view over that period.)

The results also show that the US economy on average grows only 1.6% annually over the past 13 years and nearly all that growth comes from increase in productivity (as interpreted from the production function) and not from the higher outputs generated by increased aggregate asset. The results also suggest that production attribute is most important in driving real output growth, as a consistent positive production attribute would lead to stable growth path. Trade imbalance and government expenditure are just counter-cyclical. Investment itself may lead to over-investment.

The intent of this paper is not to analyze the historical GDP growth rate in detail. Applying the attribution model to study the production of an economy, such as studies of over-investment, demographic shifts, is beyond the scope of this paper. In this section, we just want to illustrate how the attribution model can provide insight into the risk drivers of the stochastic behavior of the GDP growth rate. Further discussion will be left for future research.

6 Summary and conclusions

Extending the Ho *et al.* (2012, 2013) framework, this paper first presents a macro-financial model, which we then use to formulate a GDP growth rate attribution model. We also provide some empirical results that may suggest that the model can provide insights into the real sector performance.

Our macro-financial model shows that the financial sector influences a positive feedback to the real output. This paper uses a flow of risk methodology to determine the structure of this feedback effect and we show that the stochastic GDP growth rate has a lagged term, the feedback effect. And this lagged term depends on the household leverage and financial leverage in a non-linear way. This feedback effect can be destabilizing to economic growth as market participants tend to increase both household leverage and financial leverage when the economy grows steadily. Higher leverages lead to stronger feedback effects.

The GDP attribution model has eight economic factors. Productivity, production risk, feedback effect, labor, government expenditure, trade imbalance, investment, and residuals. We propose that the most important attribute to sustain stable economic growth is productivity. The production factor is most important to identify the performance of an economy.

We use the attribution model to study the GDP quarterly growth rate from 2000 Q1 to 2013 Q3. The empirical results of the attribution show that the model has reasonable explanatory power with only 1.8% of the historical GDP quarterly growth rate variations unexplained. The production and feedback effects account for 29% of the variation.

The attribution results also present the sources of the real output risks over the sample period of 13 years. The results show that during the Great Moderation period, much of the GDP growth came from increases in investment and labor, without significant increase in production. In fact, the production attribute turned negative in 2006, several cycles ahead of the beginning of the Great Recession. The result also shows that the first three cycles of 2013 seem to follow a more stable growth path. These results suggest that the attribution model may be able to provide useful insight into the performance of the economy.

This paper has many usable applications. The model can be applied to investment. Ho and Lee (2014a) show that broad-based indices' returns do not depend on the GDP growth rate per se but on the risk drivers of the GDP growth rate, taking the feedback effect of the credit market into account. Since market fragility may result from "irrational exuberance" of unjustified expectation of economic performance, it is important to tie prospective analysis (equity market valuation) and retrospective analysis (equity return attribution) in a coherent framework. Today, return attribution is an important risk management tool. The return attribution of broad-based equity indices enables the analysis of market returns.

Our model can also be applied to evaluate the efficacy of financial regulations. Ho and Lee (2015b) extend this model to a structural macro-finance model, endogenizing the optimal household leverage and financial leverage to study the appropriate level of risk capital that should be embedded in an economic network system. Our model results provide an important component to a macrostructure theory that describes the dynamic and stochastic relationships of real output, financial sector, and regulations (Ho and Lee, 2015a).

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Appendix A

Proposition 2. *The default rate from the financial sector is given by*

$$\delta_F = \beta^F \beta^H l_F l_H (1 - \beta^H l_F \varepsilon_n - \varepsilon_n). \quad \text{(A.1)}$$

The second pathway leads to a flow of default rate back to the real sector. The result shows that the flow of funds is proportional to the product of the financial leverage and the household leverage. However, the flow of risk is a quadratic form of the financial leverage with a multiplicative factor of the household leverage.

Proof. By assumption, the financial sector default rate is proportional to the financial leverage:

$$\delta_F^- = \beta^F l_F \delta_H \quad \text{(A.2)}$$

Substituting Equation (10) into the above equation, we get

$$\delta_F^- = \beta^F l_F (\beta^H l_H - \beta^H l_H \varepsilon_n)$$

Or

$$\delta_F^- = \beta^F l_F \beta^H l_H (1 - \varepsilon_n) \quad \text{(A.3)}$$

By definition, the financial leverage through the period n and just before the production risk ε_n is $l_F = L/C$. The uncertain increase in K (\$) is given by $K_n \varepsilon_n$. Therefore, the production risk impact on the aggregate credit market size L is given by $K_n \beta^H l_H \varepsilon_n$. But the capital is assumed to be the senior tranche to the changes in the credit market value. Therefore, the uncertain household default that affects the capital is given by $K_n \beta^H l_H \varepsilon_n$ and as the capital absorbs the losses and gains in step with the production risk, the capital is therefore given by

$$C^* = C + K_n \beta^H l_H \varepsilon_n, \quad \text{(A.3a)}$$

Since

$$K = \left(\frac{K}{L}\right)\left(\frac{L}{C}\right) C = (l_F / l_H) C \quad \text{(A.4)}$$

Substituting Equation (A.4) into Equation (A.3a) the above equation:

$$C^* = C(1 + \beta^H l_F \varepsilon_n) \quad \text{(A.5)}$$

Immediately after the production risk resolved, and noting l_F is the ratio of L to C, the financial default rate, using the change of C in Equation (A.5), is given by:

$$l'_F = l_F (1 - \beta^H l_F \varepsilon_n) \quad \text{(A.6)}$$

Substituting Equation (A.6) into Equation (A.3), replacing l_F by l'_F in Equation (A.3)

$$\delta_F = \beta^F \beta^H l_F l_H (1 - \beta^H l_F \varepsilon_n)(1 - \varepsilon_n).$$

Rearranging the above equation, we can derive Proposition 2 result. □

Appendix B: Attribution results during the period of 2000 Q3 to 2013 Q4

	2000Q3	2000Q4	2001Q1	2001Q2	2001Q3	2001Q4	2002Q1	2002Q2
Productivity	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005
Production	0.001163	−0.00367	−0.00219	0.002423	−0.00533	0.004804	−0.00072	−0.0035
Feedback	−0.00078	0.000382	−0.00125	−0.00075	0.000787	−0.00183	0.001543	−0.00023
Labor	−0.00079	0.002041	0.001205	−0.00199	−0.00116	−0.00156	−0.00038	0.000809
Investment	−0.00162	8.07E−05	−0.00593	−0.00046	−0.00229	−0.00632	0.004599	0.001293
Trade balance	−0.00199	−0.00093	0.000905	0.000566	−0.00135	−0.0008	−0.0014	−0.00139
Government	−0.00046	0.000688	0.003301	0.004371	−0.00016	0.003233	0.003281	0.002128
Unexplained	0.001745	0.002726	−0.0029	−0.00286	0.002458	0.000932	−0.00153	0.002344
GDP growth rate	0.001275	0.005325	−0.00286	0.005303	−0.00305	0.002455	0.009401	0.005461

	2002Q3	2002Q4	2003Q1	2003Q2	2003Q3	2003Q4	2004Q1	2004Q2
Productivity	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005
Production	−0.00295	0.003115	−0.00147	0.004122	−0.00011	0.004052	0.002054	0.000371
Feedback	−0.00119	−0.00101	0.001008	−0.00049	0.001317	−3.6E−05	0.00129	0.000658
Labor	0.001416	−0.00049	0.002504	0.000666	−0.00035	0.002289	0.000675	0.001288
Investment	−1.7E−05	−0.00023	0.001091	0.000679	0.004557	0.004545	7.48E−05	0.004612
Trade balance	−0.00106	−0.00241	0.000582	−0.00141	0.000558	−0.00072	−0.00106	−0.00262
Government	0.001719	0.001583	−0.00072	0.003493	1.36E−06	0.001208	0.000606	0.00127
Unexplained	0.002882	−0.00409	−0.00193	−0.00161	0.00692	−0.00401	−0.00168	−0.00193
GDP growth rate	0.004806	0.000486	0.00507	0.009463	0.016896	0.011335	0.005972	0.007662

	2004Q3	2004Q4	2005Q1	2005Q2	2005Q3	2005Q4	2006Q1	2006Q2
Productivity	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005
Production	0.000484	0.00396	−0.00026	0.001276	0.001158	0.001292	0.001663	−0.0024
Feedback	0.000121	0.000159	0.00126	−8.4E−05	0.000416	0.000377	0.00042	0.000538
Labor	0.001981	0.001517	0.001231	0.003378	0.002323	0.00095	0.002572	0.001864
Investment	0.001984	0.002599	0.003672	−0.00164	0.001342	0.003865	0.001704	−0.00084
Trade balance	−0.00072	−0.00084	−0.00016	0.000316	−0.0002	−0.00117	0.000266	7.11E−05
Government	0.000849	−0.00098	0.000463	0.000383	0.001735	−0.0008	0.001787	0.000827
Unexplained	6.95E−05	−0.00199	0.000728	−0.00218	−0.00263	−0.00298	−0.00029	−0.00095
GDP growth rate	0.008776	0.008432	0.010943	0.005454	0.008149	0.005541	0.012131	0.003113

	2006Q3	2006Q4	2007Q1	2007Q2	2007Q3	2007Q4	2008Q1	2008Q2
Productivity	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005
Production	0.000437	0.002649	−0.00092	−0.00015	−0.00087	−0.00257	−0.00387	−0.00244
Feedback	−0.00081	0.000145	0.000852	−0.0003	−5E−05	−0.00029	−0.00087	−0.00133
Labor	0.001436	0.003165	0.001569	−0.00084	0.000275	0.000968	−0.00019	−0.00083
Investment	−0.00131	−0.00305	−0.00119	0.001768	−0.0008	−0.00256	−0.00434	−0.0023
Trade balance	−0.00091	0.002493	−0.0007	0.000769	0.002318	0.003621	1.16E−17	0.00365
Government	0.000489	0.001443	−0.00049	0.00187	0.001605	0.000892	0.00092	0.001729
Unexplained	−0.00246	−0.00303	−0.00249	0.000583	0.00026	−0.00042	−0.00238	0.002483
GDP growth rate	0.000876	0.007817	0.000651	0.007703	0.006738	0.003649	−0.00672	0.004961

	2008Q3	2008Q4	2009Q1	2009Q2	2009Q3	2009Q4	2010Q1	2010Q2
Productivity	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005
Production	−0.00459	−0.00728	−0.00138	0.001713	0.002193	−0.00017	0.001013	−0.00288
Feedback	−0.00083	−0.00159	−0.00261	−0.00047	0.000559	0.000706	−5.5E−05	0.000331
Labor	−0.00194	−0.00351	−0.00775	−0.00368	−0.00279	−0.00318	0.000749	0.001942
Investment	−0.00359	−0.01146	−0.01485	−0.0078	−0.00112	0.010348	0.004185	0.006659
Trade balance	0.001594	−0.00071	0.004597	0.006088	−0.00176	−7.4E−05	−0.00264	−0.0046
Government	0.003073	0.001451	0.000341	0.004049	0.001219	−0.00044	−0.00161	0.001578
Unexplained	−0.00267	−0.00241	0.003733	−0.00496	0.000876	−0.00164	−0.0017	0.002583
GDP growth rate	−0.00495	−0.02151	−0.01391	−0.00105	0.00318	0.009561	0.003945	0.009618

	2010Q3	2010Q4	2011Q1	2011Q2	2011Q3	2011Q4	2012Q1	2012Q2
Productivity	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005
Production	0.003002	0.002344	0.001111	−0.00129	−0.00486	0.004166	0.001645	−0.00153
Feedback	−0.00097	0.000972	0.000751	0.00036	−0.00043	−0.00168	0.001344	0.000528
Labor	0.00027	−0.00054	0.000962	0.000335	0.000879	0.002508	0.003753	0.001053
Investment	0.004189	−0.00116	−0.00249	0.004356	0.00081	0.009232	0.003244	−0.00052
Trade balance	−0.00202	0.002937	6.16E−05	0.001254	0.000314	−0.0015	0.001067	0.000278
Government	−0.00019	−0.00229	−0.0043	−0.00071	−0.00142	−0.00086	−0.00077	0.000162
Unexplained	−0.00141	0.000669	−0.00335	−0.00044	0.004076	−0.00391	−0.00513	−0.00098
GDP growth rate	0.006878	0.006944	−0.00324	0.007877	0.003382	0.011954	0.009151	0.002991

	2012Q3	2012Q4	2013Q1	2013Q2	2013Q3	2013Q4
Productivity	0.004005	0.004005	0.004005	0.004005	0.004005	0.004005
Production	0.002951	0.000132	0.003613	0.000103	0.00162	0.001386
Feedback	−0.00051	0.000953	4.31E−05	0.001154	3.36E−05	0.000527
Labor	0.000714	0.002547	0.000193	0.00144	0.001225	0.001351
Investment	0.002052	−0.00077	0.001494	0.002859	0.005089	0.002168
Trade balance	−7.9E−05	0.001742	−0.00077	−0.00017	0.000177	0.000245
Government	0.001896	−0.00371	−0.00237	−0.00022	0.000242	−0.00152
Unexplained	−0.00414	−0.00453	−0.00336	−0.00303	−0.00221	−0.00019
GDP growth rate	0.006894	0.000368	0.002847	0.006142	0.010185	0.007972

Note

1 See William H. Greene (1993). "*Econometric Analysis*," Macmillan Publishing Company, pp. 115–116. Russell Davidson and James G. Mackinnon (1993), "*Estimation and Inference in Econometrics*," Oxford University Press, pp. 281–284.

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