JOIM
www.joim.com

JOURNAL OF INVESTMENT MANAGEMENT, Vol. 10, No. 4, (2012), pp. 116–117

CASE STUDIES

"Case Studies" presents a case pertinent to contemporary issues and events in investment management. Insightful and provocative questions are posed at the end of each case to challenge the reader. Each case is an invitation to the critical thinking and pragmatic problem solving that are so fundamental to the practice of investment management.

Jack L. Treynor, Senior Editor

PROVIDING FOR RETIREMENT

What does it take to provide for 10, 15, 20, 25 or 30 years of retirement if the family is drawing down principal? Suppose a family has provided a certain amount for retirement. If their cost of living exceeds the expected return, how long will it be before they run out of money?

The answer is obviously different depending whether the money is invested in bonds or stocks. And of course the year-to-year risk is also different—twice as big if the family invests in a portfolio of diversified stocks.

Let

annual consumption $= C$
return on investment $= r$
value of portfolio $= i(t)$
value at retirement $= I$.

Then we have

$$\frac{di}{dt} = ri - C,$$

with solution

$$i = \lambda e^{rt} + \frac{C}{r},$$

where λ is an unknown constant.

At retirement, the value of the principal is I. We have

$$t = 0$$
$$I = \lambda + \frac{C}{r}$$
$$\lambda = I - \frac{C}{r}.$$

Substituting

$$i = \left(I - \frac{C}{r}\right) e^{rt} + \frac{C}{r}$$
$$= Ie^{rt} + \frac{C}{r}(1 - e^{rt}).$$

When the money runs out

$$i = 0,$$

and we have

$$Ie^{rt} = -\frac{C}{r}(1 - e^{rt})$$

$$\frac{I}{C} = -\frac{1}{r}\frac{1 - e^{rt}}{e^{rt}}$$

$$= +\frac{1}{r}(1 - e^{-rt})$$

where $\frac{I}{C}$ is the ratio of requisite initial endowment to annual consumption.

Investing in Bonds
$r = 0.016$

t (years)	rt	e^{-rt}	$(1 - e^{-rt})$	$\frac{1}{r}(1 - e^{-rt})$
10 years	0.16	0.8521	0.1479	9.2438
15 years	0.24	0.7866	0.2134	13.3375
20 years	0.32	0.7261	0.2739	17.1188
25 years	0.40	0.6703	0.3297	20.6063
30 years	0.48	0.6188	0.3812	23.825

Investing in Stocks
$r = 0.067$

t (years)	rt	e^{-rt}	$(1 - e^{-rt})$	$\frac{1}{r}(1 - e^{-rt})$
10 years	0.67	0.5117	0.4883	7.2881
15 years	1.005	0.3660	0.6340	9.4627
20 years	1.340	0.2618	0.7382	11.0179
25 years	1.675	0.1873	0.8127	12.1299
30 years	2.010	0.1340	0.8660	12.9254

How to Use the Tables

These tables are too dangerous to put into the hands of people without extensive investment experience. They should be used only by seasoned professional counselors whose clients have already experienced the risks.

Determine your client's life expectancy based on his age and health and add a margin of safety. Enter either the Stock Table or the Bond Table at the appropriate row. Estimate the annual cost to live in retirement and multiply by the number at the right end of the row. The product is how much the client needs to retire, if he realizes the average investment experience reported by Dimson, Marsh and Staunton.[1]

What happens to your client if he outlives his life expectancy?

Here is a checklist of some of those risks.

Have you conveyed the size of the investment risks, and how easily they can reduce your client's wealth? Point out that 16% of their yearly returns will be 20.2% worse than the average, if they invest in stocks. That 16% of their yearly returns will be 10% worse than their average if they invest in bonds. And the risks are greater if they do not diversify.

Do you help clients find ways to reduce their cost of living?

Some of your clients will develop cardiovascular troubles, adult onset diabetes or other chronic diseases. Does they know how much an expensive illness can cost? A full care retirement home?

Note

[1] Elroy Dimson, Paul Marsh and Mike Staunton (2001). *Triumph of the Optimists: 101 Years of Global Investment Returns*. Princeton University.