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A STRUCTURAL MACRO-FINANCIAL MODEL AND MACRO-RISK MANAGEMENT

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This paper provides a structural macro-financial model that can be used for the cost and benefit analysis of alternative financial regulatory regimes. The model solves for the optimal financial sector size to the real aggregate asset (household leverage) and to the aggregate capital (financial leverage) that maximize the expected real output. This paper suggests that macro-risk management is necessary and managing the aggregate capital in the financial sector is important.

We illustrate the impact of some regulatory policies on the real outputs with some numerical examples. Our model shows that holding 2.39% in excess of the optimal capital ratio would lower the GDP growth rate by 0.61%. Since the model shows that higher financial leverage would result in higher expected growth rate and volatility of real outputs, we suggest that macro-risk management also needs to determine a risk and return tradeoff of real output.

1 Introduction

This paper provides a structural macro-financial model that can be used for a cost and benefit analysis of alternative financial regulatory regimes, for example, alternative levels of aggregate capital in the financial system. The model solves for the optimal financial sector size to the real aggregate asset (household leverage) and to the aggregate capital (financial leverage) that maximize the expected real output.

This paper's macro-financial model is based on that used by Ho *et al.* (HPS, 2012, 2013) and Ho and Lee (HL, 2015). HPS describe a dynamic stochastic macro-financial model and the impact of the credit market on real production risk, and its implication on regulatory issues. HL derive an empirical model from the HPS framework and provide some empirical evidence of the reasonableness of the model. HL results suggest that the uncertain real sector output has a positive marginal impact on the performance of the credit market. An increase (decrease) in the size of the credit market in turns leads to an increase (decrease) in the real production of an economy.

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That is, the credit market induces a feedback effect on the real sector. The empirical results indicate that this feedback effect is empirically significant and can be used to interpret the underlying causes of economic boom–bust cycles and provide insights into a sustainable GDP growth pattern. These papers have described how the financial sector and real sector are related in a stochastic dynamic way and have shown the empirical relevance of the model.

This paper extends the HPS macro-financial model by endogenizing the financial sector. A structural macro-financial model presented in this paper provides insight into the role of macro-risk management. In particular, we show that macro-risk management can adjust the contribution of financial intermediation in lowering cost of funding to maximize expected real outputs.

We show that if regulators raise the aggregate capital (lowering the financial leverage) by constraining the role of financial intermediation, then the aggregate credit held by the household (the size of the financial sector) would shrink. Therefore, the model suggests that an increase in the capital ratio of financial institutions would lower the default risk of financial institutions and may also lower the real sector outputs, when the capital ratio exceeds its optimal level.

We illustrate the impact of some regulatory policies on the real outputs with some numerical examples. Our model numerical simulation assumes an optimal capital ratio of 8%. Our model shows that holding 2.39% excess capital ratio would lower the GDP growth rate by 0.61%. Since the model shows that higher financial leverage would result in higher expected growth rate and higher volatility of real outputs, this paper suggests that macro-risk management may use a structural model to evaluate the risk and return tradeoff of real output using the aggregate capital as a control variable.

There is a growing literature in determining the optimal size of the financial sector. For example, Arcand *et al*. (2012), Deidda and Fattouh (2002), and Rioja and Valev (2004) have recently focused on estimating a non-monotone relationship between financial and economic development empirically across global economies. Spencer (2008), Tsang (2013), and others have studied macro-financial models empirically identifying factors in the real sector that may affect the capital markets. Woo (2010) empirically studies the impact of regulations on economic performance. They have focused on formulating the econometric models which are not based on theoretical models.

By way of comparison, this paper provides a theoretical structural model that provides insights into the determinants of the size of the financial sector, the relationship between the financial sector and the economy, and the importance of measuring the real output volatility and aggregate capital in the financial sector for macro-risk management. This model can provide empirically testable hypotheses.

The paper proceeds as follows: describing the assumptions of the model and then derives the structural macro-financial model, and presenting some numerical simulation results and implications of the model on macro-risk management.

2 The structural model

The structural model begins with assumptions on the real sector output based on the aggregate real asset K.[1] The financial sector consists of the aggregate household liability and aggregate household asset. In this structural macro-financial model, the credit market consists of one-period bonds. Each dollar funded is matched by a dollar invested. The financial market consists of contracts between two parties: borrowers and lenders. The financial sector transforms every dollar from the liability side of the household balance sheet to the asset side of the household balance sheet. Since the financial sector holds capital to absorb portion of the credit risk, the asset side of the household balance sheet is partitioned into senior and junior tranches, where the junior tranche, the capital C, absorbs the credit losses up to the capital level. The senior tranche, called investment, has minimal credit risk. The investment sector and the financial sector sizes are denoted by A and L, respectively. Therefore, the sum of the capital C and the investment A equals the financial sector size L.

The structural model assumes asymmetric transaction cost for the senior and junior tranches, as the financial sector performs the risk transform. We assume that the cost of capital in lending to the household without any efficiency gained by the financial sector is constant r_L^0. To the extent that the financial sector does not lower the cost, r_L^0 is constant as the financial leverage changes. The risk of the projects is allocated between the financial capital and the returns of investments of the household. For simplicity, we can think of the returns of investment is "risk free" rate because much of the risk is absorbed by the financial capital C.

The cost of capital in lending equals the weighted average of the costs of funds using investment A and capital C. We have the following equations:

$$r_L^0 = \left(\frac{A}{L}\right) r_A + \left(\frac{C}{L}\right) r_C = \text{constant}. \quad (1)$$

But the financial sector includes institutions like the banks which reduce the transaction costs. Financial institutions hold capital and their liabilities are collateralized by assets with loan to value ratio lower than 100%, for example. Collateralization lowers the information cost for the investors in determining the credit exposures of their investment. At the same time, financial institutions have the special knowledge and economy of scale to monitor the risk exposures of the capital. Collateralization reduces the information cost resulting in a lower cost of capital.

We will call this spread between the investment rate with and without financial intermediation the gain-in-efficiency spread s. For the purpose of intuitive explanation, such a spread can be thought of as the spread between a typical mortgage loan rate without financial intermediation to a loan rate based on securitization.

The actual cost of funding is lowered by the weighted average cost of capital calculation, as the financial institutions' processes lower the transaction cost. The loan rate can therefore be expressed as:

$$r_L = \left(\frac{A}{L}\right)(r_A - s) + \left(\frac{C}{L}\right) r_C \quad (2)$$

In simplifying the above equation, we get:

$$r_L = r_L^0 - s\left(\frac{A}{L}\right) \quad (3)$$

Expressing A in terms of L and C and rearranging the equation to be expressed as financial leverage, we have

$$r_L = r_L^0 - \left(1 - \frac{1}{l_F}\right) s \quad (4)$$

Or in rearranging the terms, we get:

$$r_L = (r_L^0 - s) + \left(\frac{s}{l_F}\right) \quad (5)$$

Equation (5) shows that the cost of funding can be lowered by collateralization of financial institutions but increases with the financial leverage. Equation (5) gives the required rate of returns from the investor's perspective, the "supply" of funds. Next, we discuss the demand for funds.

2.1 *Demand for funds*

We assume that there is a downward sloping demand function specifying the negative relationship between the cost of funding r_L to the household debt per unit real total aggregate asset, which is the household leverage l_H.

We assume a simple demand function of funds, where lower cost of funding would lead to a higher demand to borrow, or higher household leverage. Given some constant δ, we have

$$r_L = \delta / l_H \tag{6}$$

For the market to clear, the required rate of return should equal the cost of funding, and therefore by combining Equations (5) and (6) we have

$$(r_L^0 - s) + \left(\frac{s}{l_F}\right) = \delta / l_H \tag{7}$$

After total differentiating both sides, we get the differential relationship between the two leverages:

$$\frac{s}{l_F^2} = \delta (dl_H / dl_F) / l_H^2 \tag{8}$$

Equations (7) and (8) provide the relationship between the value and the marginal changes of financial leverage and the household leverage, enabling us to determine the optimal leverages jointly in an economy.

2.2 *Clearing conditions: Optimal financial leverage and household leverage*

We assume that economy seeks to maximize the expected outputs by adjusting the household leverage l_H, where the household leverage and the financial leverage l_F are related by Equation (7). The expected real output is given by Equation (A.7) of the basic model, described in Appendix A. Note that the linear transform of the function does not affect the optimal solution of an objective function.

$$h = c' + b' l_H - \kappa l_F l_H \tag{9}$$

where c' is the net yield, the real sector output net of depreciation and consumption; b' the marginal output from household leverage; and κ marginal bankruptcy cost to financial leverage. Therefore, at the optimal output level, we have a clearing condition:

$$\frac{dh}{dl_h} = b' - k\left(l_F + \frac{l_H dl_F}{dl_H}\right) = 0 \tag{10}$$

Substitute the differential of the leverages (Equation (8)) to Equation (10) and simplify, we get the following equation:

$$b' = \kappa l_F + \left(\frac{\kappa\delta}{s}\right)\frac{l_F^2}{l_H} \tag{11}$$

We will use Equations (7) and (11) to determine the optimal financial leverage (l_F) and the household leverage (l_H) uniquely in Proposition 1.

Proposition 1. *The household leverage l_H and financial leverage l_F uniquely determined, specified by Equations* (12) *and* (13) *below*

$$l_H = \frac{b'\delta}{b'(r_L^0 - s) + s\kappa + \sqrt{s\kappa[b'(r_L^0 - s) + s\kappa]}} \tag{12}$$

$$l_F = \frac{b's}{s\kappa + \sqrt{s\kappa[b'(r_L^0 - s) + s\kappa]}} \tag{13}$$

Proof. Using Equations (7) and (11), we can solve for the financial leverage and household leverage. In particular they can be expressed in terms of the loan demand function parameter δ, the marginal real output gain net of expected bankruptcy costs with credit funding technology b', the gain-in-efficiency spread s, the cost of funding without credit market r_L^0, and marginal bankruptcy cost from financial leverage κ. □

The result shows that given an economic environment, the leverages are uniquely determined and Equations (12) and (13) enable us to analyze the comparative statics of these optimal solutions.

Proposition 2. *The marginal changes in the leverages with respect to the gain-in-efficiency* spread s are given by:

$$\frac{dl_H}{ds} = \frac{b'\delta\left\{b' - \kappa + \frac{\kappa[b'(r_L^0 - 2s) + 2s\kappa]}{2\sqrt{s\kappa[b'(r_L^0 - s) + s\kappa]}}\right\}}{\left\{(r_L^0 - s) + s\kappa + \sqrt{s\kappa[b'(r_L^0 - s) + s\kappa]}\right\}^2} \tag{14}$$

$$\frac{dl_F}{ds} = \frac{b'^2 r_L^0 s\kappa}{2\sqrt{s\kappa[b'(r_L^0 - s) + s\kappa]}\left\{s\kappa + \sqrt{s\kappa[b'(r_L^0 - s) + s\kappa]}\right\}^2} \tag{15}$$

The proof of Proposition 2 *can be directly computed from Equations (12) and (13).*

If b' the marginal benefit of holding credit is greater than the marginal cost to the economy in bankruptcy cost κ, then the household leverage and financial leverage are positively related to the spread. (Note that b' is independent of the gain-in-efficiency and therefore is not related to the impact of financial intermediation.) That is, as the gain-in-efficiency spread increases, both the household and financial leverages would increase. Therefore, whenever financial intermediation can increase efficiency, the financial and household leverages would increase. This result underscores the importance of financial innovations that seek to improve market efficiency by increasing the gain-in-efficiency spread.

If regulators can engineer an increase in the capital ratio (lowering the financial leverage), by reducing the gain-on-efficiency spread (s), then cost of funding would increase according to our model, and that in turn would reduce the demand for funding, shrinking the credit market.

This result may offer some insights into our current credit market size. Our current regulations induce an increase in the capital ratios of financial institutions and that in turn, according to our results, would result in lowering the household leverage. This implication of the model is consistent with the current market experience where homeowners have difficulties getting mortgage loans. While the higher underwriting standard is often cited as the reasons for lowering demands for mortgage loans, the higher capital ratio imposed on financial institution, as this paper suggests, may be an additional reason.

Our paper suggests that regulators may develop methods and procedures to measure and manage the aggregate risk capital in the financial sector, and let the micro-decisions on capital structure be made by the financial institutions, avoiding our current command and control regulatory paradigm, a paradigm shift that is discussed in Ho *et al.* (2013). For example, the regulators may introduce a "capital credit" to be traded, analogous to the introduction "carbon credit" to manage the aggregate use of capital as carbon credit used to manage the aggregate output of carbon compound. The discussion of which is beyond the scope of this paper.

2.3 *Explanations of the optimal household and financial leverages' determinants*

Proposition 3. *The significant economic determinants of the optimal household and financial leverages are*: (1) *demand for funding* (δ), (2) *technological return* (b'), *the marginal real output gain with credit funding but without financial intermediation*, *and* (3) *the gain-in-efficiency spread* (s).

Proof. Equations (14) and (15) show that the household leverage and financial leverage are determined by technological return (b'), the demand for funding (δ), and the gain-in-efficiency spread (s). In addition, the leverages also depend on the default cost k and the funding rate r_L^0. The higher the default cost k, the lower would be the optimal leverages. □

We now discuss the interpretation and the impact of these three parameters on the credit market performance in turn. Financial technology (b') represents the impact of the financial sector on the real sector output in the presence of financial intermediation. If there is no impact, then the financial sector would add no value to optimal allocation of resources, capital formation, and other important contribution of the financial sector to the real economy. The parameter *technology* b' determines the marginal impact of the household leverage on real output. When b' is large, the financial sector would seek to grow to generate higher real output.

Demand parameter of funds (δ) is an important factor to the performance of a financial sector. Given a constant debt level, a unit increase in δ would result in a unit increase in the funding rate. Ho *et al.* (2013) have discussed market fragility in time of crisis. The financial sector is a network, where flow of funds and risks pass from the household liability to the household asset via the capital markets. In this network structure with a ring topology, there are many situations that these flows can be disrupted, leading to market failure. This will be particularly so when the demands for funding shift upward, resulting in market illiquidity. HPS show that this market fragility may result from market opaqueness, in appropriate regulatory actions and the network design.

Therefore, market participants' behavior is an important factor in determining the functioning of the financial sector. And market participants actions depend on the risk of real production. The risk creates uncertainty that can lead to market illiquidity via the demand function. This effect is clearly demonstrated by the financial crisis, with the freezing of the money markets, breakdown of the subprime mortgage markets and more, resulting in market illiquidity and fragility.

The gain-in-efficiency spread (s) is the lowering of investment required return as a result of financial intermediation. The model shows that for a complex real sector, the needs of customized risk management, as mentioned above, grow. To illustrate, consider a simple economy where capital formation process is transparent. One would think that the impact of a financial sector of such an economy would not have significant impact on the real outputs. That is, if all the agents can transact in a town hall, then the financial sector would not be able to add much more efficiency to this economy. The spread s would be tight.

However, if an economy is large and complex, then the spread s would be large. In this case, the financial institutions would participate, using their special knowledge and economy of scale to overcome the informational costs and operational costs. The creation of agency mortgage-backed securities to enable a broader participation in mortgage loan lending is an example of this process in lowering the mortgage loan rate to the borrowers. Proposition 2 suggests that regulators can adjust the gain-in-efficiency spread to change the financial and household leverages.

3 A numerical simulation of the model and implications

In this section, we provide some numerical estimation of the cost of holding excess risk capital in an economy. In this simulation, we assume that the GDP, Y_n is the returns of aggregate real asset K based on HPS model. That is,

$$Y_{n+1} = rK_n \tag{16}$$

We can then show that the dynamics stochastic real output model is given by

$$Y_n = Y_{n-1} + (w + b' l_H - \kappa l_F l_H) Y_{n-1} + Y_{n-1}\varepsilon_{n-1} + Y_{n-2}(\beta + \kappa l_F + \gamma l_F^2) \times l_H \varepsilon_{n-2} \quad (17)$$

The model, Equation (17), has eight parameters: w net yield; b' marginal output from household leverage; β marginal output from household leverage; κ marginal bankruptcy cost to financial leverage; γ feedback multiplier; δ demand function; s funding transaction cost; and r_L^0 equity funding rate. The last seven parameters are risk factors.

The main purpose of the simulation of the model is to illustrate the salient features of the model and to illustrate the impact of financial regulation on the real sector output performance. We will use a parsimonious model, using some estimates of the parameters in this paper. Explanations of the choice of the parameters are given in Appendix B.

Given the estimated parameters summarized above, we can now provide the model simulation results. Note that the sizes of the household leverage and the financial leverage in Table 1 are endogenized, and are not exogenously determined.

Table 1 Numerical values of the model: an example.

	Notations	Model parameters	Value
1	ω	Net yield	0.00538
2	b'	Marginal output from household leverage	0.000184279
3	β	Marginal bankruptcy cost to household leverage	0.00379561
4	κ	Marginal bankruptcy cost to financial leverage	0.00000101671
5	γ	Feedback multiplier	0.00209865
6	δ	Demand function	0.0162
7	s	Funding transaction cost	0.015
8	r_L^0	Funding rate	0.03
9	l_F	Financial leverage	12.5
10	l_H	Household leverage	1

3.1 Simulation results

To study the impact of regulations, we simulate the household leverage, financial leverage, GDP growth rate, feedback multiplier, and the GDP volatility with decreasing s values. The results are reported in Appendix C.

First, we estimate the cost of holding excess capital to the real output. We have assumed that currently our financial leverage 12.5, with the capital ratio 8%. Suppose the regulation seeks to deleverage the financial system so that the capital ratio would increase to 10.39%. The results above show that the GDP growth rate will drop from 110 basis points to 61 basis points, a decrease of 12 basis points per quarter, 48 basis points annually. Based on \$15 trillion GDP, that is \$72 billion loss in output per year. At the same time, the household leverage would deleverage from 1.00 to 0.583, a drop of 41.7%, or the financial sector size would be reduced from 12.5 to 9.62. These estimates seem reasonable, given the current shrinkage of the financial sector and the rise of capital ratio.

Under an increasingly restrictive regulatory environment, the optimal financial leverage and household leverage will fall. Their relationship as a function of (s) is depicted in Figure 1.

As noted in the previous section, both the financial leverage and the household leverage are

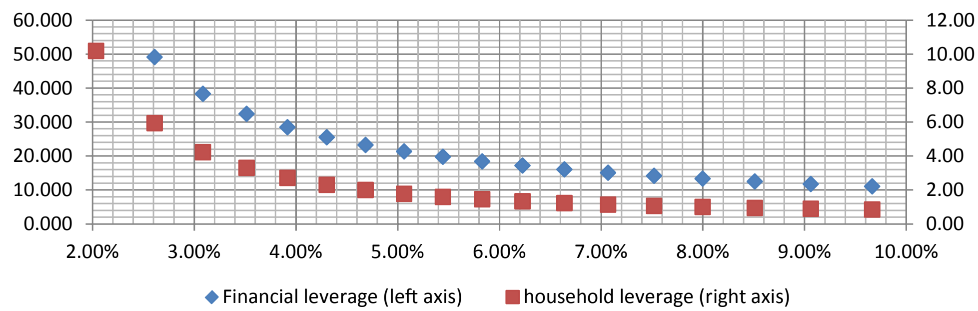

Figure 1 Financial leverage and household leverage over a range of capital ratios.

positively related to s which in turn is negatively related to the capital ratio. The above results show that as the capital ratio increases, the household leverage would decrease at a higher marginal rate than that of the financial leverage. The result suggests that in monitoring macro-risk exposure, it is important to measure the changes in household leverage, not just the financial leverage.

We have discussed that the size of the financial sector affects the expected real output. Also, our paper shows that it also affects the risk of the expected output. In the cost and benefit analysis of regulations, we can consider the regulatory impact on the expected real growth rate and the risk of the growth rate. Figure 2 depicts such a tradeoff. The values are taken from Appendix B.

Vol is the volatility of the stochastic real output and g is the expected growth rate of the real output. As the capital ratio, defined as the risk capital to the total financial asset outstanding, decreases, both the expected growth rate and the volatility increase. When the capital ratio falls below 6%, the volatility increases faster than that of the growth rate.

The result shows that raising the capital ratio is costly to the economic system, but it does lower the output uncertainty. This is particularly the case when the capital ratio is below 8%. Policy makers have to balance the growth with the increase of output uncertainty.

This result provides insight into the problem of market fragility. HL (2014) have shown that the size of the financial sector is non-linear, magnifying the effect on the real output risk; when the real sector does well, the positive feedback effect further enhances to the real sector output. But when the real output fails, the financial sector has a negative effect. Since the multiplier effect

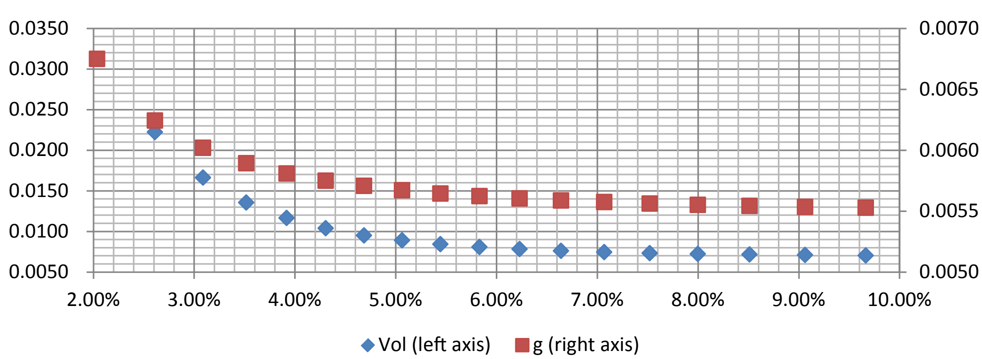

Figure 2 Real output expected growth rate and risk related to the capital ratio.

is non-linear, the economy can experience a rapid drop in output with a loss in output in one period and when the financial leverage is sufficiently high, a moderate drop in real output may trigger a financial crisis.

4 Implications on macro-risk management

Our model results suggest that if regulators use the past real sector performance to predict future performance (an increase in real sector performance would be followed by another positive output), then there is a tendency for the regulators to allow the financial sector to grow when the economy performs well. When the real output underperforms unexpectedly, a negative feedback effect will result. This macro-risk management behavior is anecdotally supported by historical experience, which shows that an economy tends to experience a period of financial deregulation and then a period of increased financial regulations. We provide three examples here showing the regulatory regime changes with the performance on the real economy.

The first example is the Glass–Steagall Act of Banking Act 1933 which was put into effect to separate investment banking and commercial banking. However, during the boom period, the Act was repealed through the Gramm–Leach–Bliley Act of 1999, and the separation was eliminated because the financial sectors can perform better without the separation. But after the 2008 financial crisis, the Volcker rule of the Dodd–Frank Act once again imposes a separation of the two businesses.

The second example is the establishment and abolishment of the Office of Thrift Supervision (OTS). OTS was established after the 1989 financial crisis to regulate community banks in their interest rate risks exposures. The federal agency then developed an extensive interest rate risk monitoring system to measure the interest rate risks of all regulated institutions. However, the risk system cannot be extended to measure credit risk because that was considered regulatory burden on the financial institutions at the time when the economy was performing well. A few community banks fail during the financial crisis because of their exposure to credit risk. This is one of the reasons that the Office of Thrift Supervision was eliminated in October 2011 resulting from the Dodd–Frank Act.

The third example is the growth and the demise of the Government Sponsored Enterprises (GSE), Fannie Mae (1938) and Freddie Mac (1970). These GSEs were established to securitize residential mortgages, extending from the Federal Housing Agency and GNMA programs to more homeowners. From 1996 to 2006, these GSEs grew significantly. Mortgage loans which are qualified for securitization by these GSEs are called conforming loans. Over this period, broader definitions of conforming loans were made, qualifying more loans for securitization. For single family, 2-family, 3-family, and 4-family, the conventional loan limits all grew 55% from 1986 to 1996, but grew 101% from 1996 to 2006.

From the macro-risk management perspectives, such experience suggests that: when the real sector does well, the public would prefer an expanded role of the financial sector to increase outputs. But when the real sector fails to perform as expected, the policy makers would tend to delever the financial market, but often after a financial crisis. This observation is somewhat inconsistent with our optimal macro-risk management results.

Our model suggests that ideally macro-risk management should increase the financial leverage when the economy expected to do well with minimal output uncertainty. The growth of the financial sector in this situation will enhance the real sector growth further. However, policies to adjust the financial leverages are difficult to implement without relatively accurate foresight. For this reason, we argue that managing macro-risk is similar to managing trading risk, where the senior management cannot optimally adjust the risk constraints put on traders. Both macro-risk and micro-financial risk management seek to lever up the position in anticipation of a "bull" market and deleverage ahead of a "bear" market—a winning strategy that is difficult to achieve consistently. The voluminous research on managing trading risk suggests, by analogy, that in macro-risk management, transparency of risk is important, and that increasing leveraging is prudent only if the growth is not accompanied by the increase in risk. That is, the observed growth rate has to be ascertained to be consistent with the expected growth rate and not idiosyncratic risk, "trading luck" if the Bayesian approach in managing risk and return is used.

5 Conclusions

This paper provides a structural macro-financial model based on the framework of Ho *et al.* (2013) and the model of Ho and Lee (2015). The model determines the optimal level of financial and household leverages in an economy. The results have regulatory implications in setting capital ratio requirements for financial institutions, design of a financial sector, and macro-risk management.

Our structural macro-financial model can be intuitively described as follows. This model provides a description of the dynamic relationship between the real sector and the financial sector. When the aggregate asset of the real sector grows, the financial sector has more asset to collateralize and therefore can offer funding at a lower cost, given financial intermediation, resulting in a growth in the financial sector, the household financial assets and liabilities. The growth of the financial sector results in more positive net present value projects, resulting in a growth of the real sector. This is the positive feedback effect of the financial sector.[2]

The size of the growth will depend on the demand for funds and the funding rate. The latter depends on the capital ratio of the financial sector. The optimal capital ratio is balanced by the total efficiency gained by financial intermediation and the expected deadweight loss of financial institutions' default and market failure. For this reason, the financial leverage and household leverage are related.

Our model shows the importance of macro-risk management and we suggest that a risk and return analysis may be used by macro-risk management. In particular, macro-risk management can be implemented by regulating aggregate capital in the financial system.

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Appendix A. Basic model assumptions

Since our paper extends Ho *et al.* (2012 and 2013) and Ho and Lee (2015) model, for clarity of exposition, we first provide a summary of the salient features of the model that are pertinent to our model.

The economy and the production process

Using a multi-period discrete time model, we assume that the aggregate real asset K is a linear stochastic process at time $t+1$:

$$K_{n+1} = K_n + (h - c')K_n + K_n\varepsilon_n \quad \text{(A.1)}$$

where h is the output per unit of physical capital, c' is the consumption rate and the depreciation rate net of investments, and ε_n is an independent and identical random variable component of output, all at time n, with a constant standard deviation of σ, where $n = 0, 1, \ldots, N$.

Institutional framework of the economy: Financial system and market frictions

The aggregate household liability is supported (collateralized) by the aggregate real asset, K. Therefore, the total asset of the household is sum of the financial asset and the aggregate real asset, $(L+K)$, and the total liability is L. So, by accounting identity, the net worth is K. Since the net worth is K, we can define the household leverage l_H to be the ratio of the total liabilities to net worth:

$$l_H = L/K \tag{A.2}$$

The bank has asset A^B, liability L^B, and capital C. By definition of capital, which is asset net the liability,

$$A^B = L^B + C \tag{A.3}$$

But the flow of funds from the household liability to the household asset must pass through this aggregate bank. And therefore, each household debt (liability) is the bank's loan (asset). That is,

$$L = A^B \tag{A.4}$$

Substituting Equation (A.3) to Equation (A.4), we get:

$$L = L^B + C$$

Since the household aggregate asset equals the household aggregate liability, we can then conclude that the aggregate household assets are separated into two classes: capital C and investments A, which is the aggregate bank's liability L^B. For example, bank deposit is household assets and is also the financial sector liability. Capital is the total asset net of the total liabilities of all the financial institutions in the financial system. And therefore,

$$L = A + C \tag{A.5}$$

The bankruptcy cost in the household sector has to pass from the aggregate household liability side of the household balance sheet to the aggregate household asset side of the balance sheet via the financial sector. The capital can be viewed as a junior tranche of the aggregate household asset that absorbs the default costs first. Therefore, C is a buffer to credit losses. For this reason, we can define the financial leverage l_F to be the aggregate bank's total asset (equaling the aggregate household liability) to its capital,

$$l_F = L/C \tag{A.6}$$

The pathway of the flow of risk via household sector

The production risk triggers the flow of risk, starting from the household aggregate liabilities, passing through the financial sector to the household assets, raising or lowering the financial risk capital of the household sector. Based on these pathways, we can proceed to specify the dynamic model of the aggregate real asset. Let K_n be the aggregate real asset value at time n. The stochastic movement of K_n is derived as a linear stochastic process with a drift term and two stochastic terms. The dynamics stochastic aggregate asset model is given by

$$\begin{aligned} K_{n+1} = K_n &+ (h - c' + (b - \beta)l_H \\ &- \kappa l_F l_H)K_n + K_n\varepsilon_n \\ &+ K_{n-1}(\beta + \kappa l_F + \gamma l_F^2)l_H\varepsilon_{n-1} \end{aligned} \tag{A.7}$$

where:

h = output per unit of the aggregate real asset
c' = consumption and depreciation rate net of investments
b = positive effect of the household leverage

β = bankruptcy cost rate on household leverage
κ = combined financial and household bankruptcy cost
γ = compounding dead weight loss of bankruptcy feedback effect
ε_n = idiosyncratic output

This result specifies the stochastic production risk ε_n without the credit market, when l_H and l_F have no value. The lagged stochastic term of the aggregate asset $(K_{n-1}(\beta + \kappa l_F + \gamma l_F^2) l_H \varepsilon_{n-1})$ will be noted as the "feedback effect."

Ho and Lee (2015) show that the GDP Y_{n+1} expected growth rate is linearly proportional to the aggregate real asset growth, when labor growth rate is constant. Therefore, the expected GDP growth rate is given by, for some constant z and z':

$$\frac{(Y_n - Y_{n-1})}{Y_{n-1}} = z + z'(w + b' l_H - \kappa l_F l_H) \quad \text{(A.8)}$$

where $w(= h - c')$, net yield, $b'(= b - \beta)$ marginal output from household leverage.

Appendix B. Derivation of the input parameters

The quarterly time series of the GDP are obtained from the Federal Reserve Board. The bank credit of all commercial banks and the total credit market debt data are from the Federal Bank of St. Louis. The federal debt data are from U.S. Department of the Treasury from 2005Q1 to 2012Q4. Data of GDP output by sector are from the Department of Commerce from 2005Q1 to 2012Q4.

Derivation of the input parameters for the model

$$Y_n = Y_{n-1} + (w + b' l_H - \kappa l_F l_H) Y_{n-1} + Y_{n-1}\varepsilon_{n-1} + Y_{n-2}(\beta + \kappa l_F + \gamma l_F^2) \times l_H \varepsilon_{n-2} \quad \text{(B.1)}$$

	Notations	Model parameters	Value	Remarks
1	ω	Net yield	0.00538	Historical data
2	b'	Marginal output from household leverage	0.000184279	Equations (11) and (A.7)
3	β	Marginal bankruptcy cost to household leverage	0.00379561	Equation (A.1)
4	κ	Marginal bankruptcy cost to financial leverage	0.00000101671	Equations (11) and (A.7)
5	γ	Feedback multiplier	0.00209865	Equations (11) and (A.7)
6	δ	Demand function	0.0162	Equation (7)
7	s	Funding transaction cost	0.015	Assumption
8	r_L^0	Funding rate	0.03	Assumption
9	l_F	Financial leverage	12.5	The BIS ratio
10	l_H	Household leverage	1	Historical data

We first estimate parameters (l_F, l_H) with the data: aggregate debt, household net worth, and the BIS ratio. Parameters (s, r_L^0) are specified from the observed market interest rates and required returns on capital. We assume that the sum of the total credit market debt and the bank credit is the loan size and the proxy variable of capital is the net worth of the household. The ratio of the loan size to the capital is historically estimated to be around 1. Therefore, l_H is assumed to be 1. Since the BIS ratio is 8% and l_F is the reciprocal number of the BIS ratio, l_F is assumed to be 12.5.

We estimate ω and b $(= b' + \beta)$ using GDP and its component growth rates. We first calculate the growth rate of each sector which constitutes the GDP. Note that the weighted average of the sector output growth rate is the GDP growth rate. We classify each sector into the financial sector and non-financial sector. We calculated the contributions of the financial sector and the non-financial sector to the GDP growth rate to be 0.009364446 and 0.0280884, respectively. The sum is the average GDP growth rate over the period of 2005–2012, which is 0.03745282. Also, since c' is 70% of the GDP growth rate, c' is estimated to be 0.00655424 $(= \frac{0.03745282}{4} \times 0.7)$ and ρ is 0.0119376 $(= \frac{0.0280884}{4} + 0.00655424 \times \frac{0.0280884}{0.03745282})$. The reason that we have to add back the impact of c' to ρ is that h is defined on a before-consumption basis. Since ω is equal to $\rho - c'$, ω is equal to 0.00538. Similarly, b is estimated to be 0.00397989 $(= \frac{0.00936445}{4} + 0.00655424 \times \frac{0.00936445}{0.03745282})$. Note that $\rho - c' + bl_H = 0.03745282$, which is the average GDP growth rate over the sample period.

We empirically estimate g and g' from Equation (A.7). After replacing $h - c' + b'l_H - \kappa l_F l_H$ and $(\beta + \kappa l_F + \gamma l_F^2) l_H$ with g and g', we express the error terms (i.e., ε_n and ε_{n-1}) in terms of K_{n+1}, K_n, and K_{n-1} for each period and minimize the sum of squared error terms in terms of g and g'. Once we have the estimated g and g' we have two more constraints to determine five parameters (b', β, κ, γ, δ).

Once we estimate or assume five parameters such as l_F, l_H, s, r_L^0, and ω, we solve the other five parameters (b', β, κ, γ, δ) with the following three equations and the two constraints above.

$$(r_L^0 - s) + \left(\frac{s}{l_F}\right) = \delta / l_H \quad \text{Equation (7)}$$

$$b' = \kappa l_F + \kappa\delta \frac{l_F^2}{l_H} \frac{1}{s} \quad \text{Equation (11)}$$

$$w + b' l_H - \kappa l_F l_H = g \quad \text{Equation (A.7)}$$

$$\beta + \kappa l_F + \gamma l_F^2 = g' \quad \text{Equation (A.7)}$$

$$b - \beta = b' \quad \text{Definition}$$

Note that the numerical simulations are used simply for illustrative purpose. The results should not be viewed as empirical estimation of the model.

Appendix C. Numerical simulations

Simulation results of the macro-financial model state variables over a range of gain-in-efficiency spread values efficiency spread values

States	1	2	3	4	5	6	7	8	9
s	0.029	0.026	0.023	0.020	0.017	0.014	0.011	0.008	0.005
l_F	49.085	28.434	21.338	17.144	14.143	11.749	9.681	7.763	5.824
l_H	10.183	3.296	2.005	1.451	1.141	0.942	0.805	0.703	0.626
V	3.54%	1.35%	0.95%	0.89%	0.74%	0.71%	0.70%	0.69%	0.69%
CR	2.04%	3.52%	4.69%	5.83%	7.07%	8.51%	10.3%	12.8%	17.2%
g	0.67%	0.59%	0.57%	0.56%	0.56%	0.55%	0.55%	0.55%	0.55%
g'	5.060	1.701	0.959	0.621	0.424	0.293	0.201	0.130	0.075

where s is the gain-in-efficiency spread, l_F the financial leverage, l_H the household leverage, g the GDP expected growth rate, g' the feedback effect, V the GDP growth rate volatility $(\sqrt{1+\sigma^2 g'})$, and CR the capital ratio $(\frac{1}{l_F})$.

Notes

[1] A more detail description of HPS model assumptions pertinent to this paper is provided in Appendix A.

[2] HL (2014) provide an alternative explanation of the positive feedback effect. A growth in the real sector would reduce the default deadweight loss of default in the financial sector, resulting in a positive impact on the real sector the following period. The modeling framework applies to both explanations.

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