
CASE STUDIES

“Case Studies” presents a case pertinent to contemporary issues and events in investment management. Insightful and provocative questions are posed at the end of each case to challenge the reader. Each case is an invitation to the critical thinking and pragmatic problem solving that are so fundamental to the practice of investment management.

Jack L. Treynor



HOUSEHOLD RISK

Every year end after computing his taxes, Joe Smith computed his household risk. In order to make the computation, he assumed that his various casualty risks were independent of each other, and that his investment risk was independent of his casualty risks. The latter included fire insurance on his house, various kinds of liability.

Presumably the insurance company had enough policy holders that it did not to worry about the variance—just the mean. But Joe needed to worry about the variance.

For his investment risk he used a number from Ibbotson and Brinson¹: a standard deviation on return on the US stock market since 1940 to 1985 of 17.1%. Joe had his savings of \$10,000 invested in a popular index fund. The variance on its annual return was

$$(0.171)(100,000)^2 = 17100^2 = 292.41 \times 10^6.$$

Regarding his casualty risks, Joe assumed

(1) his insurance companies could estimate the risks better than he could.

(2) Adjusted downward to allow for the agent’s sales commission, overhead and profit, the insurance company’s annual fee was the product of

- (a) amount at risk and
- (b) frequency of loss.

Joe recognized that the actuaries were contemplating a more complicated reality, with a range of amounts at risk and a corresponding range of frequencies, but he did not need the actuaries’ precision.

(3) The probability of a house fire, for example, is proportional to the time interval and independent of whether the fire, etc., had occurred in the past. The probability of two (or more) fires occurring simultaneously was zero.

(4) But if the distributions of Joe’s casualty risks are consequently Poisson, the variance of their frequencies equals their mean. (If the mean for 12 months is 12 times the mean for one month, the variance for 12 months will be 12 times the variance for one month. But if the months are independent events, then the variance for 12 months should be 12 times the variance for

one month. So it is not unintuitive that the Poisson should behave this way.)

- (5) If the various insured risks are indeed statistically independent, not only of each other, but also of Joe's investment risk, the variances add.

What the insurance company charged Joe probably depended on the product of frequency and the value at risk. Presumably they charged more for fire insurance on big houses than on small houses. But if the insurance company's premiums reflect the product of the frequency and the size of potential losses, the two factors affect the variance differently. If the distribution of the occurrences is Poisson, then the variance of the frequency equals the mean frequency. But the effect of the size of the exposure on the variance is the square of its effect on the fee. Indeed, the variance of each casualty risk depended on both the amount at risk and the frequency of losses in the following way:

Variance for individual risk

$$\begin{aligned} &= (\text{amount at risk})^2(\text{variance of frequency}) \\ &= (\text{amount at risk})^2(\text{mean of frequency}) \end{aligned}$$

if the distributions were Poisson.

If the insurance company's fee was based on

$$\begin{aligned} \text{fee} &= (\text{amount at risk}) \\ &\quad \times (\text{mean of frequency}) \end{aligned}$$

Then Joe could express the mean

$$\text{mean of frequency} = \frac{\text{fee}}{\text{amount at risk}}.$$

Substituting, Joe had for the variance of his risk

$$\begin{aligned} &(\text{amount at risk})^2 \left(\frac{\text{fee}}{\text{amount at risk}} \right) \\ &= (\text{amount at risk})(\text{fee}). \end{aligned}$$

Joe performed this simplified calculation for each of his casualty policies and then added the results

together to obtain the variance of his total casualty risk. For example, if it would cost \$1,523,500 to rebuild his house and his annual premium is \$1480, then the frequency is 0.00097 and the variance for house fires is

$$1,523,500(1480) = 2255 \times 10^6$$

Some investors will obviously have bigger casualty risks than other investors. But the range of investment risks will often be far greater, if only because some investors' portfolios are far larger, raising the following question: how big does the investor's portfolio have to be before it swamps his casualty risks?

Is the importance of the casualty risks sensitive to the size of Joe's investment portfolio? The following table shows the increase in Joe's overall household risk for a range of ratios of investment risk to casualty risk (everything measured in standard deviations).

Ratio of investment risk to casualty risk	Ratio of household risk to investment risk
7.05	1.01
3.12	1.05
2.18	1.10
1.50	1.20
1.20	1.30
1.02	1.40
0.89	1.50
0.80	1.60
0.73	1.70
0.67	1.80
0.62	1.90
0.58	2.00

Questions

1. Are standard deviations appropriate measures for the kind of risks Joe is insuring against?

2. If Joe's portfolio grows over his career, will his investment risk ultimately dominate his casualty risks?
3. Don't insurance companies have their own investment risks? Are they a potential problem for Joe? Won't there be a correlation between Joe's portfolio value and the insurance company's portfolio value?
4. If Joe had more than one car, wouldn't the assumption of independence break down? If he had a summer house?
5. Is it really legitimate to combine casualty risks and investment risks?
6. How sensitive is Joe's calculation to his assumptions about the dollar amounts at risk?
7. Are the Poisson assumptions reasonable for the casualty risks Joe is insuring against?
8. Are the risks of a regional casualty company really insurable? Hurricanes? Earthquakes? Forest fires?
9. Should his insurance agents make the calculation for Joe?
10. Won't Joe have to recompute every time the value of his portfolio changes? Isn't his investment risk more volatile than his casualty risk?
11. Should households care about their household risk?

Note

- ¹ Roger G. Ibbotson and Gary P. Brinson, *Investment Markets*, p. 76, McGraw Hill (1987).