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PUT OPTION EXERCISE AND SHORT STOCK INTEREST ARBITRAGE

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U.S.A. exchange-traded stock options are exercisable before expiration. While put options should frequently be exercised early to earn interest, they are not. In this paper, we explain an early exercise decision rule and then examine actual exercise behavior during the period January 1996 through September 2008. We find that more than 3.96 million puts that should have been exercised early remain unexercised, representing over 3.7% of all outstanding puts. We also find that failure to exercise cost put option holders $1.9 billion in forgone interest income and that this interest is systematically captured by market makers and proprietary firms.

Stock options traded on exchanges in the U.S.A. may be exercised before contract expiration. The decision to exercise an American-style call option on a stock early is relatively straightforward. If the stock pays a dividend during the call's life, holders of deep in-the-money (ITM) call option positions may find it optimal to exercise just prior to the ex-dividend day because of the impending stock price decline. Otherwise, early exercise is suboptimal.[1] The decision to exercise an American-style put option on a stock early is more complicated. Put options may be optimally exercised early for both dividend-paying and non-dividend-paying stocks, and on almost any day prior to the option's expiration day.

The intuition underlying the put option early exercise decision is as follows. A deep ITM put has no time value remaining and is priced at its floor value. Upon exercise, the put option holder receives the exercise price in cash. Each day the put option holder defers exercising the deep ITM put, he forgoes the interest income that can be earned on the cash proceeds, but retains an option to exercise the put on the following day. The difference between forgone interest income and the value of future exercise opportunities determines whether the put should be exercised early or not. In this paper, we describe a rule for deciding whether to exercise the put early. Using a sample of put options on stocks during the period January

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1996 through September 2008, we show that more than 3.96 million put options, or over 3.7% of all put options outstanding, are not exercised when it is optimal to do so and that the failure to exercise cost long put option holders over $1.9 billion.

The failure of long put option holders to exercise early has given rise to a trading game. Since the interest income being forfeit by long put option holders is being earned by short put option holders, the game involves capturing short open interest. The game, dubbed "short stock interest arbitrage," involves simultaneously buying and selling a large (relative to existing open interest), but equal, number of deep ITM puts and then immediately exercising the long puts. Since exercises are randomly assigned to open short positions, the arbitragers systematically capture the dominant share of the total short open interest and thereby earn the dominant share of the forfeit interest. Using actual exercise data, we document short stock interest arbitrage activity.

In this paper, we present the key results from our longer and more comprehensive study, Barraclough and Whaley (2012). In Section 1, we explain a decision rule for exercising a put option early. In Section 2, we examine actual early exercise decisions under plausible exercise cost assumptions, and in Section 3, we discuss the short stock interest arbitrage strategy. Section 4 contains a summary and the main conclusions.

1 Early exercise of put options on stocks

The early exercise decision rule for an American-style put involves solving for the critical stock price, S^* below which it is optimal to exercise the put early. To do so, we equate the immediate exercise proceeds of the put to its value if left unexercised, that is,

$$X - S^* = P(S^*, X, T), \tag{1}$$

where $P(S^*, X, T)$ is the value of an American-style put with exercise price X and time to expiration T assuming that the underlying stock pays no dividends during the option's life.[2] Note that Equation (1) will be satisfied by a range of stock prices for which the delta of the put on the right-hand side equals -1. The solution of interest, however, is the highest level of stock price for which Equation (1) is satisfied.

From a practical standpoint, the early exercise decision of the long put option holder (hereafter, the "long") is to either exercise the put immediately or defer the decision for one more trading day. The net benefit from early exercise at the end of the next trading day is

$$(X - S)e^{r\Delta t} - \max\left[P(S_{\Delta t}, X, T - \Delta t), X - S_{\Delta t}\right], \tag{2}$$

where Δt is the number of calendar days between adjacent trading days.[3] The first term in Equation (2) is the immediate exercise proceeds carried forward for one trading day at the risk-free interest rate. The second term is the value of the open put position left unexercised. Note that the second term explicitly recognizes that the stock price may rise above the critical stock price by the end of the next trading day. If so, the put is worth more alive than dead. To isolate the interest income component in Equation (2), re-write the expression as

$$(X - S)e^{r\Delta t} - (X - S_{\Delta t}) - \max\left[p(S_{\Delta t}, X, T - \Delta t) - (X - S_{\Delta t}), 0\right]. \tag{3}$$

Note that the third term in Equation (3) is the payoff of a call option whose terminal value at the end of one day is

$$c_{\Delta t} = \begin{cases} P(S_{\Delta t}, X, T - At) - (X - S_{\Delta t}) & \text{if } S_{\Delta t} > S^*_{\Delta t} \\ 0 & \text{if } S_{\Delta t} \le S^*_{\Delta t} \end{cases} \tag{4}$$

Applying risk-neutral valuation, the present value of the long's expected net benefit from early exercise over the next trading day is

$$NII = X(1 - e^{-r\Delta t}) - c(P_{\Delta t}, X - S_{\Delta t}, \Delta t), \quad (5)$$

where $S = e^{-r\Delta t}E(S_{\Delta t})$ and $c(P_{\Delta t}, X - S_{\Delta t}, \Delta t)$ is the value of a one-day European-style call option on an American-style put (dubbed a "caput") with exercise price $X - S_{\Delta t}$. We define Equation (5) as "net interest income (NII)." If $NII > 0$, the American-style put option should be exercised immediately. Failure to do so implies that the long chooses to forfeit (whether deliberately or not) today's *NII* to the short put option holder (hereafter, the "short"), only to face the same decision again tomorrow.

The economic intuition underlying Equation (5) is easiest to understand from the perspective of someone who is short a deep ITM put and short the stock. If the long chooses to exercise early when it is optimal to do so, he receives X in cash and delivers the stock. The short is then assigned the exercise, pays X in cash, receives the stock, and covers his short stock position. If the long fails to exercise, the short defers the payment of X and thereby earns one day's interest income (i.e., the first term on the right-hand side of Equation (5)). Since the put is deep in the money, the short put/short position is riskless for small changes in stock price (i.e., the deltas of the put and the stock positions sum to zero). If the stock price spikes upward, however, the put's delta rises above -1 and the hedge loses value. The second term on the right-hand side (i.e., the caput) is the present value of the expected cost of this contingency. Hence, we defined Equation (5) to be *net* interest income.

2 Early exercise behavior of puts

The empirical analyses in this study are based on all exchange-traded stock options traded in the U.S.A. during the period January 1996 through September 2008. The option data are drawn from OptionMetrics. Closing bid-ask quotes for stocks are drawn from the CRSP daily files.[4] The proxy for the risk-free interest rate is based on the zero-coupon yield curve of rates for overnight, seven-day, 30-day, 90-day, 180-day, and one-year Eurodollar time deposits downloaded from Datastream. The Options Clearing Corporation (OCC) provided us data on the number of contract exercises for each option series each day for the subperiod July 2001 through September 2008. The exercise data are classified by customer (C), market maker (M), and firm (F), which makes it possible to observe the behavior of different market participants. The customer category consists of retail traders and hedge funds. The firm category refers to proprietary trading by various financial institutions.

In addition to the information described above, an estimate of the expected future volatility rate is required to implement the early exercise decision rule. We use the historical volatility over the 60 trading days prior to the valuation date. To proxy for the amount and timing of the expected dividends paid during an option's life, we use the actual dividend payments.[5] We use the Black–Scholes (1973)/Merton (1973) assumption that the underlying stock price follows geometric Brownian motion and value the put option in Equation (1) using the Cox *et al.*'s (1979) binomial method.[6]

2.1 Early exercise decisions in the absence of exercise costs

The analysis in this section begins by showing that the total forgone net interest income from the failure to exercise puts early is substantial. For expositional convenience, we define an "option series day" as a trading day on which a put option series has nonzero open interest. Each option series day is earmarked as being

(a) out-of-the-money (OTM), (b) ITM but suboptimal to exercise (ITM-S), or (c) ITM and optimal to exercise (ITM-O). OTM puts are those whose exercise price is below the end-of-day stock price quote midpoint, and ITM puts are those whose exercise price is above. The early exercise decision is based on whether *NII*, as determined by Equation (5), is positive, which is equivalent to whether the prevailing stock price is below its critical level as determined by Equation (1). If the put is in the money and $NII > 0$, it is earmarked as ITM-O; if not it is classified as ITM-S.

The present value of the maximum potential gain from the early exercise of the put is $X(1 - e^{-rT})$. In a sense, it determines the size of the pie held by the long. If the long fails to exercise today, a slice of the pie is eaten by the short, with the size of each slice determined by Equation (5). If, on the next trading day, the long again fails to exercise when it remains optimal to do so, the short eats another slice, and so on through the remaining life of the option. In the event the long steadfastly refuses to exercise, the short fully consumes the pie.

To measure the total amount of forgone net interest income, we sum across all put option series within each option class each day, and then across all option classes across all days of the sample period. Table 1 contains summary statistics. In all, put options on 5,571 stocks are considered. The total put option open interest across all days in the sample period is 108.0 billion. Of these contracts, 4.0 billion (or approximately 3.7%) should have been exercised but were not. As a result, $1.87 billion of net interest income is forfeit. The results in Table 1 show that the failure of the longs to exercise their deep ITM puts resulted in extraordinary gains for the shorts.[7]

Table 1 Total put option contracts outstanding, total contracts that should be exercised early, and forgone net interest income during the sample period January 1996 through September 2008.

Year	Total contracts outstanding	Total number of series with failed exercise	Total contracts with failed exercise	Total foregone net interest income	Average interest rate
1996	1,085,561,323	228,716	42,295,177	26,601,147	0.0540
1997	1,634,115,599	345,898	63,873,723	43,204,128	0.0566
1998	2,256,237,120	591,748	93,704,631	65,678,494	0.0559
1999	3,177,505,949	541,498	116,361,789	75,320,483	0.0530
2000	4,565,690,003	926,859	221,587,188	239,185,634	0.0646
2001	5,558,526,528	857,214	243,916,947	127,985,604	0.0390
2002	6,792,243,580	739,008	262,298,717	44,699,877	0.0177
2003	7,078,914,066	288,615	118,772,457	12,704,494	0.0119
2004	11,479,962,762	715,706	324,415,804	49,664,312	0.0149
2005	13,785,885,948	1,055,310	538,051,126	185,464,902	0.0341
2006	15,518,138,225	1,066,471	484,671,996	290,802,984	0.0513
2007	19,692,753,188	1,221,214	781,290,261	474,654,290	0.0530
2008	15,400,003,638	978,821	668,464,125	231,939,701	0.0297
Total	108,025,537,929	9,557,078	3,959,703,941	1,867,906,049	

The average interest rate is the average 30-day Eurodollar rate across all trading days in each year. The number of option classes is 5,571.

2.2 *Early exercise decisions in the presence of exercise costs*

The evidence provided thus far does not account for the trading/exercise costs faced by the long put option holders. We now turn to re-examining the forgone economic benefits when exercise costs are considered. From a cost perspective, the difference between exercising today and exercising tomorrow is not the amount of the exercise costs from immediate exercise *per se* but the present value of the interest that can be earned on the exercise costs by deferring payment of the costs for one day. In other words, exercise costs must be incorporated not only into the interest income component of Equation (5) but also into the caput component. The decision rule for exercising the put in the presence of exercise costs is

$$NII(k) = (X - k)(1 - e^{-r\Delta t}) - c(P_{\Delta t}(k), X - k - S_{\Delta t}, \Delta t) > 0, \quad (6)$$

where k is the per-share exercise cost.[8]

The most cost-effective means of exercising the put can be determined by examining actual costs faced by put option holders. Broadly speaking, traders can be categorized into two groups—hedgers and speculators. Hedgers who buy puts, for example, typically have a long position in the underlying stock. Assuming hedgers are not restricted to holding the underlying stock, "exercise" can be accomplished by either (a) exercising the put and delivering the stock, or (b) reversing the put in the marketplace. In the former case, the hedger pays a fixed commission per exercise/assignment, independent of the number of contracts. In the latter case, the hedger pays an option trade execution commission as well as half of the put's bid-ask spread in selling the put and a stock trade execution commission as well as half the stock's bid-ask spread in selling the stock.[9]

Speculators who are long puts have a directional view that the stock price will fall and want to profit from their prediction. Assuming that they are correct in their prediction and that the stock price happens to fall below its critical level, the speculator, like the hedger, can exercise or reverse. To exercise, the speculator must buy the stock, exercise the put, and then deliver the stock. The total costs include paying an option exercise commission, and paying a stock commission and half the stock's bid-ask spread to acquire the stock for delivery. To reverse, the total costs include paying the option trade execution commission and half the put's bid-ask spread.

For commission levels, we use the online customer rates currently charged by Charles Schwab.[10] For option exercise/assignment, Schwab charges a flat fee of $8.95 for option exercise/assignment, and for option trade execution, they charge $8.95 plus $0.75 per contract. For stock trade execution, they charge a flat fee of $8.95. Since we require trading costs to be on a per-share basis, we compute the commission per-share based on different assumptions regarding the number of contracts exercised.

	Commission per-share		
Number of contracts	1	10	20
Hedger exercises put and delivers stock	0.0895	0.0090	0.0045
Hedger reverses put and sells stock	0.0970	0.0165	0.0120
Speculator exercises put, and buys and delivers stock	0.1790	0.0179	0.0090
Speculator reverses put	0.0970	0.0165	0.0120

To account for the effects of bid-ask spreads, we use actual closing bid-ask price quotes for put options and their underlying stocks each

day during the period January 1996 through September 2008.[11] Combining our estimates of commissions and bid-ask spreads and the average bid-ask spreads from the overall sample period, we find that the average exercise costs per-share for retail customers are as follows:

	Total costs per-share		
Number of contracts	1	10	20
Hedger exercises put and delivers stock	0.0895	0.0090	0.0045
Hedger reverses put and sells stock	0.4660	0.3049	0.2959
Speculator exercises put, and buys and delivers stock	0.2513	0.0902	0.0813
Speculator reverses put	0.3042	0.2236	0.2191

Clearly reversing the put in the marketplace is more expensive from a trading cost standpoint. Put option bid-ask spreads are simply too high. Consequently, in the subsequent analysis we use the trading costs associated with exercising the put and ignore the alternative of reversing.

In determining the per-share exercise cost, we assume a worst-case scenario where the hedger or speculator exercises only a single contract. For a hedger who exercises the put, k in Equation (6) is assumed to be \$0.0895 based on the information provided above. For a speculator who exercises the put and buys and delivers the stock, k is assumed to be \$0.1790 plus half the prevailing bid-ask spread for the stock.

Table 2 summarizes the total number of contracts with failed exercise and the total forgone net interest benefits in the presence of exercise costs. The first row of the table matches the figures reported in Table 1. The second and third rows document the effects of exercise costs. As the results show, the effects are relatively modest. Accounting for the exercise costs of a hedger, the number of contracts with failed exercise drops by only 128 million, with the total forgone net interest income remaining at an astonishing \$1,823 million. Increasing exercise costs to those of a speculator, the number of failed exercises drops by only another two million. In other words, failure to exercise is pervasive. Even after accounting for plausible exercise costs, the longs forfeit \$1,816 million over the 13-year sample period.

2.3 Actual exercise behavior

The evidence provided thus far in this section suggests that long put option holders are failing to exercise when they should even after accounting for exercise/trading costs. As a result, a substantial amount of money is "left on the table".

Table 2 Total number of series days on which put option should be exercised, total open interest across series days, and total forgone net interest income during the sample period January 1996 through September 2008.

Trading cost per share assumption	Total number of series with failed exercise	Total contracts with failed exercise	Total forgone net interest income
No exercise costs	9,557,078	3,959,703,941	1,867,906,049
Hedger exercises and delivers stock	9,383,407	3,831,981,849	1,823,059,269
Speculator exercises, and buys and delivers stock	9,378,630	3,829,674,250	1,815,550,790

The number of option classes is 5,571. The cost of the hedger exercising and delivering the stock is assumed to be \$0.0895 per share. The cost of a speculator exercising, and buying and delivering the stock is assumed to be \$0.1790 per share plus half of the prevailing bid-ask spread.

While this evidence is important, it does not provide a complete picture of exercise behavior. To complete the picture, we turn to our subsample of 4,011 option classes for which we have actual exercise data during the period July 2001 through September 2008, and determine whether observed put option exercises are, in fact, optimal and whether other put option series, which should have been exercised, are not. Table 3 contains a summary of our results.

Table 3 contains actual numbers of contracts exercised and separates them by whether they "Should be exercised" or "Should not be exercised". Of the put options that should be exercised, about 6.2 million series had open interest totaling 3.2 billion contracts.[12] Recall that since $NII > 0$ for these series, all remaining long (and, hence, implicitly short) open interest should disappear. Equally perplexing is the trading volume for the ITM-O puts—more than 201 million contracts. These trades are unlikely to be attributable to hedgers/speculators reversing their option positions since, as we have already documented, bid-ask spreads are so much higher in the option market than in the stock market. It is much cheaper to exercise than reverse. At the same time, the trades are unlikely to be newly established positions. Since these options are trading at their floor values, it is more cost-effective to short the stock than to buy the put. In the next section of the paper, we show that this trading volume is largely attributable not to position reversals or new positions but rather to an activity called short stock interest arbitrage.

Table 3 also summarizes the numbers of contracts exercised by market participant. Of the contracts that should be exercised, only 184.9 million contracts are exercised, accounting for less than 6% of the previous day's open interest. Of the contracts exercised, market makers are the most active of market participants, accounting for 47.4% of all exercise activity. While proprietary firms are almost as active, accounting for 42.6% of all exercise activity, retail customers account for only 9.9%. This stylized fact suggests that retail customers, as nonprofessional traders, are not as sophisticated as professional traders (i.e., market makers and proprietary firms), so

Table 3 Optimal and suboptimal put option exercises during the sample period July 2001 through September 2008.

Panel A. Actual exercises

	Should be exercised ($S_t < S^*$)		Should not be exercised ($S_t \geq S*$)			
	ITM-O		ITM-S		OTM	
Number of option series days	6,198,164		49,117,376		51,594,734	
Total open interest	3,191,601,879		25,619,699,835		60,393,758,470	
Previous day's open interest	3,247,528,057		25,493,688,859		59,790,672,931	
Number of contracts traded	201,454,795		860,709,189		1,902,760,394	
Number of contracts exercised by:						
Customer	18,325,606	9.9%	8,244,992	18.8%	125,687	21.6%
Market maker	87,748,104	47.4%	21,776,742	49.7%	275,824	47.5%
Proprietary firm	78,860,356	42.6%	13,772,026	31.4%	179,699	30.9%
Total	184,934,066		43,793,760		581,210	

The number of option classes is 4,011. The months of November 2001, January and July 2002, and January 2006 are missing.

they may be unaware that early exercise is optimal, either because they do not fully understand the decision rule or do not have time to constantly monitor their positions.

The exercise summary for the ITM-S and OTM put options is also informative. About 44 million puts that are in the money but not optimal to exercise are in fact exercised. Of these, retail customers accounted for 18.8% of the exercise activity in the ITM-S category, while proprietary firms and market makers accounted for 31.4% and 49.7%, respectively. About 0.6 million OTM puts are exercised. Of these, 21.6% are by retail customers, 47.5% by market makers, and 30.9% by proprietary firms. On the one hand, the relative frequency of exercise reflects the fact that professional traders know the early exercise decision rule and constantly monitor their positions. It is their primary line of business. On the other hand, the fact that exercises take place in this ITM-S category is worthy of note. One possible reason is that we use end-of-day stock price quote midpoint in determining whether exercise is optimal. It may be the case that it was optimal to exercise the put earlier in the day when the stock price was lower.[13] Another reason is that there may be noise in our model-based exercise decision rule due to option valuation model misspecification and/or noise in the estimate of expected future volatility.

3 Short stock interest arbitrage

The failure of long put option holders to exercise deep ITM puts has given rise to what amounts to a trading game—short stock interest arbitrage (hereafter, "SSIA"). SSIA involves "… the purchase, sale and exercise of ITM options of the same class" on the same trading day.[14] SSIA allows arbitragers to systematically capture the open interest of deep ITM puts from existing shorts and thereby capture the interest income being left on the table by long put option holders' failure to exercise.

SSIA works as follows. Assume that a deep ITM put has open interest of m contracts. Anticipating the longs' failure to exercise, arbitragers step in front of the existing shorts to earn the forgone net interest income by simultaneously buying and selling n deep ITM puts with the same exercise price (where n is significantly greater than m) and then immediately exercising the n long puts.[15] Under clearinghouse rules, exercises are randomly assigned to open short positions at the end of the trading day. After random assignment, the preexisting shorts' proportionate share of the total open interest is expected to drop from 100% to $\frac{m}{m+n}$%, while the new shorts' goes from 0% to $\frac{n}{m+n}$%. Note that larger the SSIA trade relative to the preexisting open interest, the greater the capture. The new shorts are "arbitragers" in the sense that the strategy is risk-free for small changes in the stock price.

Before showing an actual example of the execution of SSIA, it is worthwhile to note that many stock option exchanges implicitly encourage this type of trading activity by capping fees. On the NASDAQ OMX PHLX, for example, the exchange fees for nonelectronic trades are \$0.25 per contract. Thus, to buy and sell n contracts, the total cost would be \$0.50n. The NASDAQ OMX PHLX, however, caps the fee on SSIA trades at \$1,000.[16] Consequently, for trades greater than 2,000 contracts, the per-contract cost of executing the SSIA spread begins to fall. So, not only does a very large trade garner more short open interest, but it also provides cost savings.

To illustrate the practice of short stock interest arbitrage, consider the daily trading/exercise activity of a single deep ITM put option series—the January 2006 65-put written on Wal-Mart's stock—during the period August 17 through November 9, 2005. Table 4 contains a summary.

Table 4 Trading volume, open interest, and number of exercises of the January 2006 65-put option on Walmart during the period August 17 through November 9, 2005.

Date	Closing stock price	Days to expiration	Trading volume	Open interest	Change in open interest	Term interest rate	Overnight interest rate	Present value of maximum interest income	Net interest income	Number of contracts exercised		
										Customer	Market maker	Proprietary firm
8/17/2005	47.11	156	0	4,578	0	0.03932	0.03501	495,935	2,854	0	0	0
8/18/2005	47.24	155	10,002	4,578	0	0.03976	0.03548	498,160	2,893	0	0	10,002
8/19/2005	46.58	154	2,100	6,678	2,100	0.03978	0.03532	722,439	12,600	0	0	0
8/22/2005	46.67	151	1,002	6,678	0	0.03967	0.03548	706,506	4,220	0	0	1,002
8/23/2005	46.34	150	0	6,678	0	0.03996	0.03533	706,953	4,201	0	0	0
8/24/2005	45.55	149	0	6,678	0	0.03962	0.03533	696,455	4,201	0	0	0
8/25/2005	45.29	148	0	6,678	0	0.03971	0.03548	693,357	4,220	0	0	0
8/26/2005	45.70	147	0	6,678	0	0.03989	0.03564	691,774	12,713	0	0	0
8/29/2005	45.65	144	0	6,678	0	0.03992	0.03564	678,224	4,239	0	0	0
8/30/2005	45.19	143	0	6,678	0	0.03987	0.03596	672,715	4,276	0	0	0
8/31/2005	44.96	142	0	6,678	0	0.03976	0.03596	666,217	4,276	0	0	0
9/1/2005	45.00	141	0	6,678	0	0.03907	0.03628	650,264	4,314	0	0	0
9/2/2005	44.55	140	0	4,578	−2,100	0.03842	0.03556	435,308	11,594	0	2,100	0
9/6/2005	45.69	136	0	4,292	−286	0.03839	0.03596	396,220	2,748	0	286	0
9/7/2005	45.86	135	0	4,292	0	0.03869	0.03533	396,388	2,700	0	0	0
9/8/2005	45.86	134	0	4,292	0	0.03899	0.03548	396,504	2,712	0	0	0
9/9/2005	45.89	133	0	4,292	0	0.03914	0.03564	395,006	8,171	0	0	0
9/12/2005	45.89	130	0	4,292	0	0.03917	0.03548	386,490	2,712	0	0	0
9/13/2005	45.07	129	15,850	4,292	0	0.03925	0.03548	384,280	2,712	0	10	15,840
9/14/2005	44.70	128	8,115	6,405	2,113	0.03932	0.03533	570,173	4,029	0	0	6,002

Table 4 (*Continued*)

Date	Closing stock price	Days to expiration	Trading volume	Open interest	Change in open interest	Term interest rate	Overnight interest rate	Present value of maximum interest income	Net interest income	Number of contracts exercised		
										Customer	Market maker	Proprietary firm
9/15/2005	44.32	127	0	4,305	−2,100	0.03938	0.03786	380,773	2,902	0	2,100	0
9/16/2005	43.87	126	4,002	4,305	0	0.03952	0.03743	379,167	8,608	0	0	4,002
9/19/2005	44.01	123	0	4,224	−81	0.03976	0.0377	365,389	2,836	0	81	0
9/20/2005	43.21	122	12,057	4,224	0	0.03963	0.03754	361,300	2,824	0	4,019	8,038
9/21/2005	42.49	121	10,502	4,224	0	0.04009	0.0377	362,508	2,836	0	500	10,002
9/22/2005	43.19	120	0	4,224	0	0.03992	0.03786	357,988	2,848	0	0	0
9/23/2005	43.20	119	0	4,224	0	0.04022	0.03817	357,684	8,613	0	0	0
9/26/2005	43.11	116	0	4,224	0	0.04048	0.03802	350,938	2,860	0	0	0
9/27/2005	43.10	115	12,657	4,224	0	0.04062	0.03865	349,118	2,907	0	4,219	8,438
9/28/2005	43.13	114	10,002	4,224	0	0.0406	0.03802	345,967	2,860	0	0	10,002
9/29/2005	43.54	113	18,000	4,224	0	0.04113	0.03834	347,390	2,884	0	6,000	12,000
9/30/2005	43.82	112	12,002	2,224	0	0.04112	0.03907	344,231	8,816	0	0	12,002
10/3/2005	43.76	109	40	4,211	−13	0.04133	0.03897	335,760	2,922	0	40	0
10/4/2005	43.85	108	25,002	4,211	0	0.04141	0.03802	333,309	2,851	0	4,000	21,002
10/5/2005	43.50	107	14,040	4,211	0	0.04139	0.0377	330,121	2,827	0	4,019	10,021
10/6/2005	43.93	106	6,002	4,211	0	0.04154	0.03802	328,176	2,851	0	0	6,002
10/7/2005	44.03	105	11	4,220	9	0.04157	0.03823	326,084	8,617	0	0	0
10/10/2005	44.54	102	0	4,220	0	0.04152	0.03802	316,403	2,857	0	0	0
10/11/2005	45.02	101	23,559	4,220	0	0.04163	0.03834	314,204	2,881	0	4,519	19,040
10/12/2005	44.94	100	10,002	4,220	0	0.04164	0.03786	311,120	2,845	0	0	10,002

Table 4 (*Continued*)

Date	Closing stock price	Days to expiration	Trading volume	Open interest	Change in open interest	Term interest rate	Overnight interest rate	Present value of maximum interest income	Net interest income	Number of contracts exercised		
										Customer	Market maker	Proprietary firm
10/13/2005	44.76	99	0	2,720	−1,500	0.04193	0.03818	199,931	1,849	1,500	0	0
10/14/2005	45.04	98	0	2,720	0	0.04191	0.03839	197,839	5,577	0	0	0
10/17/2005	45.24	95	0	2,720	0	0.04186	0.03865	191,567	1,872	0	0	0
10/18/2005	45.13	94	14,459	2,720	0	0.042	0.03818	190,185	1,849	0	2,819	11,640
10/19/2005	45.99	93	0	2,720	0	0.04181	0.03802	187,345	1,841	0	0	6,002
10/20/2005	45.60	92	0	2,720	0	0.04196	0.0377	186,002	1,826	0	0	0
10/21/2005	45.72	91	0	2,711	−9	0.04194	0.03838	183,299	5,558	0	9	0
10/24/2005	46.21	88	0	2,711	0	0.04203	0.03818	177,665	1,843	0	0	0
10/25/2005	45.39	87	5,438	2,711	0	0.04201	0.03786	175,591	1,828	0	2,719	2,719
10/26/2005	45.58	86	7,002	2,711	0	0.04215	0.03818	174,141	1,843	0	0	7,002
10/27/2005	44.74	85	0	2,711	0	0.04257	0.03913	173,846	1,889	0	0	0
10/28/2005	45.50	84	0	2,711	0	0.04253	0.04013	171,642	5,811	0	0	0
10/31/2005	47.31	81	0	2,711	0	0.04249	0.04071	165,385	1,965	0	0	0
11/1/2005	46.99	80	14,002	2,711	0	0.04242	0.04071	163,059	1,965	0	4,000	10,002
11/2/2005	47.56	79	6,002	2,711	0	0.04267	0.04071	161,996	1,965	0	0	6,002
11/3/2005	47.45	78	0	2,711	0	0.04251	0.0404	159,369	1,950	0	0	0
11/4/2005	47.69	77	0	2,711	0	0.04257	0.0406	157,552	5,880	0	0	0
11/7/2005	48.05	74	11,002	2,711	0	0.04259	0.0404	151,488	1,950	0	2,500	8,502
11/8/2005	47.61	73	6,002	2,711	0	0.04275	0.0404	150,021	1,950	0	0	6,002
11/9/2005	48.20	72	0	311	−2,400	0.04272	0.0404	16,962	224	0	0	2,400

Exercises are classified by market participant, and the classifications are customer, market maker, and proprietary firm.

For all days during the period, the put should be exercised immediately. But, as the table shows, the open interest of the series does not disappear. Indeed, it remains in the thousands of contracts throughout the period.

At the end of the day on August 17, 2005, open interest in the January 65-put was 4,578 contracts. This implies that the 4,578 longs behaved suboptimally. The present value of the maximum interest income that they could have earned over the remaining life of the option if they exercised immediately was $495,935. As noted earlier, this amount can be thought of as the size of the potential pie that the longs are making available to the shorts at the close of August 17. The net interest income that can be earned over the next trading day less the value of the caput is $2,854. This represents the size of each slice of the pie that the shorts earn from the longs' failure to exercise.

On August 18, short stock interest arbitrage takes place. This strategy has a number of tell-tale signs. One sign is that trading volume is high but open interest remains unchanged from the previous day. The second sign is that trading volume equals the number of exercised contracts on that day. While determining exactly how many market participants are involved in the trading on August 18 is not possible from the available data, a likely scenario is that Proprietary Firm A simultaneously bought and sold 5,001 65-puts from Proprietary Firm B at, say, the midpoint between the bid and ask prices.[17] This activity produces a daily trading volume of 10,002 contracts.[18] Absent other considerations, open interest should increase by 10,002 contracts. But the trading game also involves both firms immediately exercising their long positions, an activity documented in the rightmost column of the table, which indicates that 10,002 contracts were exercised that day.

With the clearinghouse randomly assigning the exercises to the shorts, the newcomers (in this case the proprietary firms) wind up with about $\frac{10,002}{4,578+10,002} = 68.6\%$ of the open interest after assignment,[19] and consequently capture about 68.8% of the forgone interest income, $2,893. While we do not know the identity of the shorts on August 17, we know that on August 18 about 69% are proprietary firms who will continue to earn their proportionate share of the forgone interest income, provided that long positions remain open and no one steps in front of them to capture their open interest.

On the next day, 2,100 contracts are traded, no contracts are exercised, and open interest rises by 2,100 contracts to a level of 6,678. In theory, the buyer(s) of these contracts should exercise immediately. Yet no contracts are exercised. This trading activity seems irrational in the sense that it simply increases the available pool of forgone interest income available to the shorts. From August 22 through September 1, the activity in the 65-put is dormant—no trading and no exercises. This means that on August 22 the short put option holders are allowed to earn interest on the exercise proceeds of the 6,678 open long contracts for 10 days. Exercise activity occurring on September 2, September 6, September 15, September 19, October 13, and November 9 is considered normal and not part of a short stock interest arbitrage since it is unaccompanied by commensurate trading volume. But what remains a mystery is why the long put holders deferred exercise. The market maker exercising the 2,100 contracts on September 2, for example, was holding the long put position at least as far back as August 23. By exercising earlier, more interest income would have been earned.

Aside from the normal exercises noted in the previous paragraph, all other exercise activities appear to be short stock interest arbitrage executed by both market makers and proprietary firms. In nearly all cases, trading volume equals

the sum of the number of exercises across market makers and proprietary firms and open interest remains unchanged from the previous day. Proprietary firms are the single largest player of the game. Only once during the period did market makers play at an equal level, while customers did not play the game at all.

3.1 Aggregate short stock interest arbitrage activity

Table 4 suggests that proprietary firms are the most active short stock interest arbitragers. The evidence is modest, however, considering that the table contains only one put option series class for a three-month period. We now turn to examining aggregate SSIA exercise behavior across all put option series in all option classes across all days during the sample period July 2001 through September 2008. To do so, we need to apply a rule for identifying SSIA trades. The rule that we use is that the number of contracts exercised for a particular put option series on a given day is earmarked as SSIA trading activity if (a) the total number of exercises by market makers and proprietary firms exceeds 1,000 contracts and (b) the trading volume exceeds 1,000 contracts. Note that this rule accurately identifies all SSIA trades in Table 4.

The aggregate exercise activity results are reported in Table 5. The "No constraint" columns

Table 5 Number of put option contracts exercised early by market participant and trading activity during the sample period July 2001 through September 2008.

	No constraint		Less than $1,000		Greater than or equal to $1,000	
	Number	Percent	Number	Percent	Number	Percent
Panel A. Total exercises						
Customer	18,325,606	9.9%	9,283,416	13.3%	9,042,190	7.9%
Market maker	87,748,104	47.4%	35,081,494	50.2%	52,666,610	45.8%
Proprietary firm	78,860,356	42.6%	25,588,148	36.6%	53,272,208	46.3%
Total	184,934,066		69,953,058		114,981,008	
Panel B. SSIA exercises						
Customer	2,209,460	2.1%	459,501	2.1%	1,749,959	2.0%
Market maker	44,913,815	41.9%	7,714,900	35.6%	37,198,915	43.5%
Proprietary firm	60,142,396	56.1%	13,506,313	62.3%	46,636,083	54.5%
Total	107,265,671		21,680,714		85,584,957	
Panel C. Non-SSIA exercises						
Customer	16,116,146	20.7%	8,823,915	18.3%	7,292,231	24.8%
Market maker	42,834,289	55.2%	27,366,594	56.7%	15,467,695	52.6%
Proprietary firm	18,717,960	24.1%	12,081,835	25.0%	6,636,125	22.6%
Total	77,668,395		48,272,344		29,396,051	

Panel A contains the total number of exercises. Panel B contains number of exercises associated with trades earmarked as short stock index arbitrage (SSIA) trades, and Panel C contains numbers of exercises in trades not associated with SSIA. The column headings "Less than $1,000" and "Greater than or equal to $1,000" refer to the size of the net interest income of the option series on that day. The number of option classes is 4,011.

correspond to the case in which the arbitrager pays no exchange fees. The results are telling in two respects. First, note that of the total exercises reported in Panel A, 107,265,671 (see Panel B) or 58.0% are associated with SSIA activity. In other words, nearly six-tenths of all put option exercises are "game playing," designed to capture the short open interest from existing shorts. Second, the game is being played almost exclusively by market makers (41.9%) and proprietary firms (56.1%).

The columns headed "Less than $1,000" and "Greater than or equal to $1,000" categorize the number of exercises by net interest income, where $1,000 corresponds to the exchange fee cap discussed earlier. The most noteworthy result is that SSIA activity grows as the net interest income grows, which should come as no surprise. Proprietary firms are the single largest market participant, followed by market makers. For non-SSIA trades, market makers tend to exercise most frequently.

In summary, long put option holders fail to exercise when they should and leave substantial sums of money on the table. Knowing this, market makers and proprietary firms step in front of the holders of existing short open interest to earn the net interest income proceeds, and, from the analysis in this section, are very successful in their efforts.

4 Summary and conclusions

In the U.S.A., exchange-traded stock options may be exercised at any time before contract expiration. Unlike call options, which may be exercised optimally only on the day before the ex-dividend day, put options may be optimally exercised on any day up to and including the expiration day. This study describes a put option early exercise decision rule and then reconciles the rule in light of actual early exercise behavior in U.S.A. stock options markets.

Using a sample of put options on stocks over the period January 1996 through September 2008, we find that over 3.96 million put option contracts remain unexercised on days when they should be and that the cost to long put option holders was nearly $1.9 billion. We find that the main beneficiaries of this forgone net interest income are professional traders (i.e., market makers and proprietary firms) who know when to exercise and constantly monitor their positions. By simultaneously buying (and exercising) and selling thousands of deep ITM put options, they systematically capture available short open interest and earn the interest income being forfeit by the longs. Thus, not only are the longs implicitly paying a premium for the ability to exercise early that they rarely use, but the potential gains to the original shorts are being appropriated by market makers and proprietary firms. Among other things, this raises fundamental concerns regarding contract design and market integrity. If many option buyers pay for the right to early exercise but either cannot or do not take advantage of it as a result of exercise costs, unawareness of appropriate decision rules, inability to continuously monitor open positions, or irrationality, would the integrity of the market not be better preserved with stock option contracts that are of European-style?

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Notes

[1] See, for example, Pool *et al.* (2008).

[2] Barraclough and Whaley (2012) provide a more detailed explanation for the early exercise rule and extend the rule for the early exercise of puts on dividend-paying stocks.

[3] Interest is earned on a daily basis, so typically the time increment is either one day or three days depending upon whether it falls within the trading week or over the weekend. Where necessary, we also adjust the time increment for holidays.

[4] Where the CRSP daily file has missing bid-ask quotes, NYSE TAQ data are used.

[5] Naturally, the expected dividend stream is adjusted for any stock splits and stock dividends paid during the option's life.

[6] The CRR binomial method is modified to account for cash dividend payments using the procedure described in Harvey and Whaley (1992).

[7] In our analysis the early exercise decision is based on a *model-based* rule whereby the put should be exercised immediately if *NII*, as defined by Equation (5), is greater than zero. A number of *market-based* early exercise rules exist, however, unlike the model-based rule, market-based rules offer no insight into the amount of economic benefit that will be realized if the put is exercised early. Since a key objective in this study is to estimate the magnitude of the cost of failure to exercise, we use a model-based approach in our analyses.

[8] When k is zero, Equation (6) reverts back to Equation (5).

[9] If the long put option holder is also long the underlying stock, an alternative to exercising or reversing the put is to sell a call option with the same exercise price. In this case, the option holder would incur a commission plus half the bid-ask spread for selling the call. In practice, however, most deep OTM calls have bid prices equal to zero, indicating that market makers are unwilling to buy.

[10] Developing accurate estimates of brokerage commission rates is virtually impossible since rates are negotiable and often embed services other than trade execution (e.g., an online trading platform with real-time data). Moreover, rates have undoubtedly fallen over the sample period due to increased competition in securities markets.

[11] The closing bid-ask spreads for options are from the OptionMetrics database, and the closing stock spreads are from the CRSP daily file. Where the CRSP daily file has missing stock price quotes, we use NYSE's TAQ quotes.

[12] This total, of course, includes open put positions that should have been exercised before day t.

[13] While the OCC only does exercise assignment at the end of the day, the long put option holder can, in effect, exercise his option earlier in the day by buying the stock. When she exercises at the end of the day, the exercise proceeds equal the difference between the exercise price and the closing stock price plus the difference between the closing stock price and purchase price of the stock earlier in the day. For this reason, we repeat the analysis using the lowest daily stock price rather than the closing price to determine whether early exercise is optimal and still find results are nearly the same.

[14] See NASDAQ OMX PHLX, Inc. Fee schedule, February 2, 2009.

[15] It is worth noting that SSIA may involve buying and exercising n puts at one exercise price and selling puts in the same option class but at a different exercise price. As long as both put series are deep in the money and should be exercised, the game may be played. Since we focus only on SSIA activity where the exercise prices are the same, we understate the amount of SSIA activity taking place.

[16] While the CBOE also caps fees at $1,000, ARCA, and AMEX cap fees at $750.

[17] Such trades cannot be accommodated in electronic markets like the ISE and must be executed on an exchange floor like CBOE or NASDAQ OMX PHLX.

[18] Proprietary firms tend to execute stock interest arbitrage in order sizes ending in the digit "1" so as to easily separate short stock interest positions from other positions in their books.

[19] With open interest remaining at 4,578 contracts, the proprietary firms were assigned delivery on 5,422 of the 10,002 put option contracts that they, themselves, exercised.

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