JOIM

CASE STUDIES

"Case Studies" presents a case pertinent to contemporary issues and events in investment management. Insightful and provocative questions are posed at the end of each case to challenge the reader. Each case is an invitation to the critical thinking and pragmatic problem solving that are so fundamental to the practice of investment management.

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INVESTING EARLY FOR RETIREMENT

To appreciate the importance of starting early, investors need to understand the role of time in preparing for their retirement.

Let

time be t,
wealth be $w(t)$,
the return on investment in the US stock market be r,
and the household's savings rate be s.

Consider the following (trivial) differential equation where $\frac{dw}{dt}$ is the household's combined gain each year from savings *and* the return on existing wealth:

$$\frac{dw}{dt} = rw + s,$$

with solution

$$w = \lambda e^n - \frac{s}{r},$$

where λ is an unknown constant. If wealth is zero at $t = 0$, we have

$$\lambda = \frac{s}{r},$$

$$w(t) = \frac{s}{r}(e^{rt} - 1).$$

In the following example,

$$s = \$1,000 \text{ a year}$$

$$r = 7\% \text{ a year},$$

the expected geometric after-tax return on the US stock market.[1]

Median of Retirement Wealth

Years	$\frac{s}{r}$	e^{rt}	$e^{rt} - 1$	$\frac{s}{r}(e^{rt} - 1)$
10	14,286	2.014	1.014	$14,482
20		4.055	3.055	$43,647
30		8.167	7.167	$102,388
40		16.445	15.445	$220,642
50		33.120	32.120	$458,878

Even a very modest level of savings can make a big difference at retirement, if only one starts early. A tempting rule of thumb is that the median retirement wealth doubles if one starts saving ten years sooner. Perhaps, more people would make the effort if they understood this.

Questions

1. How many bottles of beer would one have to give up to save $1,000 a year?
2. How many packs of cigarettes? How many visits to the local movie theater?
3. If we could give up beer, cigarettes, and movies, could we save $3,000 a year?
4. Would baby boomers be more willing to defer retirement if they knew about these results?
5. Will these results become more relevant as life expectancy increases?
6. If people start saving sooner, what will happen to the supply of jobs?
7. Are the machines that provide the assembly line jobs expensive? Or will workers be able to provide their own jobs? (Hint: A Peterbilt tractor and refrigerated trailer costs almost $100,000.)
8. If there aren't enough machines for the workers who need jobs, what do we tell the workers who are left out?
9. When we fail to provide enough machines, do we just increase the number of government jobs?
10. When we tax the households who have the self-discipline to start saving early, are we taxing their consumption or their savings?

Note

[1] See *Triumph of the Optimists: 101 Years of Investment Returns*, (Elroy Dimson, Paul Marsh, and Mike Staunton, published 2002).