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CASE STUDIES

"Case Studies" presents a case pertinent to contemporary issues and events in investment management. Insightful and provocative questions are posed at the end of each case to challenge the reader. Each case is an invitation to the critical thinking and pragmatic problem solving that are so fundamental to the practice of investment management.

Jack L. Treynor, Senior Editor

TIME DIVERSIFICATION

Because he was the oldest of their ten children, Bob Smith's parents had asked him to take some special responsibility for his siblings. But he had never expected that their plane would disappear over the Andes when he was only 20 years old. On the other hand, Bob's siblings wouldn't begin to retire for many years. He had plenty of time to translate the generous inheritance from his parents into a still more generous inheritance for his siblings.

Over the next forty years, a lot is going to happen to them, including some big surprises. And he knew even less about their futures than they knew. Some would be able to cope well with investment disappointment, others would not.

On one hand, Bob is anxious to avoid uncertainty about the value of the dollar. On the other, he knew that, over successive years, returns on the stock market multiply. It was at that point Bob had a small insight: if he could invest in the logarithm of the stock market, his total gain or loss could be the sum of the logarithms of the individual years. If the value of the stock market forty years from now is

$$\prod_{i=0}^{40}(1 + r_i),$$

then the value of the logarithm will be

$$\ln \sum_{i=1}^{40}(1 + r_i) = \sum_{i=1}^{40} \ln(1 + r_i).$$

Over the next forty years, his returns would add rather than multiply. He'd give up the possibility of spectacular upsides, but he'd have 40 years of time diversification.

A way for Bob to do that was suggested by the Taylor series. The first three terms:

$$f(x) \approx f(a) + (x - a) f'(a) + \frac{(x - a)^2 f''(a)}{2!}.$$

Substituting for the derivatives for the logarithmic function:

$$\frac{d \ln x}{dx} = \frac{1}{x}$$

$$\frac{d^2 \ln x}{dx} = -\frac{1}{x^2},$$

Bob had

$$\ln x \approx \ln a + (x - a)\frac{1}{a} - \frac{(x-a)^2}{2}\frac{1}{a^2}.$$

The second term required conventional stocks. But for the third term he required some stocks whose value varied with the square of the market level.

Such stocks weren't that hard to find. If x is the market level as before and y is the value of such stocks, then these values satisfied

$$y = x^2,$$

$$\frac{dy}{dx} = 2x,$$

and

$$\text{beta}\ \frac{x}{y}\frac{dy}{dx} = \frac{x}{x^2}(2x) = 2.$$

All Bob needed was some stocks with a beta of two—enough to give him a diversified portfolio of such stocks.

Questions

What kinds of companies vary with the square of the market level?

Are there enough of such companies to comprise a diversified portfolio?

Should investors who don't have Bob's training in math avoid such schemes?

If Bob didn't have a long time to retirement and a big initial inheritance, would time diversification be less appealing?

Should Bob run his idea past his siblings before actually investing? Will they understand?

Are the omitted terms in the Taylor series approximation going to be a problem?

Isn't Bob sacrificing a lot of upside potential?