
THE RISK THAT RISK WILL CHANGE

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Standard approaches to risk management focus on short-run risks, yet many positions are held for longer periods. Over such holding periods there is a risk that risks will change. In this note several easily implemented approaches to estimating the term structure of risk are proposed based on either statistical or economic criteria. It is argued that some portion of the financial crisis of 2007–2008 was due to the use of short-run risk measures to assess long-term risks.



1 Introduction

In reflecting on the financial crisis of 2007 and 2008, it is clear that risk management failed but it is unclear whether this is because the risks were not well measured or whether risks were ignored because decision makers had strong incentives to do so. A particularly interesting aspect of risk measurement is the time horizon for each measurement. A typical market risk measure would use value at risk (VaR) at a one or 10-day horizon, whereas most assets in most portfolios are held longer than this. Thus, it is difficult to assess the relevant risk of the portfolio based only on short-run risk measures. After all, the value of the portfolio will be dramatically affected if the risk itself changes.

An interpretation of the excessive risk taking of banks and other financial organizations can be built from this observation. In the low volatility regime

from 2003 through early 2007, leveraged equity positions were not particularly risky. Similarly, with low short-term interest rates, the cost of short-term financing was extremely low and the risk that they could not be rolled over in the short run was also very low. Consequently, any short-term risk measure would say that the leveraged portfolios held by banks and financial institutions were not very risky. However, in the long run, volatility was likely to rise, short-term rates were likely to rise and credit spreads were likely to rise. This is not just a product of hindsight, it was incorporated in the term structure of implied volatility from options prices and in the term structure of interest rates.

A similar point of view emerges from descriptions of how ratings were set for mortgage portfolios. Taking the current default rate and correlations as given, what is the probability that the super senior tranches would be impacted? Such a calculation reveals that they are as safe as a high-grade corporate

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bond. However, it again suffers from recognizing that these parameters are not given but can change.

If short-term risk is low but long-term risk is high, an investment manager may be tempted to shorten the holding period of his positions. This market timing solution leads immediately to the problem that when risk creeps up slightly, there will be a mass of sales to close positions driving prices down and risk up. This rush for the exit has frequently been examined; a recent interesting discussion is Pedersen (2009).

If decision makers were given each day a full term structure of risk as will be discussed below, it might still be difficult to choose optimal portfolios. Various methods to solve this problem have been proposed. See for example Merton (1973) and Engle (2009) via hedging and Colacito and Engle (2009) following a dynamic programming with discrete time framework. Considerably more work is needed to solve this complex problem.

In this note I will propose some measures of the term structure of risk that would provide a natural supplement to the short run measures so widely used. This approach builds on measures introduced by Guidolin and Timmermann (2006) and Engle (2009) and discussed in Colacito and Engle (2009). These measures essentially describe the term structure of risk or the VaR at a continuum of different horizons. These are computed for various models of the evolution of risk and are applied to the current financial data. In calculating long-term VaR, there is a role for economic analysis.

2 Statistical approaches

Measures of VaR are typically based on volatility or correlation models that are adjusted frequently to reflect changes in risk. The updating of these measures would not be necessary if the risks or volatilities were constant, hence the updating is

exactly why risk can change. If the updating process is sufficiently accurate, then it can be used to generate scenarios useful for assessing long-run risk.

For example, in a GARCH model or an exponentially smoothed model, the short-term forecast of volatility is given by an expression such as:

$$\begin{aligned} r_{t+1} &= \sqrt{h_{t+1}} \varepsilon_{t+1} \\ h_{t+1} &= \omega + (\alpha \varepsilon_t^2 + \beta) h_t \end{aligned} \quad (1)$$

In this equation, we can think of r as the daily return on an asset, h as the variance for today's return made yesterday, and ε as an unpredictable shock that has mean zero and variance one, which could be normal but is most likely fat tailed. The parameters (ω, α, β) are calibrated econometrically to the past experience of this data.

From this expression, a forecast of volatility one step in advance is directly available. For more steps ahead, it is simply necessary to replace the future unknown squared ε s with their expectation which is 1. However, if the future ε s are not exactly unity, then the two-step forecast will differ from the future one-step forecast in a way that captures the uncertainty. By simulating this process with random draws of ε s that might be assumed to have a specific distribution such as normal or student- t , a series of scenarios can be constructed. Even better approximations can be achieved by using a bootstrap, that is by drawing the ε s from the historical distribution of sample ε s. This incorporates the fat tails that can be expected and the skewness that is in the data.

If asymmetric volatility models are used, then the simulated data will develop tails that are especially long on the downside as pointed out in Engle (2004). Whenever a negative draw is made, volatility will increase and thus the return will be especially negative. Such asymmetries have long been observed in financial volatility and are natural consequences of investors who avoid volatility unless it comes with higher expected returns. A common

model is the TARCH model:

$$\begin{aligned} r_{t+1} &= \sqrt{h_{t+1}} \varepsilon_{t+1} \\ h_{t+1} &= \omega + (\alpha \varepsilon_t^2 + \gamma \varepsilon_t^2 I_{\varepsilon_t < 0} + \beta) h_t. \end{aligned} \quad (2)$$

A wide range of asymmetric models have been proposed and Engle (2009) compares their properties in this context.

3 Economic approaches

Statistical models fail to utilize additional information that may exist on the future path of volatilities or other economic variables such as business cycles or electoral cycles. In order to introduce this information into the term structure of risk, it is necessary to incorporate it into the forecast of the risk measure. A natural way to do this is based on Engle and Rangel's (2008) model of Spline GARCH. Closely related models are developed in Ghysels *et al.* (2009). In this case the volatility is composed of two components that are multiplied together.

Let the low frequency component of volatility be denoted by τ which is assumed to depend on global macroeconomic and other slow-moving variables. The high frequency component of variance is given by g_t so that the model becomes:

$$\begin{aligned} r_{t+1} &= \tau_{t+1} \sqrt{g_{t+1}} \varepsilon_{t+1} \\ g_{t+1} &= \omega + (\alpha \varepsilon_t^2 + \beta) g_t \\ \tau_{t+1} &= \exp(\theta' y_t). \end{aligned} \quad (3)$$

Here y is a vector of macroeconomic variables that are associated with future volatility. Generally, the persistence of these variables is sufficiently high that long-run forecasts of these variables can be used directly. Alternatively, extreme scenarios of these variables can be used. The parameters, θ , reflect the importance of these variables in explaining volatility.

4 Estimating long-term risk

In Engle (2009), many simulations are run using familiar volatility models specified at a daily level.

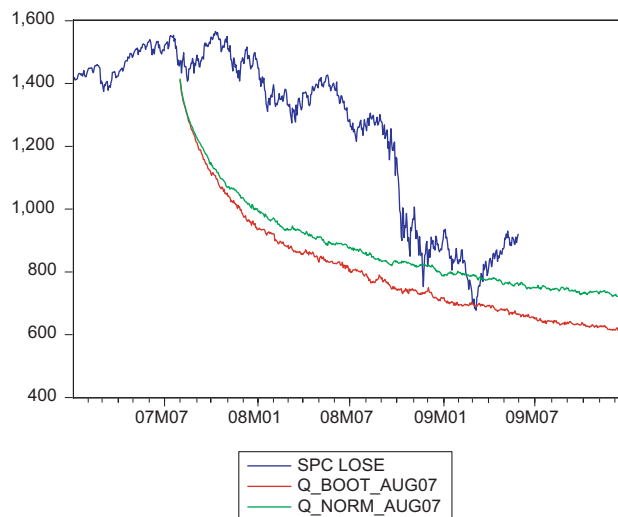


Figure 1 1% Quantile at various horizons.

For any horizon, there is a distribution of prices and correspondingly returns. The 1% quantile of these simulations is an interesting benchmark for long-term risk. In Figure 1, 10,000 simulations are run starting on the 1st of August, 2007 and continuing for 1000 days or four years. One set of simulations uses standard normal shocks and the other uses bootstrapped shocks. The model is given in (2) with parameters estimated from 1990 to 7/30/2007. The 1% quantile is computed for each horizon. The lowest line is the bootstrapped risk measure while the second line is based on normal innovations. As can easily be seen, the events just over one year and a half later were inside the bootstrapped 1% interval but not the normal interval.

The same quantile graph constructed on January 1, 2008 and September 1, 2008 is shown in Fig. 2. The quantiles forecast from the end of the sample in June are also shown. Clearly, the interval beginning on the first of September 2008 is quite severely crossed by future prices. Of course, the 1% line is drawn so that 1% of the scenarios at any particular horizon will touch the border so that the interval is only supposed to be accurate on average. A systematic analysis by Brownlees *et al.* (2009) shows that

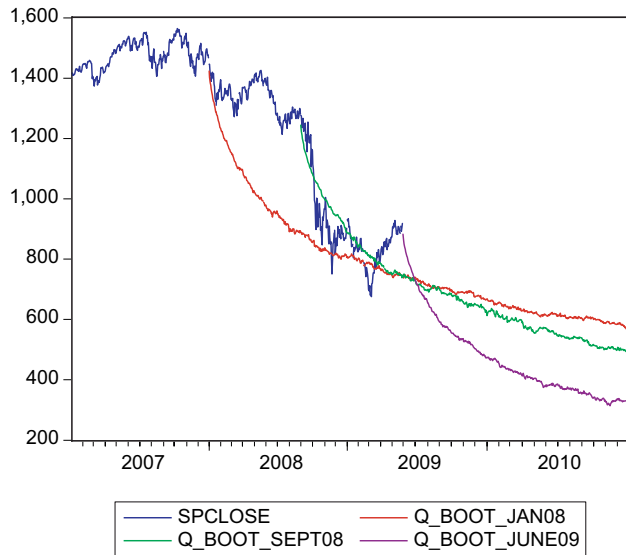


Figure 2 1% Quantiles from various time periods.

the one-step forecasts through this time period are as accurate as in other time periods. The 30-day forecasts are within 1% of the average performance but are less accurate than an average 30-day forecast. As shown in Engle (2009), similar results follow from various volatility models.

These quantiles can be compared with more standard measures of long-run risk. If the return scenarios are generated by independent normals or by independently bootstrapped samples from the historical distribution, then the quantiles are given in Figure 3. As can be seen, the normal and bootstrapped quantiles are very similar but are very far from the TARCH quantiles. After one year, the 1% quantile of the bootstrapped TARCH process is a decline of 61% while for the bootstrapped independent scenarios it is only 37%. Clearly, the long-run risk is much greater if the process allows changes in volatility than if it does not.

In order to incorporate additional information such as the term structure of volatility into the long-run risk measure, the spline garch method from Eq. (3) can be adopted. For example, suppose at the beginning of August 2007, a risk manager notices

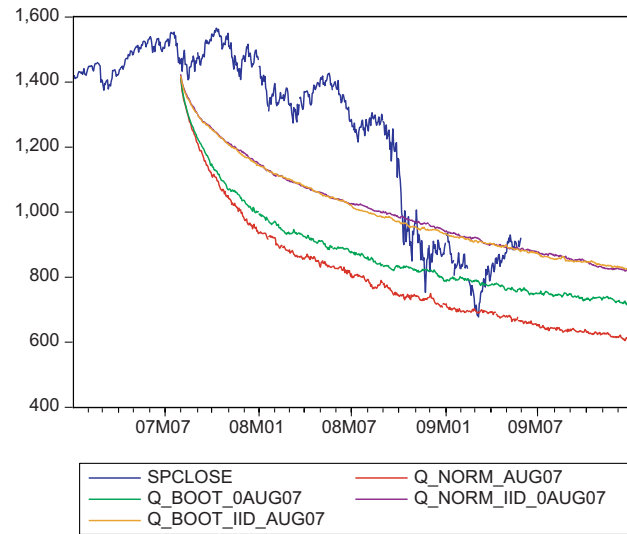


Figure 3 1% Quantiles for IID returns.

the upward slope of the term structure of implied volatility which is confirmed by similar forecasts from volatility models and macroeconomic analysis. Then he might construct a function τ that doubles in some number of years. For example, the same quantiles are calculated where the multiplier τ is given by:

$$\tau_t = \exp(\log(2)/504). \tag{4}$$

Thus, the volatility will increase by a factor of 2 every two years over what would be expected from simply the time series structure of the TARCH model. In the context of August 2007, this is a prediction that means volatility will rise from 15% to 30% in two years. It actually did this in two weeks according to the VIX and in the fall of 2008 reached 80%. Nevertheless such a scenario might be a reasonable *ex ante* forecast.

The result is given in Figure 4. In this figure the lowest line reflects the anticipated rise in volatility and correspondingly higher long-run risk. Instead of a 61% decline one year forward, the spline scenario finds the 1% quantile reflect a 77% decline over the following year which is more extreme than actually occurred.

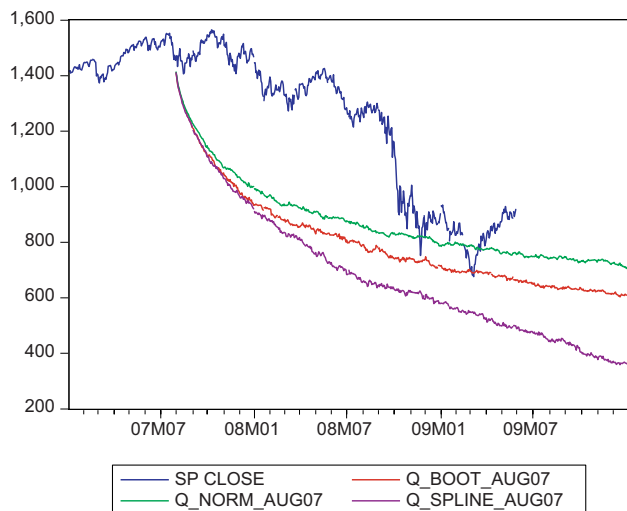


Figure 4 1% Quantile with Spline GARCH and rising volatility.

5 Implications and conclusions

Measures of long-term risk are very important for assessing the risk in many positions that are held over a period of months or even years. Standard risk measures are not designed for this task and this paper suggests some approaches to constructing measures with a continuum of risk horizons. It is clear that a major component of the risk one or more years in the future is the risk that the risk will change. Whenever volatility rises, so does the risk. Hence, the expected distribution of future volatilities is central to estimating long-term risks. It is hoped that these examples will encourage more research into measures of long-term risk.

References

- Brownlees, C. (2008). "On Variable Selection for Volatility Forecasting: The Role of Focused Selection Criteria" (with Giampiero M. Gallo) *Journal of Financial Econometrics* 6(4), 513–539.
- Colacito, R. and Engle, R. F. (2008). "Term Structure of Risk, the Role of Known and Unknown Risks and Non-stationary Distributions." In: Diebold, F. X., Doherty, N., Herring, R. *The Known, the Unknown and the Unknowable in Financial Risk Management*, forthcoming, Princeton University Press.
- Ghysels, E., Engle, R. F. and Sohn, B. (2009). *Stock Market Volatility and Macroeconomic Fundamentals*. AFA 2008 New Orleans Meetings Paper.
- Guidolin, M. and Timmermann, A. (2006). "Term Structure of Risk Under Alternative Econometric Specifications." *Journal of Econometrics* 131, 285–308.
- Engle, R. (2004). "Risk and Volatility: Econometric Models and Financial Practice." *American Economic Review, American Economic Association* 94(3), 405–420.
- Engle, R. (2009). "Long Run Skewness and Systemic Risk." *Presidential Address for SoFiE in Geneva*, manuscript in preparation.
- Engle, R. F. and Rangel, J. G. (2008). "The Spline-GARCH Model for Low-Frequency Volatility and Its Global Macroeconomic Causes." *Review of Financial Studies* 21(3), 1187–1222, 36p.
- Merton, R. C. (1973). "An Intertemporal Capital Asset Pricing Model." *Econometrica* 41(5), 867–887.
- Pedersen, L. (2009). "When Everyone Runs for the Exit." *International Journal of Central Banking*, forthcoming.

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