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GREAT MOMENTS IN FINANCIAL ECONOMICS: II. MODIGLIANI–MILLER THEOREM

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Franco Modigliani and Merton Miller are almost universally credited with the theorem that bears their name. In fact, the theorem was stated and proven 20 years earlier by John Burr Williams, to which he gave the name: "the Law of the Conservation of Investment Value." However, Modigliani–Miller deserve credit for clearly laying out a formal arbitrage proof and popularizing the subsequent use of arbitrage arguments in financial economics. Even after their work (1958) and subsequent simplified proof (1969), there were still issues that needed to be clarified and led finally to a more modern proof of the theorem based on state-prices.

1 John Burr Williams

For our third *Great Moment* in financial economics, John Burr William's (1899–1989) book *The Theory of Investment Value* (1938) contains what is probably the first exposition of the Modigliani–Miller proposition on the irrelevancy of capital structure, which Williams poetically calls "the Law of the Conservation of Investment Value." Williams writes with borrowed nineteenth century elegance:

> "If the investment value of an enterprise as a whole is by definition the present worth of all its future distributions to security holders, whether on interest or dividend account, then this value in no wise depends on what the company's capitalization is. Clearly, if a single individual or a single institutional investor owned all of the bonds, stocks and warrants issued by the corporation, it would not matter to this investor what the company's capitalization was (except for details concerning the income tax). Any earnings collected as interest could not be collected as dividends. To such an individual it would be perfectly obvious that total interest- and dividend-paying power was in no wise dependent on the kind of securities issued to the company's owner. Furthermore no *change* in the investment value of the enterprise as a whole would result from a change in its capitalization. Bonds could be retired with stock issues, or two classes of junior securities could be combined into one, without changing the investment value of the company as a whole. Such constancy of investment value is analogous to the indestructibility of matter or energy: it leads us to speak of the Law of the Conservation of Investment Value, just as physicists speak of the Law of the Conservation of Matter, or the Law of the Conservation of Energy." (pp. 72–73)

Although this exposition does not use the magic word "arbitrage," in his next paragraph on the subject Williams says that his Law will not hold exactly in practice (he had not yet absorbed later notions

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of informationally efficient markets). But, he says, that simply leaves open "opportunities for profit by promoters and investment bankers." From his analysis of United Corporation, it is clear that he sees "promoters" profiting by taking advantage of naïve techniques used by investors to value the separate securities in the recapitalization, who, had they but understood the Law of the Conservation of Investment Value, would have defeated the promoters' efforts.

2 Franco Modigliani and Merton Miller

Twenty years later in 1958 Franco Modigliani (1918–) and Merton Howard Miller (1923–2000), in "The Cost of Capital, Corporation Finance and the Theory of Investment," attempt to extend to uncertainty Irving Fisher's (1930) idea that the financing and production decisions of a firm can be separated. It is also the first formal treatment of William's (1938) "Law of the Conservation of Investment Value," showing that in a perfect market, the value of a firm is independent of its capital structure (M&M's "Proposition I"). Although this result was clearly anticipated by Williams, Modigliani–Miller (M&M) argue that Williams does not really prove his Law because he has not made it clear how an arbitrage opportunity would arise if his Law were to fail. To quote M&M's complete comment with regard to Williams:[1]

"A number of writers have stated close equivalents of our Proposition I although by appealing to intuition rather than by attempting a proof and only to insist immediately that the results are not applicable to the actual capital markets.[13]"

"(footnote)[13]See, for example, J.B. Williams [21, esp. pp. 72–73]; David Durand [3]; and W.A. Morton [14]. None of these writers describe in any detail the mechanism which is supposed to keep the average cost of capital constant under changes in capital structure. They seem, however, to be visualizing the equilibrating mechanism in terms of switches by investors between stocks and bonds as the yields get out of line with their 'riskiness'. This is an argument quite different from the pure arbitrage mechanism underlying our proof, and the difference is crucial." (p. 271)

While this criticism, it seems to me, is questionable with respect to Williams, it does seem on the mark with respect to Walter A. Morton who writes in "The Structure of the Capital Market and the Price of Money" (1954):

"The essential difference between the obligations of the same company lies in the priority of claim to earnings and assets. If only one security is issued, it bears all the risk whether it be called a bond, preferred stock, or common stock, and would have the same value provided that the security could share in all the earnings. (I ignore at this point the difference that might be made by the fact that interest payments are tax deductible as a cost before computing federal income taxes whereas preferred and common stock dividends are not.) Similarly, if one individual owned all of the various types of securities issued, his risk would be the same. Legal differences in the event of insolvency or reorganization and tax policy will modify this result. If all the securities were sold in 'packages' of bonds, preferred and common, the risk to each owner would be the same as if it were all common stock. It follows accordingly that the over-all cost of money would be unaffected by capital structure if individuals could not differentiate risks." (p. 442)

Morton then goes on to argue that once investors can specialize their portfolios in one type of the firm's securities, so that one investor only owns its bonds say, and another its stock, then clientele effects (some value safety, and others value the higher return that inures to higher risk) can cause the sum of the values the investors place on their positions to exceed the value had they not been able to specialize.

To show this would not happen in a "perfect market," in their 1958 article with lengthy argument, M&M prove William's Law using a number of assumptions that their own later work and the work of several others show to be unnecessary. For example, M&M assume the debt of the firm is riskless and that, to use their terminology, two firms must

exist that are in the same "risk class." This means that, as in the proof below, the random variables X_U and X_L are equal in all states of the world.

In 1969, M&M strip their proof to its essentials and come full circle, it seems to me, to William's original insight. They assume there are two otherwise identical firms (i.e. with the same total future cash flows from assets), one unlevered and one not. They then show that if the sum of the current values of the stock and bonds of the levered firm were not equal to the current value of the stock of the unlevered firm, there would be an arbitrage opportunity ("Proposition I").

Here is their proof (sometimes called the "homemade leverage argument"):

$X \equiv X_U = X_L$ (future operating income of unlevered and levered firms (same "risk class" assumption))

$V_U = S_U$ (unlevered firm relation between current total firm value and its current equity value)

$V_L = D_L + S_L$ (levered firm relation between current total firm value and the current values of its riskless debt and equity)

r (return on riskless bonds)

Compare the costs and payoffs of the following two portfolios:

	Current cost	Future payoff
Buy α% shares of U	$\alpha S_U = \alpha V_U$	αX
Buy α% bonds of L	αD_L	$\alpha r D_L$
Buy α% shares of L	αS_L	$\alpha(X - rD_L)$
Total	$\alpha D_L + \alpha S_L = \alpha V_L$	$\alpha r D_L + \alpha(X - rD_L) = \alpha X$

The two portfolios have the same payoff (αX). Therefore, if there are no riskless arbitrage opportunities, their current costs must be the same so that $\alpha V_U = \alpha V_L$, implying that $V_U = V_L$.

Although Williams did not spell out this arbitrage mechanism in detail, his one paragraph verbal proof, particularly coupled with his subsequent use of the Law in his book, seems implicitly to have this in mind.

M&M's "Proposition II" says that the expected return on equity $E(r_S)$ equals the expected return on the portfolio of debt and equity ($E(r_V)$) plus the difference between $E(r_V)$ and the return on debt r times the debt-to-equity ratio (D/S): $E(r_S) = E(r_V) + (E(r_V) - r)(D/S)$. Today, this seems obvious since this is equivalent to saying that the expected return of a portfolio with two securities equals a weighted average of the expected returns of the securities in the portfolio, where the weights are the value-weighted proportions: $E(r_V) = (D/V)r + (S/V)E(r_S)$ where $V = D + S$. $E(r_V)$ is M&M's "weighted average cost of capital," which, of course, will be invariant to capital structure since by assumption the numerator of r_V is independent of capital structure and by proof ("Proposition I") its denominator is as well.

3 After Modigliani and Miller

In both their 1958 and their streamlined 1969 proofs, M&M require riskless debt. For example, in order to capture the limited liability of stock, $\alpha(X - rD_L) \geq 0$ so that $X \geq rD_L$, which implies that there must always be sufficient operating income to meet debt payments. On the other hand, it is equally immediately clear from Williams' "proof" that risky debt (provided there are no bankruptcy costs) does not alter his Law. But what both M&M initially and Williams failed to notice, and only became clear later in Joseph E. Stiglitz (1969, 1974), is that if risky debt is created as the capital structure is shifted more toward debt, in an incomplete market, a fundamentally new security can be created or old security destroyed (one that cannot be replicated by a portfolio of pre-existing

securities in the economy) and this may alter state prices that will, in turn, change the discount rates used to determine the present value of the sum of the cash flows to debt and equity.[2] In practice, this influence on discount rates, if any, will typically be negligible, but it is a refinement to the proof that could in rare circumstances prove significant. It may very well provide the motivation for some of the highly innovative recapitalizations we see in practice. Charitably interpreted, this seems similar to Morton's argument.

A second, more modern way to look at the Law is to see it as a special case of present value additivity: the present value of the sum of two potentially uncertain income streams equals the sum of their separate present values. Reading Williams' "proof" one can hardly fail to notice that, as applied to a firm's capitalization, this is exactly what he is saying.

This proof goes like this. Say a firm has two financial claims (possibly stock and bonds) against its assets, A and B with future random payoffs A_s and B_s, across the exhaustive set of states $s = 1, 2, \ldots, n$ and the contractual arrangements of these claims are such that in every state of the world s, the sum of the payoffs to both of these claims exactly exhaust the operating income X_s of the firm in that state. That is, $X_s = A_s + B_s$ for all states s. Let π_s be the economy-wide state price for state s. Then, the value of the firm:

$$V = \Sigma_s \pi_s A_s + \Sigma_s \pi_s B_s = \Sigma_s \pi_s (A_s + B_s) = \Sigma_s \pi_s X_s$$

Now suppose the firm changes its capital structure in such a way that leaves its operating income unchanged state by state. In other words (assuming for simplicity that a third claim is not thereby created), in state s, Δ_s is now added to the payoff of asset A and correspondingly, to leave X_s unchanged, Δ_s is subtracted from the payoff of asset B. Also assume, as Stiglitz suggests, that the change in capital structure does not create a new desired pattern of payoffs across states (or destroy an old desired pattern across states). In that case, since agents in the economy continue to face the same opportunity sets for desired investments, the state prices π_s will remain unchanged. Then the new value of the firm will be:

$$\Sigma_s \pi_s (A_s + \Delta_s) + \Sigma_s \pi_s (B_s - \Delta_s) = \Sigma_s \pi_s (A_s + B_s) = \Sigma_s \pi_s X_s = V$$

clearly unchanged from its previous value. Note that since the payoffs of the two claims are quite arbitrary, one could be risky debt and the other (limited liability) stock.

A similar proof, under the assumption of "complete markets" (the economy has as many different securities as states of the world), so that the state prices π_s exist and are unique, first appeared in Hirshleifer (1966). As Hirshleifer points out, "the single-price law of markets" implies that a dollar received in the same state but from the payoffs of different securities must have the same state price to convert it into its present value (an assumption embodied in the above proof). Later, Rubinstein (1976) argues and Ross (1977) clearly proves that, even in the absence of complete markets, although state prices will not generally be unique, state prices will nonetheless exist if and only if there are no riskless arbitrage opportunities. This latter result is sometimes referred to as "the first fundamental theorem of financial economics." So the existence and application of the same state price to a dollar payoff received from different securities in the same state merely requires that there be no riskless arbitrage opportunities.

Another confusion in the literature, and I think in the original M&M paper itself, is the difference between the irrelevancy of capital structure for (1) firm value and for (2) the stock price (per share). M&M assert at the outset that they want to prove the latter, but only end up proving the former. Clearly, from the standpoint of stockholder-centered corporate financial theory, it is the latter proposition that is paramount. It is easy to see how even if (1) is true, (2) need not be. Consider a firm with one risky debt issue, and then the firm issues new debt senior to the original debt, using the proceeds to repurchase stock. This will increase the risk of the original (now junior) debt holders and transfer value to the stockholders. Fortunately, the jump from (1) to (2) is, with academic hindsight, easy to see: given (1), as long as with each recapitalization,

the original debt holders can intelligently renegotiate their debt contracts, then (2) will hold as well.

To see this, start with an unlevered firm (with n outstanding shares):

$$V_U = nS_U$$

Now suppose the firm alters is capital structure by buying back m shares and replacing them dollar for dollar with debt:

$$V_L = (n - m)S_L + D_L = (n - m)S_L + mS_L = nS_L$$

where, in general, the stock of the levered firm may sell at a different price per share S_L.

However, since $V_L = nS_L$, $V_U = nS_U$, and $V_L = V_U$, then $S_L = S_U$.

It should be kept in mind that the connection of capital structure irrelevancy to present value additivity (which itself must hold if there are no riskless arbitrage opportunities in perfect markets) is made with the benefit of hindsight. Academics, perhaps even as late as 1978, remained unclear about the exact assumptions needed for the Law to hold, and there was considerable confusion, particularly in the earlier literature. The field figuratively "cut-its-teeth" on the M&M's Proposition I in a rite of passage from childhood to adulthood.

So what finally are the assumptions needed for William's Law or the M&M Proposition I to hold?

(1) there are no riskless arbitrage opportunities (i.e. "all equal-sized bites of the pie have the same taste");
(2) operating income (from assets) is not affected by capital structure (i.e. "the total pie is fixed");
(3) the proportion of operating income that is jointly allocated to stock and bonds is not affected by the firm's capital structure (i.e. roughly, "only stockholders and bondholders eat the pie"); and
(4) the present value function (the economy-wide state prices) is not affected by capital structure (i.e. "the taste per bite of the pie is fixed").

Assumption (1) insures the existence, but not necessarily the uniqueness, of state prices. Assumption (2) rules out (a) bankruptcy costs, (b) differential transactions costs in issuing or trading stock and bonds, (c) managerial incentives to alter operating income that are changed by capital structure such as occurs with employee stock options or capital structure effects on managerial salaries and perks, (d) stockholder incentives to accept high risk negative net present value projects that shift value from bondholders to stockholders, and (e) conveyance of information about the operating income of the firm to the market by signaling via capital structure. Assumption (3) rules out differential taxes for income from stock and bonds. And assumption (4) disallows the possibility of creating or destroying desired patterns of returns not otherwise existing in the market by changing capital structure.

The second related proposition that capital structure does not affect the stock price requires an additional assumption:

(5) there are no pure transfers between bondholders and stockholders, and between new and old shareholders.

This assumption rules out:

(a) pure asset substitution, that is, *ex-post* risk-changing projects that can shift value between bondholders and stockholders even though they leave the total market value of the firm unchanged;
(b) use of contractual arrangements between bondholders and stockholders that permit *ex-post* transfers between them such as in the junior debt example above; and

(c) violations of "strong-form market efficiency" that permit stockholders or managers to use inside information to issue bonds or stock in response to the failure of the market to reflect that information in their relative prices (e.g. issue stock when it is over-priced and bonds when the stock is under-priced).

Indeed, it has become commonplace to view the Modigliani–Miller Theorem not as a realistic proof that capital structure is "irrelevant," but rather as a way of obtaining the list of reasons that make it relevant. Since the publication of the original M&M paper in 1958, each of these reasons has given rise to its own virtual cottage industry of academic research. Tracing these developments to the present will have to await another opportunity.

4 Conclusion

I have argued that the Great Moment in the history of the Modigliani–Miller Theorem ironically belongs to the publication of John Burr Williams' 1938 book, *The Theory of Investment Value*. Not only did M&M not "invent" the "Modigliani–Miller Theorem," M&M did not even invent arbitrage reasoning or proof, as it is sometime claimed. As if it were needed, I mention just a few examples. According to Swan (2000), evidence from cuneiform tablets inscribed in ancient Mesopotamia indicates that forward transactions have existed at least since 1750 BC. A forward transaction (agree today to make an exchange tomorrow) is such an obvious idea, often motivated by necessity, that this is hardly surprising. What is more, by the sixteenth century, secondary markets in forward contracts had developed across much of Europe with forwards on stock trading on the Amsterdam Exchange. Commodity traders for perhaps many centuries and academics from at least early in the twentieth century clearly understood the role of arbitrage in determining the relation between spot and forward prices, as well as triangular currency arbitrage.[3] The Dutch market for calls and puts (or "opsies" as they were then called), colorfully described in 1688 by Joseph de la Vega, surely included put–call parity arbitrage trading in practice. As noted in the previous article in this series, Irving Fisher, in his 1907 book, used an arbitrage argument to justify why the present value of cash flows from a capital project must be the same as the present value of the cash flows from a portfolio of securities constructed to match the project. In another context, Hotelling (1931) explains that the price of an exhaustible resource should grow at the rate of interest to prevent profitable shifting of extraction between two periods.

With this in mind, we can now see that M&M's real and enduring contribution was to point others in the direction of arbitrage reasoning, as the most fundamental tool to derive results in financial economics. Both Modigliani in 1985 and Miller in 1990 won Nobel Prizes in Economic Science, Modigliani "for his pioneering analyses of saving and financial markets" and Miller primarily for his role in their 1958 paper.

Notes

[1] As far as I can discern, this is the one and only time in any of Modigliani's or Miller's publications on capital structure that they comment on or reference William's earlier work.

[2] It is thus possible in M&M's proof that with changes in capital structure, although $V_U = V_L$ continues hold, both could be higher or lower than they were before the change. However, in many cases, the securities created by changing capital structure are not new since the patterns of their returns across states can be replicated by forming a portfolio of pre-existing securities. In other cases, although firms may be able to creatively innovate new securities, investors do not desire these patterns; so these actions should have no effect on their total value. For example, in the mean–variance capital asset pricing model, since all investors divide their investible wealth between cash and the "market portfolio," investors do not desire other patterns of

returns across states. Or consider an economy, as outlined by Hakansson (1978), with a full set of state securities on the market portfolio macro-states and homogeneous beliefs about the residual return of the securities of individual firms conditional on the macro-state. In that case, all investors are perfectly happy restricting themselves to a portfolio containing only macro-state securities; so capital structure changes that create new patterns of residual returns for firms are of no interest to investors. Finally, consider the economy described by Ross (1976) in which the returns of all securities are approximately spanned by a small set of priced factors, and where the cross-sectionally uncorrelated residual return of any security is "small" relative to the size of the market and hence can be approximately diversified away. As long as changes in capital structure do not create new priced factors or create "large" residual return, to a good approximation, capital structure changes will not affect value.

[3] This elementary form of arbitrage takes advantage of the mispricing of three currencies relative to each other. For example, the dollar/pound ($/£), pound/yen (£/¥) and dollar/yen ($/¥) exchange rates have to satisfy ($/£) × (£/¥) = ($/¥) for there to be no riskless arbitrage opportunities.

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