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ESTIMATING DEFAULT PROBABILITIES IMPLICIT IN EQUITY PRICES

Tibor Janosi,[a] *Robert Jarrow,*[b,*] *and Yildiray Yildirim*[c]

This paper uses a reduced-form credit risk model to estimate default probabilities implicit in equity prices. For a cross-section of firms, a time-series regression of monthly equity returns is estimated. We show that it is feasible to infer the firm's probability of default implicit in equity returns. However, the existence of price bubbles and the difficulty in modeling equity price risk premium confound the estimation of these default probabilities, generating potentially biased estimates with large standard errors. Comparing these default intensities with those obtained from historical data or implicitly from debt prices confirms this result.

1 Introduction

Given the recent exponential growth in the credit derivatives market (see *Risk Magazine*, 2000), credit risk modeling and estimation has become a topic of current interest. The theoretical literature is quite extensive (see Bielecki and Rutkowski, 2000, for a review). The empirical literature estimating reduced-form credit risk models has concentrated on using debt prices (see Duffee, 1999; Duffie and Singleton, 1997; Duffie *et al.*, 2000; Janosi *et al.*, 2002; Madan and Unal, 1998), credit derivative prices (see Hull and White, 2000, 2001), or bankruptcy histories (see Altman, 1968; Chava and Jarrow, 2002; Shumway, 2001; Zmijewski, 1984). Equity prices have only been used to estimate default parameters for structural models (see Delianedis and Geske, 1998). The purpose of this paper is to use equity prices in conjunction with a reduced-form credit risk modeling approach to estimate default probabilities. The approach utilized is a slight generalization of the model contained in Jarrow (2001).

The data used for this investigation are equity prices from CRSP and debt prices from the University of Houston's Fixed Income Database over the time period May 1991–March 1997. The observation interval is 1 month. Debt prices consist of bids taken from Lehman Brothers trading sheets on the last calendar day in each month, see Warga (1999) for additional details.

[a]Computer Science Department, Cornell University, Ithaca, NY, USA
[b]Johnson Graduate School of Management, Cornell University, Ithaca, NY, and Kamakura Corporation, USA
[c]School of Management, Syracuse University, Syracuse, NY, USA
*Corresponding author. Johnson Graduate School of Management, Cornell University, Ithaca, NY 14853, USA. Tel.: +1 607 255 4729; e-mail: raj15@cornell.edu

Fifteen different firms are included in this study, where the firms are chosen to stratify various industry groupings: financial, food and beverages, petroleum, airlines, utilities, department stores, and technology. The same 15 firms as in Janosi *et al.* (2002) are included so that a comparison of the different estimation procedures can be performed.

Eight different models for equity returns are investigated herein, the simplest models containing no default. Using a rolling estimation procedure, for each month during this observation period, the equity model's parameters (including the bankruptcy parameters) are estimated using a time-series regression on monthly equity returns. In this procedure, only information available to the market at the time that the equations are estimated is utilized.

First, the analysis supports the feasibility of estimating default probabilities implicit in equity returns. In a relative comparison of the eight models, in-sample root mean squared error goodness-of-fit tests and out-of-sample generalized cross-validation statistics support the necessity of including default parameters into the equity return model. The best performing default intensity depends on the spot rate of interest but not on an equity market index, confirming similar results that Janosi *et al.* (2002) obtained when using debt prices.

Second, we find that equity returns contain a bubble component not captured by the Fama and French (1993, 1996) four-factor model for equity's risk premium. This bubble component, proxied by the firm's P/E ratio, is significant for many of the firms in our sample.

Third, due to the possible existence of equity price bubbles and the difficulty in modeling equity risk premium, the default intensity estimates obtained appear to confound these quantities. Indeed, a comparison of the default intensity estimates obtained herein with those obtained using either historical data or implicitly from debt prices indicates that the equity-based default intensities are significantly larger. By extrapolation, this possible model misspecification also casts doubt on the reliability of the equity-based default probability estimates obtained using structural models as in Delianedis and Geske (1998), confirming the previous conclusions of Jarrow and van Deventer (1998, 1999) and Jarrow *et al.* (2002) in this regard. This is also consistent with the inability of structural models, using equity price information, to explain credit spreads in corporate debt, see Collin-Dufresne *et al.* (2001), Eom *et al.* (2002) and Huang and Huang (2002).

The previous literature estimating reduced-form credit risk models using debt prices include Duffee (1999), Duffie and Singleton (1997), Duffie *et al.* (2000), Janosi *et al.* (2002), Madan and Unal (1998). Duffie and Singleton (1997) estimate swap spreads, Madan and Unal (1998) estimate yields on thrift institution certificates of deposit, and Duffie *et al.* (2000) estimate credit and liquidity spreads for Russian debt. Both Duffee (1999) and Janosi *et al.* (2002) estimate default intensities using US corporate debt. As mentioned earlier, we will compare our estimated default intensities with those from Janosi *et al.* (2002). Bankruptcy prediction models using historical bankruptcy data include Altman (1968), Chava and Jarrow (2002), Shumway (2001) and Zmijewski (1984), among others. A structural model for estimating default intensities is Delianedis and Geske (1998).

An outline of this paper is as follows. Section 2 presents the model structure. Section 3 provides a description of the data and Section 4 estimates the state variable process parameters. The equity model parameter estimation is discussed in Sections 5–10. Section 11 compares the default parameter intensities estimated using the equity model to those obtained using debt prices by Janosi *et al.* (2002). A conclusion is provided in Section 12.

2 The model structure

This section introduces the notation and provides a generalization of the reduced-form credit risk model for equity returns contained in Jarrow (2001). Default-free zero-coupon bonds of all maturities and a firm's common stock are traded. Markets are assumed to be frictionless with no arbitrage opportunities, but equity prices may contain bubbles.

Let $p(t, T)$ represent the time t price of a default-free dollar paid at time T where $0 \leq t \leq T$. The instantaneous forward rate at time t for date T is defined by $f(t, T) = -\partial \log p(t, T)/\partial T$, with the spot rate of interest $r(t) = f(t, t)$.

Consider a firm issuing equity. This firm may default. Let τ be the random variable representing the first time of default and let

$$N(t) = \begin{cases} 1 & \text{if } \tau \leq t \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

denote its point process. We assume that this point process has an intensity, denoted by $\lambda(t)$. $\lambda(t)\Delta$ gives the approximate probability of the firm's default over the time interval $[t, t + \Delta]$.[1]

Equity pays dividends and has a liquidating payoff at time T_L. The time t value of all of these payments equals the value of the equity (per share), denoted by $\xi(t)$. These promised dividends and liquidating payoff are made, unless the firm defaults. If default occurs, the equity holders lose everything.[2]

We need to develop some notation for these promised payments to equity. The regular dividends D_t are paid at times $t = 1, 2, \ldots, T_D$. We assume that these dividends are *deterministic* quantities, placed in an escrow account and paid for sure.[3] The requirement that these dividends are deterministic implicitly determines the date T_D. For many equities T_D will be a month or less. The liquidating dividend $L(T_L)$ is paid at time T_L unless default occurs prior to this date. The liquidating dividend consists of the time T_L (future) value of all unannounced and random dividends paid over (T_D, T_L), plus the remaining resale value of the firm at time T_L.

Let $S(t)$ represent the time t present value of the liquidating dividend, conditional upon no default prior to time t. There is some evidence, for example, the recent price growth of internet stocks,[4] that stock prices contain a "bubble" or "monetary value" component, see Jarrow and Madan (2000). For simplicity, we model the bubble component as a random process that is proportional to the present value of the liquidating dividend:

$$S(t)\left(e^{\int_0^t \mu_\theta(u)\,du} - 1\right) \tag{4}$$

where $\mu_\theta(u) \geq 0$ is the continuous return in the stock price due to the bubble component.

Given this set-up, it is easy to see[5] that the per share equity value at time t is given by

$$\xi(t) = \begin{cases} S(t)e^{\int_0^t \mu_\theta(u)\,du} + \sum_{j \geq t}^{T_D} D_j p(t, j) \\ \quad \text{if } t < \tau \\ 0 \quad \text{if } t = \tau. \end{cases} \tag{5}$$

The share price consists of the present value of the liquidating dividend ($S(t)$) compounded by the bubble ($\mu_\theta(t)$), and the (announced) deterministic dividends (D_j for $j = t, \ldots, T_D$).

To obtain an empirical formulation of the above model, more structure needs to be imposed on the stochastic nature of the economy. Exactly following Jarrow (2001),[6] we consider an economy that is Markov in three state variables: the spot rate of interest, the cumulative excess return on an equity market index, and the liquidating dividend process. For the spot rate of interest, we use a single-factor model with deterministic volatilities, sometimes called the extended Vasicek model. This model has two parameters: a mean reversion parameter (a) and the spot rate's volatility (σ_r). The second state variable $Z(t)$ is the cumulative excess return on an equity market index. The equity market index follows a geometric Brownian motion with volatility (σ_m). The correlation coefficient between the return on the market index and changes in the spot rate of interest is (φ_{rm}). The third state variable is the liquidation value of the firm's equity, denoted by $L(t)$. This liquidation value is assumed to follow a geometric Brownian motion with volatility (σ_L) and with φ_{mL} and φ_{rL} representing the correlation of the liquidation value with the market index and changes in the spot rate of interest, respectively.

For analytic tractability, the default intensity process is assumed to be linear in the spot rate of interest and the cumulative excess return on the equity market index, that is,

$$\lambda(t) = \lambda_0 + \lambda_1 r(t) + \lambda_2 Z(t) \quad \text{where } \lambda_0, \lambda_1, \lambda_2 \text{ are constants} \tag{6}$$

Under this structure, it is shown in Jarrow (2001) that the present value of the liquidating dividend can be rewritten as

$$\begin{aligned} S(t) &= \frac{L(t)}{p(t, T_L)} e^{-\lambda_1 \sigma_1^2(t,T_L) - \lambda_1 \sigma_L \varphi_{rL} \int_t^{T_L} b(u,T_L^*)\,du} \\ &\quad \times e^{-\lambda_2 \varphi_{rm} \eta(t,T_L) - \lambda_2 \sigma_L \varphi_{mL} (T_L - t)^2/2} \nu(t, T_L) \end{aligned} \tag{7}$$

where

$$\begin{aligned} \nu(t, T) &= p(t, T) e^{-\lambda_0 (T-t) - \lambda_1 \mu_1(t,T)} \\ &\quad \times e^{+(2\lambda_1 + \lambda_1^2)\sigma_1^2(t,T)/2 - \lambda_2 Z(t)(T-t)} \\ &\quad \times e^{+(1+\lambda_1)\lambda_2 \varphi_{rm} \eta(t,T) + [T-t]^3 \lambda_2^2/6} \end{aligned}$$

$$\mu_1(t, T) = \int_t^T f(t, u)\,du + \int_t^T b(u, T)^2\,du/2,$$

$$\sigma_1^2(t, T) = \int_t^T b(u, T)^2\,du$$

$$b(u, t) = \sigma_r (1 - e^{-a(t-u)})/a$$

and

$$\begin{aligned} \eta(t, T) = &- (\sigma_r/a^3)[1 - e^{-a(T-t)}] \\ &+ (\sigma_r/a^2) e^{-a(T-t)} (T - t) \\ &+ (\sigma_r/2a)[T - t]^2 \end{aligned}$$

Substitution of (7) into the stock price expression (5) yields the final valuation formula.

Unfortunately, observing only a single value for the stock price at each date leaves this system under-determined as there are more unknowns $(L(t), \lambda_0, \lambda_1, \lambda_2)$ than there are observables $(\xi(t))$.[7] To overcome this situation, the stochastic process for $L(t)$ is used to transform expression (5) into a time-series model for the firm's equity returns. Unfortunately, this transformation introduces the equity price's risk premium into the estimation procedure. In this regard, it is shown in the appendix that[8]

$$\begin{aligned} &\log\left(\frac{\left[\xi(t) - \sum_{j\geq t}^{T_D} D_j p(t,j)\right]}{\left[\xi(t-\Delta) - \sum_{j\geq t-\Delta}^{T_D} D_j p(t-\Delta,j)\right]}\right) - r(t-\Delta)\Delta \approx \\ &+ \lambda_0 \Delta + \lambda_1 \left(\left(\frac{b(t-\Delta, T_L)^2}{2}\right)\Delta + \log\left(\frac{p(t, T_L)}{p(t-\Delta, T_L)}\right)\right) \\ &- \lambda_2 [Z(t)(T_L - t) - Z(t-\Delta)(T_L - t + \Delta)] \\ &+ \lambda_1 \lambda_2 \varphi_{rm} b(t, T_L)(T_L - t)\Delta \\ &+ [\sigma_L \Theta_L(t-\Delta) + \mu_\theta(t-\Delta) - (1/2)\sigma_\xi^2]\Delta \\ &+ \varepsilon(t-\Delta) \end{aligned} \tag{8}$$

where $\varepsilon(t - \Delta) \equiv \sigma_L(w_L(t) - w_L(t - \Delta))$ and $\Theta_L(t)$ is the liquidation value's risk premium.

Expression (8) gives a time-series expression for the stock's return over the time period $[t - \Delta, t]$. This is a generalization of the typical asset-pricing model to include a firm's default parameters. This expression forms the basis for our empirical estimation in the subsequent sections.

One can think of this model for equity returns in three different, but related ways. The first interpretation of expression (8) is that it is equivalent to a reduced-form credit risk model for the firm's equity. This interpretation follows the method of derivation. The second interpretation of expression (8) is that it is a type of structural model for the firm's equity where the firm's liquidation value (assets less liabilities) is exogenously given and default occurs according to a default intensity process correlated with the randomness inherent in the firm's liquidation value. Finally, the third interpretation of expression (8) is that it is a generalized asset-pricing model with bankruptcy explicitly parameterized within the equity's return process. Given this perspective, expression (8) makes explicit the bond market factor discussed in Fama and French (1993).

3 Description of the data

The time period covered in this study is May 1991–March 1997. The interval for computing equity returns is 1 month. For each firm, and for each month in the observation period, we will be fitting a time-series regression of equity returns going back in time 4 years (48 months). Thus, we lose the first 4 years of our observation interval, giving time-series regressions for each firm and for each month from May 1995–March 1997.

All individual firm equity data (including earnings, dividends, etc.) are obtained from CRSP. For the equity market index, we used the S&P 500 index. For estimating an equity risk premium, we will employ the Fama–French benchmark portfolios (book-to-market factor (HML), small firm factor (SMB) and a momentum factor (UMD). These monthly portfolio returns were obtained from Ken French's webpage.[9]

The US Treasury prices used for this investigation were obtained from the University of Houston's Fixed Income Database. The data consists of monthly bid prices of all outstanding bills, notes, and bonds taken from Lehman Brothers' trading sheets on the last calendar day in each month; see Warga (1999) for additional details. Being such a large database (containing over two million entries), the potential for data errors is quite large. Indeed, a careful examination of the data confirmed this suspicion. Hence, we filtered the data to remove obvious data errors. We excluded Treasury bonds with inconsistent or suspicious issue/dated/maturity dates and matrix prices. Lastly, using a median yield filter of 2.5%, we also removed US Treasury debt listings whose yields exceeded the median yield by this percent. After filtering, there are approximately 29 100 US Treasury prices left in the sample set.

The same 20 firms as in Janosi *et al.* (2002) were initially selected for analysis. These firms were selected to stratify various industry groupings: financial, food and beverages, petroleum, airlines, utilities, department stores, and technology. Due to unavailability of balance sheet data or stock prices, five of these companies were eliminated. The remaining 15 firms included in this study and the industry represented by each firm are provided in Table 1. The Moody's and S&P's ratings for each company's debt issues at the start of our sample period (May 24, 1991) are also included.

As mentioned previously, the interval for equity returns is 1 month. The monthly return interval was chosen for two reasons. First, the default parameter estimation using debt prices in Janosi *et al.* (2002) was based on monthly data, so monthly equity returns will provide an equivalent comparison. Second, and more importantly, it is believed that the use of monthly data for equities eliminates market-microstructure noise more prevalent in smaller return intervals (daily or weekly) (see Dimson, 1979; Schwartz and Whitcomb, 1977a,b; Smith, 1978).

Table 1 Details of the firms included in the empirical investigation

	Ticker symbol	SIC code	First date used in the estimation	Last date used in the estimation	Number of bonds	Moodys	S&P
Financials							
Bankers Trust NY	bt	6022	01/31/1994	04/30/1994	3	A1	AA
Merrill Lynch & Co	mer	6211	12/31/1991	03/31/1997	14	A2	A
Food & beverages							
Pepsico Inc	pep	2086	12/31/1991	03/31/1997	8	A1	A
Coca-Cola Ent. Inc	cce	2086	12/31/1991	06/30/1994	3	A2	AA−
Airlines							
AMR Corporation	amr	4512	02/29/1992	08/31/1994	2	Baa1	BBB+
Southwest Airlines	luv	4512	05/31/1992	03/31/1997	3	Baa1	A−
Utilities							
Carolina Power Light	cpl	4911	08/31/1992	01/31/1993	3	A2	A
Texas Utilities Ele Co	txu	4911	04/30/1994	03/31/1997	4	Baa2	BBB
Petroleum							
Mobil Corp	mob	2911	12/31/1991	02/29/1996	3	Aa2	AA
Department stores							
Sears Roebuck + Co	s	5311	12/31/1991	08/31/1996	7	A2	A
Wal-Mart Stores, Inc	wmt	5331	12/31/1991	03/31/1997	3	Aa3	AA
Technology							
Eastman Kodak Company	ek	3861	01/31/1992	09/30/1994	3	A2	A−
Xerox Corp	xrx	3861	12/31/1991	03/31/1997	4	A2	A
Texas Instruments	txn	3674	10/31/1992	03/31/1997	3	A3	A
Intl Bus Machines	ibm	3570	01/31/1994	03/31/1997	3	A1	AA−

Ticker symbol is the firm's ticker symbol. SIC is the standard industry code. Number of bonds corresponds to the number of distinct bond issues used in the estimation. Moodys refers to Moodys' debt rating for the company's senior debt on the first date used in the estimation. S& P refers to S& P's debt rating for the company's debt on the first date used in the estimation.

4 Estimation of the state variable process parameters

To implement the estimation of the equity return process, we use a two-step procedure. In step one, we first estimate the parameters for the state variable processes. Step two uses these estimates as constants in the equity return estimation. Step two is discussed in Section 5.

4.1 Spot rate process parameter estimation

The inputs to the spot rate process evolution are the forward rate curves over an extended observation period ($f(t, T)$ for all months $t \in$ January 1975–March 1997) and the spot rate parameters (a, σ_r).

For the estimation of the forward rate curves, a two-step procedure is also utilized. First, for a given time

t, the discount bond prices ($p(t, T)$ for various T) are estimated by solving the following minimization problem:

$$\begin{cases} \text{choose} & (p(t,T) \text{ for all relevant} \\ & \quad T \leq \max\{T_i : i \in I_t\}) \\ \text{to minimize} & \sum_{i \in I_t} [B_i(t, T_i) - B_i(t, T_i)^{\text{bid}}]^2 \end{cases} \tag{9}$$

where

$$B_i(t, T_i) = \sum_{j=I}^{n_i} C_{t_j} p(t, t_j)$$

is a US Treasury security with coupons of C_j dollars at times t_j for $j = 1, \ldots, n_i$ where $t_{n_i} = T_i$ is the maturity date, I_t is an index set containing the various US Treasury bonds, notes, and bills available at time t, and $B_i(t, T_i)^{\text{bid}}$ is the market bid price for the ith bond with maturity T_i.

The discount bond prices' maturity dates T coincide with the maturities of the Treasury bills, and the coupon payment and principal repayment dates for the Treasury notes and bonds.

Step 2 is to fit a continuous forward rate curve to the estimated zero-coupon bond prices ($p(t, T)$ for all $T \leq \max\{T_i\colon i \in I\}$). We use the maximum smoothness forward rate curve as developed by Adams and van Deventer (1994) and refined by Janosi and Jarrow (2002). Briefly, we choose the unique piecewise, fourth-degree polynomial with the left and right end points left "dangling" that minimizes

$$\int_t^{\max\{T_i\colon i \in I_t\}} |\partial^2 f(t,s)/\partial s^2| \mathrm{d}s$$

For the estimation of the spot rate parameters (a, σ_r), the procedure follows that used in Janosi *et al.* (2002). A rolling estimation of the parameters using only information available at the time of the estimation is performed, making the parameter estimates (a_t, σ_{rt}) dependent on time t as well. The procedure is based on an explicit formula for the variance of the default-free zero-coupon bond prices under the extended Vasicek model (see Heath *et al.*, 1992). For $\Delta = 1/12$ (a month), the expression is

$$\begin{aligned} \text{var}_t[\log(P(t+\Delta, T)/P(t,T)) - r(t)\Delta] \\ = \left(\sigma_{rt}^2 (\mathrm{e}^{-a_t(T-t)} - 1)^2 / a_t^2\right) \Delta \end{aligned} \tag{10}$$

First, we fix a time to maturity $T - t \in$ {3 months, 6 months, 1 year, 5 years, 10 years, the longest time to maturity of an outstanding Treasury bond closest to 30 years}. Then, we fix a current month $t \in$ {May 1991–March 1997}. Going backwards in time 60 months (5 years), we compute the sample variance, denoted ν_{tT}, using the smoothed forward rate curves previously generated. Note that the sample variance depends on both the date of estimation and the bond's maturity. Then, for each month $t \in$ {May 1991–March 1997}, to estimate the parameters (σ_{rt}, a_t) we run a nonlinear regression

$$\nu_{tT} = (\sigma_{rt}^2 \left(\mathrm{e}^{-a_t(T-t)} - 1)^2 / a_t^2\right) \Delta + e_{tT} \tag{11}$$

across the bond time to maturities $T - t \in$ {1/4, 1/2, 1, 5, 10, longest time to maturity closest to 30} where e_{tT} is the error term.

The parameter estimates are

	Min	Mean	Max	Std. dev.
a_{rt}	0.0109	0.0282	0.0428	0.0101
σ_{rt}	0.0100	0.0109	0.0117	0.0004

The R^2 for each of these monthly nonlinear regressions (not reported) exceeded 0.99 in all cases. The spot rate volatility (σ_{rt}) is nearly constant over this period. In contrast, the mean reverting parameter (a_{rt}) appears to be more volatile.

To test for the time series stability of these parameter estimates, a unit root test was performed.[10] For the volatility σ_{rt}, the test rejects a unit root, implying the time series is stationary. In contrast, one cannot reject a unit root for the mean reverting parameter a_t.

4.2 *Market index parameter estimation*

Although the equity returns are monthly, for estimating the parameters of the market index we use daily data. This is done because daily data is available for the market index, and the higher frequency data will provide less noisy estimates since market microstructure considerations are less important for an index (than they are for individual firms). Daily observations of the market return and the 3-month T-bill yield are available from CRSP. Using the daily S&P 500 index price data and the daily 3-month T-bill spot rate data, we estimate the parameters of the market index process (σ_m, φ_{rm}).

As previously mentioned, this estimation is based on daily data ($\Delta = 1/365$). As before, the procedure involves a rolling estimation of the parameters using only information available at the time of the estimation. For a given day $t \in$ {May 24, 1990–March 31, 1997}, we go back in time 365 business days and estimate the time-dependent sample variance and correlation coefficients $(\sigma_{mt}, \varphi_{rmt})$ using the sample moments, that is,

$$\sigma_{mt}^2 = \mathrm{var}_t\left(\frac{M(t) - M(t-\Delta)}{M(t-\Delta)}\right)\frac{1}{\Delta}$$

and

$$\varphi_{nnt} = \mathrm{corr}_t\left(\frac{M(t) - M(t-\Delta)}{M(t-\Delta)}, r(t) - r(t-\Delta)\right) \tag{12}$$

The parameter estimates are

	Min	Mean	Max	Std. dev.
σ_{mt}	0.0982	0.1261	0.1897	0.0270
φ_t	−0.2706	−0.0990	0.1262	0.1142

The market volatility is relatively constant between 0.1 and 0.2 over this observation period. The correlation coefficient appears to be more variable. As before, to test for the stability of the parameters a unit root test was performed. The results show that a unit root can be rejected at the 90% percent confidence level for the market volatility but not for the correlation coefficient.[11]

With the parameter estimates for the market volatility (σ_{mt}) and the daily 3-month Treasury bill yields, the excess cumulative return on the market process $Z(t)$ is computed,[12] starting the time series on May 24, 1991.

5 Equity return estimation

Given the state variable parameters as estimated in the previous sections, this section presents the estimated equity return model. The basis for this estimation is expression (8). To empirically implement expression (8), we need to specify models for both the risk premium and the equity price bubble.

Following Fama and French (1993, 1996), we use a four-factor asset-pricing model with the factors being the excess return on a market portfolio, SMB(t), HML(t), and UMD(t), that is,

$$\begin{aligned}\sigma_L\Theta_L(t-\Delta, X(t-\Delta))\Delta \\ = \beta_0\left[\frac{M(t) - M(t-\Delta)}{M(t-\Delta)} - r(t-\Delta)\Delta\right] \\ + \beta_1[\mathrm{SMB}(t)] + \beta_2[\mathrm{HML}(t)] + \beta_3[\mathrm{UMD}(t)]\end{aligned} \tag{13}$$

where SMB(t) is the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks at time t, HML(t) is the difference between the return on a portfolio of high-book-to-market stocks and the return on a portfolio of low-book-to-market stocks at time t, and UMD(t) is a momentum factor.

The equity price bubble is proxied by the price earnings ratio and possibly the stock's own variance, that is,

$$\begin{aligned} &-(1/2)\sigma_\xi^2(t)\Delta + \mu_\theta(t-\Delta, X(t-\Delta))\Delta \\ &\quad = \beta_4[\sigma_\xi^2(t)\Delta] + \beta_5\left[\frac{\text{Price}}{\text{Earnings}}(t)\right] \end{aligned} \tag{14}$$

where

$$\sigma_\xi^2(t) \equiv \text{var}\left(\frac{\xi(t)-\xi(t-\Delta)}{\xi(t-\Delta)}\right)\frac{1}{\Delta}$$

The stock's own variance is included with an arbitrary coefficient to see if it differs from its theoretical value of $-(1/2)$ in expression (8). There is also a concern that the SMB(t), HML(t), and UMD(t) factors may already include an adjustment for bubbles. For this reason, the subsequent regressions are run both with and without the P/E ratio included.

For the estimation we fix $(T_L - t) = 20$ years and we set $T_D = t$. The first restriction makes the firm's valuation horizon 20 years, making equity comparable with long-term debt. The second restriction implies that all future dividends are viewed as random. Consequently, we only need to make an adjustment for dividends in the payout month.

Substitution of the above into expression (8) yields:

$$\left.\begin{array}{l} \log\left(\dfrac{\xi(t)}{\xi(t-\Delta)}\right) - r(t-\Delta)\Delta \\ \quad \text{if no divident over } [t-\Delta, t] \\ \log\left(\dfrac{\xi(t)}{\xi(t-\Delta) - D_x p(t-\Delta, x)}\right) - r(t-\Delta)\Delta \\ \quad \text{if dividend at } x \in [t-\Delta, t] \end{array}\right\}$$

$$\begin{aligned} &\approx \Lambda_0 + \Lambda_1\left[\log\left(\frac{p(t,T_L)}{p(t-\Delta,T_L)}\right) + \left(\frac{b(t-\Delta,T_L)^2}{2}\right)\Delta\right] + \Lambda_2[Z(t)(T_L - t) \\ &\quad - Z(t-\Delta)(T_L - (t-\Delta))] \\ &\quad + \beta_0\left[\frac{M(t)-M(t-\Delta)}{M(t-\Delta)} - r(t-\Delta)\Delta\right] \\ &\quad + \beta_1[\text{SMB}(t)] + \beta_2[\text{HML}(t)] + \beta_3[\text{UMD}(t)] \\ &\quad + \beta_4[\sigma_\xi^2(t)\Delta] + \beta_5\left[\frac{\text{Price}}{\text{Earnings}}(t)\right] \end{aligned} \tag{15}$$

where

$$\begin{aligned} \Lambda_0 &= +\lambda_0\Delta + \lambda_1\lambda_2\varphi_{rm}b(t,T_L)(T_L - t)\Delta \\ \Lambda_1 &= \lambda_1 \\ \Lambda_2 &= -\lambda_2 \end{aligned}$$

To insure that the intensity process is non-negative when both λ_1 and λ_2 are zeros, we impose the constraint that $\Lambda_0 \geq 0$ in the estimation. The final computations for the default parameters are

$$\begin{aligned} \lambda_0 &= (\Lambda_0/\Delta) - \Lambda_1\Lambda_2\varphi_{rm}b(t,T_L)(T_L - t) \\ \lambda_1 &= \Lambda_1 \\ \lambda_2 &= -\Lambda_2 \end{aligned}$$

The time period covered is May 1991–March 1997. For each firm, a time-series regression is run using 48 months of historical data. Thus, the first regression estimation occurs 4 years into the data set on May 31, 1995. For each subsequent month, until March 1997, the regression is re-estimated and parameter estimates obtained. This generates 23 regressions for each firm's returns. As before, only information available to the market at the time of the estimation is utilized. This rolling estimation procedure gives a monthly time series of parameter estimates $(\lambda_{0t}, \lambda_{1t}, \lambda_{2t}, \beta_{0t}, \beta_{1t}, \beta_{2t}, \beta_{3t}, \beta_{4t}, \beta_{5t})$ based on 46 $(48 - 2)$ months of overlapping data. The choice of a 4-year estimation period was based on trading off the stability of the estimates versus larger standard errors. Although longer estimation periods are likely to make the standard errors smaller, they also imply that structural shifts are more likely to occur, making the estimated parameters less stable.

Eight different models for equity returns are estimated. The models differ with respect to the number of independent variables included in the regression. Models 1 and 2 have no default ($\lambda_0 \equiv \lambda_1 \equiv \lambda_2 \equiv 0$). They differ only in the inclusion of a P/E ratio ($\beta_5 \equiv 0$ or not). Models 3–8 include default, and they differ with respect to the default intensity investigated and the inclusion of the P/E ratio or not. In particular, models 3 and 4 have only λ_0 non-zero. Models 5 and 6 have both λ_0 and λ_1 non-zero. Models 7 and 8 have all default parameters non-zero. These eight models are nested and a relative comparison of model performance is subsequently provided.

To summarize the monthly time series estimates across all models and across all times, Table 2 provides the average values for the point estimates of the parameters and their t-statistics.[13] The average adjusted R^2 is also included. The values in Table 2 are averages over the number of months in the observation period (May 1995–March 1997) for which the linear regression estimates of the parameters are computed.[14] Summary statistics for various F-tests are also provided. The first F-test is for the null hypothesis ($\beta_{0t} = \beta_{1t} = \beta_{2t} = \beta_{3t} = \beta_{4t} = 0$). Given are the average P-scores of the F-tests (across the number of regressions). The F-tests for models 3, 5, and 7 test for the joint hypothesis that all default parameters are zero, that is, $\lambda_{0t} = 0$, $\lambda_{0t} = \lambda_{1t} = 0$, and $\lambda_{0t} = \lambda_{1t} = \lambda_{2t} = 0$, respectively. The F-tests from models 2, 4, 6, and 8 test the hypothesis that $\beta_{5t} = 0$. The subsequent sections discuss these statistics and various tests for the relative performance of the different equity models.

A typical time series graph of the default intensities for Eastman Kodak (ek) using models 3–8 is contained in Fig. 1. As depicted, all six models exhibit similar patterns in the default intensities. The magnitude of the default intensity appears to be quite large, exceeding 0.3 for all dates and models.

Table 3 provides some summary statistics (both in- and out-of-sample) regarding the quality of the models fit to the data. Included are the average root mean squared error, the average generalized cross-validation statistic, the average default intensity, the average standard error of the default intensity, and the average 1-year default probability.

6 Analysis of the time series properties of the parameters

Under the assumed model structure, the default and equity risk premium parameters ($\lambda_{0t}, \lambda_{1t}, \lambda_{2t}, \beta_{0t}, \beta_{1t}, \beta_{2t}, \beta_{3t}$) should be constant across time. The 4-year estimation period was selected to better insure this hypothesis, by minimizing the structural shifts in the economy that would more likely occur using a longer horizon interval. Given measurement error in the input data (equity prices and the state variable parameters) and its effect on the parameter estimates, we test the hypothesis that the time series variation in these parameters is solely due to random (white) noise. Alternatively stated, we test to see if the parameter estimates follow a random walk around a given mean. A unit root test is used in this regard.

Table 4 contains a summary of the unit root rejections across model types. As seen, we accept a unit root (non-stationarity) for almost all the parameters in all the models. This includes both the default parameters and the risk premium coefficients. Acceptance of the unit root implies that the model parameters may be non-stationary. This non-stationarity could be due to a confounding

Table 2 Averages of the parameter estimates and t-scores from the equity model regression

	λ_0	λ_1	λ_2	β_0	β_1	β_2	β_3	β_4	β_5	R^2	F-test
Financials											
1—Bankers Trust NY (bt)											
Model 1				1.7055**	0.0071**	0.0102**	−0.0018	−2.3971**		0.4873	0.0000
				5.5380	1.8227	2.7373	−0.4994	−2.3400			
Model 2				1.7051**	0.0068**	0.0097**	−0.0024	−2.8753**	0.0034	0.4901	0.4676
				5.5066	1.7420	2.5609	−0.6517	−2.3452	0.7640		
Model 3	0.1851			1.6377**	0.0061*	0.0088**	−0.0029	−3.4759**		0.4967	0.3083
	1.3881			5.2676	1.5567	2.2794	−0.8168	−2.6669			
Model 4	0.3244			1.6041**	0.0057*	0.0084**	−0.0028	−3.5133**	−0.0054	0.5028	0.5216
	1.2759			5.0361	1.4421	2.1534	−0.7874	−2.6798	−0.6020		
Model 5	0.1859	0.1406		1.5122**	0.0065*	0.0079**	−0.0036	−3.4779**		0.5025	0.4394
	1.3817	0.6617		4.1266	1.6377	1.8964	−0.9790	−2.6482			
Model 6	0.3446	0.1710		1.4390**	0.0062*	0.0073**	−0.0036	−3.5364**	−0.0061	0.5105	0.4674
	1.3526	0.7739		3.7553	1.5408	1.6815	−0.9708	−2.6819	−0.7244		
Model 7	0.1976	0.1214	−0.0168	−1.4618	0.0060*	0.0072**	−0.0032	−3.5981**		0.5208	0.4325
	1.4670	0.5707	1.1576	−0.3581	1.4694	1.7106	−0.8617	−2.7542			
Model 8	0.3526	0.1513	−0.0167	−1.5191	0.0057	0.0066*	−0.0032	−3.6556**	−0.0060	0.5284	0.4726
	1.4129	0.6847	1.1432	−0.3763	1.3782	1.5082	−0.8541	−2.7857	−0.7129		
2—Merrill Lynch & Co (mer)											
Model 1				2.0601**	0.0046	0.0055	0.0063*	−1.7785**		0.8292	0.0000
				5.6185	1.0160	1.2179	1.5668	−8.1829			
Model 2				1.9509**	0.0033	0.0035	0.0041	−1.9064**	0.0089**	0.8379	0.1008
				5.3521	0.7323	0.7375	0.9959	−8.2971	1.7342		
Model 3	0.2643**			1.8714**	0.0033	0.0034	0.0026	−1.9523**		0.8412	0.3857
	2.0564			5.0857	0.7421	0.7097	0.6240	−8.4074			
Model 4	0.3161			1.8729**	0.0033	0.0035	0.0024	−1.9552**	−0.0023	0.8426	0.7275
	1.0685			4.9728	0.7342	0.7279	0.5489	−8.3476	−0.2172		
Model 5	0.2584**	0.0420		1.8468**	0.0035	0.0035	0.0023	−1.9474**		0.8432	0.5592
	1.9930	0.1695		4.2686	0.7653	0.6451	0.5291	−8.2949			
Model 6	0.2970	0.0317		1.8555**	0.0034	0.0036	0.0022	−1.9505**	−0.0017	0.8441	0.7553
	0.9655	0.1165		4.2352	0.7466	0.6597	0.4903	−8.2079	−0.1569		
Model 7	0.2468**	0.0163	−0.0098	0.1971	0.0031	0.0032	0.0030	−1.9320**		0.8477	0.6244
	1.8878	0.0662	0.8235	−0.0362	0.6715	0.5695	0.6549	−8.1654			
Model 8	0.2649	0.0109	−0.0100	0.1827	0.0030	0.0033	0.0028	−1.9348**	−0.0008	0.8486	0.7547
	0.8587	0.0387	0.8120	−0.0310	0.6487	0.5670	0.6191	−8.0735	−0.0851		
Food & Beverages											
3—Pepsico Inc (pep)											
Model 1				1.1323**	−0.0058*	−0.0046	0.0053*	−0.8301**		0.5877	0.0000
				3.7001	−1.4687	−1.2377	1.5369	−4.3056			
Model 2				1.1177**	−0.0057*	−0.0046	0.0049	−0.9515**	0.0053	0.5889	0.7647
				3.5745	−1.4432	−1.2189	1.3207	−4.3600	0.2765		
Model 3	0.0841			1.0959**	−0.0059*	−0.0049	0.0046	−1.3522**		0.5887	0.6246
	0.6271			3.4971	−1.4903	−1.2944	1.2792	−4.6109			
Model 4	0.3186			1.0721**	−0.0065*	−0.0058*	0.0053	−1.2471**	−0.0436	0.5936	0.4869
	0.9245			3.3861	−1.6000	−1.4473	1.4093	−4.4420	−0.7454		
Model 5	0.0845	0.0094		1.0859**	−0.0059*	−0.0050	0.0046	−1.3592**		0.5894	0.8507
	0.6180	0.0484		2.9236	−1.4430	−1.2179	1.1967	−4.5569			
Model 6	0.3328	0.0421		1.0313**	−0.0064*	−0.0061	0.0051	−1.2688**	−0.0457	0.5944	0.4781
	0.9341	0.1931		2.7222	−1.5436	−1.3965	1.3045	−4.3840	−0.7608		
Model 7	0.0872	−0.0024	−0.0035	0.5229	−0.0063*	−0.0052	0.0049	−1.4123**		0.5956	0.9442
	0.6355	−0.0015	0.3777	0.1955	−1.5102	−1.2625	1.2656	−4.5377			
Model 8	0.3723	0.0316	−0.0061	−0.0044	−0.0070**	−0.0066*	0.0056	−1.3114**	−0.0523	0.6023	0.4356
	1.0168	0.1483	0.5364	−0.0011	−1.6455	−1.4841	1.4027	−4.3540	−0.8496		
4—Coca-Cola (cce)											
Model 1				0.9856**	−0.0028	0.0001	0.0009	0.5367		0.1675	0.0000
				2.2356	−0.5606	−0.0089	0.1999	0.9575			

Table 2 Continued

	λ_0	λ_1	λ_2	β_0	β_1	β_2	β_3	β_4	β_5	R^2	F-test
Model 2				0.9706**	−0.0025	−0.0002	0.0007	0.5427	0.0243	0.1764	0.5772
				2.1831	−0.4840	−0.0607	0.1582	0.9561	0.5613		
Model 3	0.2255			0.9211**	−0.0041	−0.0003	0.0003	−0.8996		0.1581	0.4780
	0.8963			2.0779	−0.7770	−0.0693	0.0822	−0.2253			
Model 4	0.2226			0.9103**	−0.0038	−0.0005	0.0002	−0.8706	0.0215	0.1667	0.5924
	0.8748			2.0344	−0.6907	−0.1140	0.0561	−0.2095	0.5302		
Model 5	0.2467	−0.4447		1.3239**	−0.0057	0.0027	0.0027	−1.1775		0.2043	0.2999
	0.9980	−1.4881		2.5788	−1.0530	0.4505	0.5240	−0.3747			
Model 6	0.2431	−0.4390		1.3083**	−0.0054	0.0024	0.0025	−1.1430	0.0205	0.2115	0.6180
	0.9711	−1.4567		2.5229	−0.9616	0.3948	0.4874	−0.3542	0.5049		
Model 7	0.2506	−0.4291*	0.0249	5.8008*	−0.0054	0.0035	0.0024	−1.2668		0.2368	0.3130
	1.0267	−1.4390	−1.1105	1.6003	−0.9962	0.5922	0.4650	−0.4320			
Model 8	0.2503	−0.4226	0.0255	5.8969*	−0.0050	0.0032	0.0023	−1.2459	0.0221	0.2455	0.5735
	1.0149	−1.4060	−1.1225	1.6021	−0.8961	0.5275	0.4317	−0.4194	0.5758		
Airlines											
5—AMR Corporation (amr)											
Model 1				1.7603**	0.0065	0.0008	−0.0028	−0.2679		0.3543	0.0000
				4.3989	1.3297	0.1303	−0.5812	0.2237			
Model 2				1.9060**	0.0060	0.0012	−0.0027	−0.8753	−0.0100**	0.4100	0.0724
				4.8285	1.2410	0.2359	−0.5693	−0.3361	−1.9794		
Model 3	0.0000			1.7603**	0.0065	0.0008	−0.0028	−0.2679		0.3543	0.0756
	0.0000			4.2846	1.2610	0.1285	−0.5741	0.1048			
Model 4	0.0000			1.9060**	0.0060	0.0012	−0.0027	−0.8753	−0.0100**	0.4100	0.0685
	0.0000			4.7145	1.1764	0.2328	−0.5622	−0.1961	−1.9470		
Model 5	0.0000	−0.2262		1.9712**	0.0058	0.0024	−0.0015	−0.3713		0.3669	0.3243
	0.0000	−0.8031		4.0794	1.0795	0.4104	−0.3012	0.0564			
Model 6	0.0000	−0.3201		2.2250**	0.0049	0.0036	−0.0009	−1.0727	−0.0111**	0.4331	0.0416
	0.0000	−1.1553		4.6410	0.9422	0.6597	−0.1805	−0.2787	−2.1406		
Model 7	0.0000	−0.2615	−0.0272	−2.8612	0.0048	0.0012	−0.0010	−0.3585		0.4082	0.5419
	0.0000	−0.9433	1.3987	−0.6147	0.8927	0.2143	−0.2007	0.0650			
Model 8	0.0000	−0.3658	−0.0295*	−2.9848	0.0038	0.0024	−0.0002	−1.0852	−0.0116**	0.4797	0.0294
	0.0000	−1.3552	1.6313	−0.7042	0.7245	0.4523	−0.0397	−0.2785	−2.2907		
6—Southwest Airlines Co (luv)											
Model 1				2.0166**	0.0034	0.0113*	0.0014	−1.6570**		0.4663	0.0000
				3.1244	0.4052	1.5801	0.2680	−4.0185			
Model 2				1.9524**	0.0032	0.0109*	0.0001	−1.7540**	0.0410	0.4796	0.4857
				2.9775	0.3842	1.5162	0.0848	−4.0080	0.7684		
Model 3	0.2486			1.9001**	0.0028	0.0100	−0.0007	−1.8198**		0.4909	0.4773
	1.1639			2.9004	0.3169	1.3567	−0.0306	−4.1869			
Model 4	0.4029			1.8841**	0.0026	0.0098	−0.0007	−1.8199**	−0.0424	0.4947	0.6616
	0.9320			2.8407	0.2807	1.2747	−0.0320	−4.1351	−0.4210		
Model 5	0.2167	−0.7461**		2.5738**	0.0005	0.0149**	0.0030	−1.7444**		0.5301	0.1702
	1.0517	−1.7237		3.4433	0.0318	1.8983	0.4334	−4.0043			
Model 6	0.2009	−0.7405*		2.5646**	0.0003	0.0148**	0.0028	−1.7507**	0.0047	0.5329	0.7065
	0.5453	−1.6337		3.3338	0.0041	1.8072	0.4096	−3.9784	−0.0436		
Model 7	0.2153	−0.7722**	−0.0073	1.3422	−0.0002	0.0145**	0.0033	−1.7367**		0.5338	0.2532
	1.0131	−1.7537	0.3367	0.2600	−0.0490	1.8144	0.4647	−3.7165			
Model 8	0.1939	−0.7694**	−0.0077	1.2768	−0.0004	0.0144**	0.0032	−1.7418**	0.0059	0.5370	0.6808
	0.5259	−1.6703	0.3580	0.2153	−0.0826	1.7187	0.4464	−3.6858	−0.0446		
Utilities											
7—Carolina Power Light (cpl)											
Model 1				0.8464**	−0.0031	0.0081**	0.0060**	−1.8684**		0.8275	0.0000
				3.4567	−1.0833	2.7034	2.2368	−10.7690			
Model 2				0.8692**	−0.0029	0.0084**	0.0064**	−1.6545**	−0.0039	0.8294	0.6732
				3.4627	−0.9636	2.7031	2.2443	−10.4473	−0.4180		

Table 2 Continued

	λ_0	λ_1	λ_2	β_0	β_1	β_2	β_3	β_4	β_5	R^2	F-test
Model 3	0.0015			0.8452**	−0.0031	0.0081**	0.0060**	−1.8686**		0.8276	0.6866
	0.0195			3.3190	−1.0666	2.6154	2.0881	−10.5615			
Model 4	0.0204			0.8641**	−0.0028	0.0085**	0.0065**	−1.6537**	−0.0058	0.8301	0.6597
	0.1347			3.3663	−0.9208	2.6785	2.2081	−10.3224	−0.3607		
Model 5	0.0015	0.2135		0.6614**	−0.0025	0.0069**	0.0048**	−1.8507**		0.8348	0.5397
	0.0202	1.2838		2.2047	−0.8351	2.1013	1.6044	−10.6269			
Model 6	0.0247	0.2162		0.6790**	−0.0020	0.0074**	0.0053**	−1.6434**	−0.0068	0.8367	0.5864
	0.1632	1.2784		2.2413	−0.6682	2.2060	1.7529	−10.3838	−0.4392		
Model 7	0.0014	0.1985	−0.0099	−1.0943	−0.0030	0.0064**	0.0051**	−1.8352**		0.8406	0.5845
	0.0186	1.1775	0.8539	−0.4656	−0.9768	1.9236	1.6896	−10.4632			
Model 8	0.0243	0.2028	−0.0096	−1.0226	−0.0025	0.0068**	0.0056**	−1.6729**	−0.0061	0.8416	0.5987
	0.1628	1.1816	0.8276	−0.4373	−0.8090	2.0200	1.8106	−10.2392	−0.4042		
8—Texas Utilities Ele Co (txu)											
Model 1				0.2394	−0.0040	0.0032	0.0035	−3.0857**		0.2101	0.0000
				0.9406	−1.3261	1.0673	1.2746	−1.4274			
Model 2				0.2289	−0.0041	0.0033	0.0034	−3.0423**	−0.0003	0.2278	0.3541
				0.8906	−1.3279	1.0838	1.2506	−1.3262	−0.2301		
Model 3	0.3137**			0.1219	−0.0055**	0.0016	0.0015	−9.3617**		0.2715	0.0868
	2.0807			0.4848	−1.8441	0.4914	0.5680	−2.5419			
Model 4	0.3038**			0.1159	−0.0056**	0.0017	0.0016	−9.1432**	−0.0001	0.2847	0.4086
	2.0012			0.4577	−1.8153	0.5256	0.5676	−2.4208	−0.1703		
Model 5	0.2923**	0.3495**		−0.1838	−0.0043*	−0.0004	−0.0002	−8.7331**		0.3513	0.0299
	2.0143	2.2115		−0.6802	−1.4823	−0.1872	−0.0642	−2.4598			
Model 6	0.2853**	0.3400**		−0.1828	−0.0045*	−0.0003	−0.0002	−8.6330**	0.0004	0.3594	0.4957
	1.9478	2.1258		−0.6678	−1.4826	−0.1400	−0.0486	−2.3671	−0.0639		
Model 7	0.3179**	0.3524**	0.0031	0.3425	−0.0048*	−0.0006	−0.0004	−9.2299**		0.3783	0.0084
	2.1802	2.2318	−0.4410	0.2814	−1.5949	−0.2461	−0.1406	−2.6065			
Model 8	0.3108**	0.3449**	0.0031	0.3446	−0.0049*	−0.0006	−0.0004	−9.1404**	0.0005	0.3835	0.5869
	2.1040	2.1517	−0.4214	0.2679	−1.5812	−0.2036	−0.1258	−2.5058	−0.0277		
Petroleum											
9—Mobil Corp (mob)											
Model 1				0.6879**	−0.0040**	0.0031	0.0031	0.8424		0.3943	0.0000
				3.5411	−1.7330	1.3104	1.4253	1.1439			
Model 2				0.6894**	−0.0040**	0.0029	0.0031	1.5378	−0.0028	0.4077	0.6808
				3.4993	−1.7207	1.2137	1.4223	0.9900	−0.3968		
Model 3	0.0208			0.6915**	−0.0040**	0.0031	0.0029	0.4696		0.3918	0.7177
	0.1722			3.4903	−1.7137	1.3185	1.3094	0.4455			
Model 4	0.1189			0.6977**	−0.0039**	0.0029	0.0025	0.9164	−0.0087	0.4104	0.5255
	0.6085			3.5211	−1.6896	1.1782	1.0502	0.6263	−0.7237		
Model 5	0.0148	0.1288		0.5683**	−0.0036*	0.0022	0.0023	0.7714		0.3991	0.6784
	0.1187	0.9309		2.4188	−1.5405	0.8667	0.9591	0.5468			
Model 6	0.0990	0.1062		0.5939**	−0.0037*	0.0021	0.0020	1.1824	−0.0077	0.4165	0.5572
	0.4662	0.7204		2.4838	−1.5411	0.8082	0.8308	0.7100	−0.5981		
Model 7	0.0100	0.1215	−0.0093	−1.0949	−0.0040**	0.0018	0.0025	0.9026		0.4294	0.6201
	0.0758	0.8823	0.9046	−0.4177	−1.6474	0.7026	1.0606	0.6053			
Model 8	0.0903	0.1013	−0.0095	−1.1141	−0.0039*	0.0016	0.0023	1.1509	−0.0067	0.4420	0.5932
	0.4118	0.6927	0.8999	−0.3860	−1.6236	0.6470	0.9280	0.6972	−0.5050		
Department Stores											
10—Sears Roebuck + Co (s)											
Model 1				1.7989**	0.0076**	0.0091**	0.0002	−1.6245**		0.7250	0.0000
				4.9673	1.7257	2.2308	0.0558	−6.8985			
Model 2				1.8000**	0.0077**	0.0091**	0.0002	−1.6207**	0.0004	0.7251	0.8993
				4.9109	1.7029	2.1942	0.0370	−6.8048	0.0896		
Model 3	0.2045**			1.6604**	0.0072**	0.0076**	−0.0017	−1.7049**		0.7414	0.1193
	1.7729			4.5993	1.6622	1.8643	−0.4050	−7.3206			

Table 2 Continued

	λ_0	λ_1	λ_2	β_0	β_1	β_2	β_3	β_4	β_5	R^2	F-test
Model 4	0.2060**			1.6583**	0.0071*	0.0076**	−0.0016	−1.7028**	−0.0006	0.7417	0.8567
	1.7510			4.5331	1.6097	1.8399	−0.3818	−7.2227	−0.1255		
Model 5	0.2003**	−0.0592		1.7253**	0.0070*	0.0083**	−0.0014	−1.7140**		0.7445	0.2084
	1.7218	−0.2150		3.9934	1.6030	1.8681	−0.3365	−7.1450			
Model 6	0.2022**	−0.0650		1.7283**	0.0069*	0.0083**	−0.0013	−1.7128**	−0.0008	0.7449	0.8263
	1.7029	−0.2319		3.9507	1.5384	1.8465	−0.3033	−7.0403	−0.1585		
Model 7	0.2000**	−0.0826	−0.0120	−0.3629	0.0066*	0.0079**	−0.0010	−1.7185**		0.7497	0.2301
	1.7069	−0.3101	0.8340	−0.0926	1.4739	1.7580	−0.2289	−7.1536			
Model 8	0.2011**	−0.0868	−0.0118	−0.3231	0.0065	0.0079**	−0.0009	−1.7160**	−0.0006	0.7500	0.8588
	1.6815	−0.3192	0.8139	−0.0818	1.4218	1.7356	−0.2066	−7.0422	−0.1108		
11—Wal-Mart Stores, Inc (wmt)											
Model 1				1.1142**	0.0061	−0.0023	−0.0104**	−1.6027**		0.7309	0.0000
				3.1272	1.2819	−0.6456	−2.5727	−6.4881			
Model 2				0.9708**	0.0048	−0.0042	−0.0125**	−1.6049**	0.0481**	0.7770	0.1709
				2.7181	0.9742	−1.1116	−3.0620	−6.8081	1.8077		
Model 3	0.1722*			1.0112**	0.0054	−0.0032	−0.0122**	−1.7160**		0.7688	0.3282
	1.5028			2.8215	1.1369	−0.8660	−2.9302	−6.7938			
Model 4	0.0849			0.9761**	0.0046	−0.0042	−0.0127**	−1.7035**	0.0292	0.7805	0.2938
	0.1929			2.6957	0.9270	−1.0791	−3.0526	−6.5209	0.4848		
Model 5	0.1703	0.1400		0.8920**	0.0058	−0.0039	−0.0129**	−1.7159**		0.7713	0.6799
	1.4745	0.5867		2.0960	1.2140	−1.0052	−2.9736	−6.7080			
Model 6	0.0857	0.1227		0.8732**	0.0050	−0.0047	−0.0133**	−1.7067**	0.0282	0.7824	0.3074
	0.1913	0.5196		2.0412	0.9973	−1.1817	−3.0635	−6.4437	0.4672		
Model 7	0.1744*	0.1579	0.0027	1.2975	0.0061	−0.0038	−0.0135**	−1.7124**		0.7748	0.9794
	1.5007	0.6539	−0.3628	0.7260	1.2471	−0.9672	−3.0628	−6.6328			
Model 8	0.0849	0.1409	0.0024	1.2250	0.0053	−0.0046	−0.0139**	−1.7034**	0.0288	0.7851	0.3215
	0.1871	0.5888	−0.3459	0.6983	1.0424	−1.1357	−3.1403	−6.3458	0.4718		
Technology											
12—Eastman Kodak Company (ek)											
Model 1				0.8666**	0.0011	0.0058	0.0041	−2.5096**		0.2610	0.0000
				1.9389	0.2073	1.0854	0.8540	−2.1621			
Model 2				0.8786**	0.0024	0.0058	0.0048	−3.5253**	0.0305**	0.3036	0.0839
				2.0149	0.4374	1.0978	1.0228	−2.9134	1.9362		
Model 3	0.4802**			0.7649**	0.0009	0.0059	0.0007	−4.6018**		0.3342	0.0583
	3.0286			1.8573	0.1667	1.1717	0.1475	−3.7628			
Model 4	0.4450**			0.7882**	0.0016	0.0059	0.0013	−4.7336**	0.0088	0.3436	0.4925
	2.3317			1.8946	0.2895	1.1580	0.2631	−3.8075	0.5366		
Model 5	0.4895**	0.2574		0.5459	0.0017	0.0046	−0.0007	−4.6967**		0.3502	0.0855
	3.0747	0.9315		1.1273	0.3327	0.8331	−0.1585	−3.8302			
Model 6	0.4484**	0.2738		0.5574	0.0025	0.0044	−0.0001	−4.8483**	0.0101	0.3613	0.4650
	2.3463	0.9810		1.1447	0.4718	0.8047	−0.0424	−3.8896	0.6072		
Model 7	0.4781**	0.2547	0.0108	2.5346	0.0016	0.0047	−0.0006	−4.6179**		0.3589	0.0976
	2.9428	0.9105	−0.4136	0.5916	0.3110	0.8613	−0.1344	−3.6679			
Model 8	0.4335**	0.2714	0.0119	2.7523	0.0025	0.0046	0.0001	−4.7782**	0.0109	0.3721	0.4240
	2.2397	0.9630	−0.4436	0.6245	0.4614	0.8355	−0.0017	−3.7394	0.6520		
13—Xerox Corp (xrx)											
Model 1				1.2132**	0.0103**	0.0018	−0.0045	−0.0673**		0.6048	0.0000
				3.9747	2.6915	0.5044	−1.3459	−4.4713			
Model 2				1.2573**	0.0105**	0.0024	−0.0041	−0.1420**	−0.0019	0.6125	0.5569
				3.9863	2.7132	0.6278	−1.2157	−4.4791	−0.6178		
Model 3	0.2090**			1.0930**	0.0092**	0.0007	−0.0062**	−0.8459**		0.6130	0.3091
	1.7526			3.6500	2.5085	0.2410	−1.7953	−5.8850			
Model 4	0.2316**			1.1510**	0.0095**	0.0014	−0.0059**	−1.1548**	−0.0025	0.6224	0.4468
	1.8683			3.7360	2.5493	0.4143	−1.6946	−5.9673	−0.8275		
Model 5	0.2190**	−0.1717		1.2496**	0.0087**	0.0018	−0.0054*	−1.0034**		0.6210	0.2815
	1.7784	−0.8382		3.5418	2.3048	0.5039	−1.4742	−5.9317			

Table 2 Continued

	λ_0	λ_1	λ_2	β_0	β_1	β_2	β_3	β_4	β_5	R^2	F-test
Model 6	0.2488∗∗	−0.2085		1.3552∗∗	0.0089∗∗	0.0028	−0.0048	−1.4161∗∗	−0.0030	0.6336	0.3652
	1.9319	−1.0008		3.6874	2.3474	0.7512	−1.3116	−6.0628	−0.9883		
Model 7	0.2291∗∗	−0.1694	−0.0055	0.2122	0.0085∗∗	0.0015	−0.0055∗	−1.0472∗∗		0.6248	0.4066
	1.8413	−0.8127	0.1351	0.6103	2.2263	0.4439	−1.4723	−5.9300			
Model 8	0.2580∗∗	−0.2034	−0.0047	0.4475	0.0087∗∗	0.0025	−0.0050	−1.4584∗∗	−0.0028	0.6372	0.4024
	1.9806	−0.9612	0.0851	0.7016	2.2654	0.6818	−1.3210	−6.0472	−0.9213		
14—Texas Instruments (txn)											
Model 1				1.8865∗∗	0.0121∗∗	0.0043	0.0057	−1.2969∗∗		0.5997	0.0000
				3.2322	1.6837	0.7490	0.8613	−4.3193			
Model 2				1.7710∗∗	0.0128∗∗	0.0051	0.0044	−1.3995∗∗	0.0111	0.6083	0.3454
				3.0068	1.7743	0.8591	0.6572	−4.3960	1.0504		
Model 3	0.4585∗∗			1.6216∗∗	0.0120∗∗	0.0027	0.0007	−1.5654∗∗		0.6291	0.1243
	2.3821			2.9045	1.7538	0.5159	0.0978	−5.2857			
Model 4	0.6052∗∗			1.6467∗∗	0.0115∗∗	0.0019	0.0007	−1.5630∗∗	−0.0083	0.6350	0.5676
	2.2246			2.9321	1.6593	0.4037	0.0916	−5.0617	−0.3976		
Model 5	0.4606∗∗	−0.2366		1.8318∗∗	0.0111∗	0.0040	0.0020	−1.5919∗∗		0.6331	0.1731
	2.3408	−0.6163		2.7818	1.5817	0.6849	0.2898	−5.1557			
Model 6	0.6180∗∗	−0.2677		1.8893∗∗	0.0104∗	0.0034	0.0021	−1.5895∗∗	−0.0090	0.6397	0.5253
	2.2450	−0.6848		2.8358	1.4626	0.5904	0.3036	−4.9065	−0.4403		
Model 7	0.4560∗∗	−0.1984	0.0317	7.4758∗∗	0.0125∗∗	0.0054	0.0015	−1.6093∗∗		0.6487	0.1904
	2.3349	−0.5150	−1.1841	1.7555	1.7373	0.8434	0.2132	−5.2705			
Model 8	0.5710∗∗	−0.2234	0.0296	7.1323∗∗	0.0119∗	0.0049	0.0016	−1.6197∗∗	−0.0058	0.6528	0.5866
	2.1218	−0.5680	−1.1086	1.6770	1.6216	0.7665	0.2175	−5.0215	−0.2559		
15—International Business Machines (ibm)											
Model 1				0.9832∗∗	−0.0092∗	−0.0076	−0.0113∗∗	0.4104		0.2812	0.0000
				2.0200	−1.4656	−1.2198	−2.0725	1.0783			
Model 2				1.0009∗∗	−0.0092∗	−0.0075	−0.0103∗∗	0.1364	−0.0016	0.2867	0.5534
				2.0352	−1.4598	−1.1911	−1.7640	0.7597	−0.5997		
Model 3	0.3041			0.8861∗∗	−0.0090∗	−0.0080	−0.0126∗∗	−1.0142		0.3125	0.2402
	1.3755			1.8288	−1.4502	−1.3135	−2.3075	−0.3027			
Model 4	0.2940			0.9050∗∗	−0.0090∗	−0.0079	−0.0117∗∗	−1.2047	−0.0014	0.3173	0.6189
	1.3131			1.8431	−1.4371	−1.2775	−1.9893	−0.3822	−0.5111		
Model 5	0.3426∗	−0.6866∗∗		1.5084∗	−0.0110∗∗	−0.0032	−0.0091∗∗	−1.3997		0.3841	0.0716
	1.5996	−2.1204		2.7341	−1.8386	−0.5187	−1.6619	−0.6027			
Model 6	0.3332∗	−0.6800∗∗		1.5194∗∗	−0.0110∗∗	−0.0031	−0.0084∗	−1.5464	−0.0011	0.3877	0.6688
	1.5322	−2.0763		2.7191	−1.8160	−0.5019	−1.4399	−0.6467	−0.4409		
Model 7	0.3324∗	−0.7040∗∗	0.0085	3.1610	−0.0112∗∗	−0.0030	−0.0087∗	−1.3941		0.3956	0.0928
	1.5415	−2.1559	−0.0678	0.6100	−1.8344	−0.4854	−1.5317	−0.5959			
Model 8	0.3227∗	−0.6970∗∗	0.0087	3.2185	−0.0112∗∗	−0.0029	−0.0079	−1.5573	−0.0012	0.3996	0.6456
	1.4740	−2.1109	−0.0718	0.6092	−1.8090	−0.4655	−1.3107	−0.6512	−0.4741		

In each cell under the columns $(\lambda_0, \lambda_1, \lambda_2, \beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5)$, the first number is average parameter estimates across the months in the observation period from the equity model regressions. They are presented for each company and for each model type, separated by industries. The second entry is the average the t-score for the corresponding average parameter estimate. This t-score is adjusted for the fact that the regressions contain overlapping time intervals. All t-scores test the null hypothesis that the coefficient is zero, except for β_4. For β_4, the null hypothesis is $-1/2$.

Models 1 and 2 have no default. Models 3 and 4 have a constant default intensity. Models 5 and 6 have the default intensity dependent on the spot rate of interest. Models 7 and 8 have the default intensity dependent on the spot rate of interest and a market index. The number of observations per regression is 48. The number of regressions in the average is 23. The average R^2 is given.

The F-test column contains the average P-score where the P-scores are obtained from the F-tests of the individual regressions. The P-score from an individual F-test corresponds to the probability of rejecting the null hypothesis when it is true. The first row corresponds to the null hypothesis $(\beta_0 = \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0)$. The F-tests for models 3, 5, and 7 test for the joint hypothesis that all default parameters are zero, i.e. $\lambda_0 = 0$, $\lambda_0 = \lambda_1 = 0$, and $\lambda_0 = \lambda_1 = \lambda_2 = 0$, respectively. The F-tests from models 2, 4, 6 and 8 test the hypothesis that $\beta_5 = 0$.

∗∗Significant at 10% level.

∗Significant at 15% level.

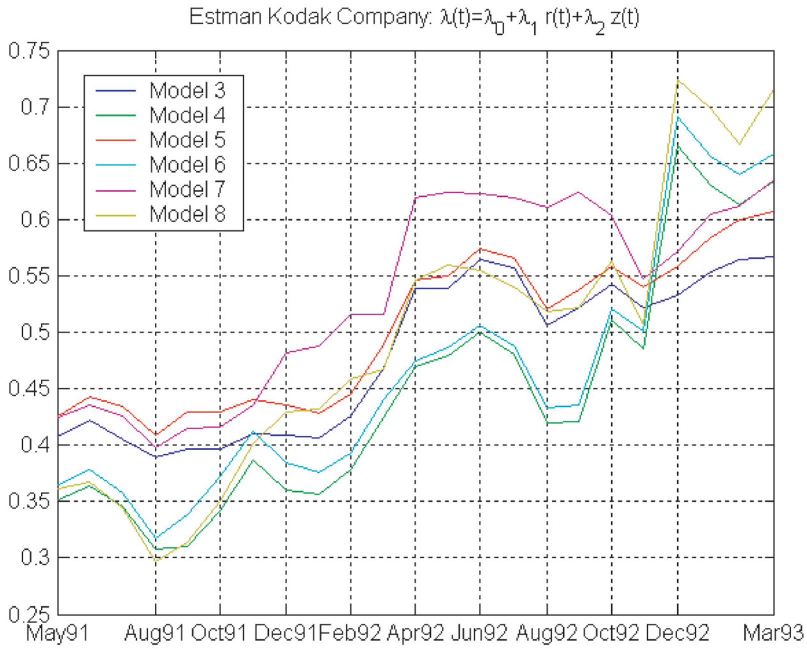

Figure 1 Time series estimates of Eastman Kodak Company's intensity function.

of the default and risk premium parameter estimates. Given the underlying variables related to both quantities are correlated, multicollinearity in the linear regression may be a problem.

7 Analysis of Fama–French four-factor model with no default

Before analyzing the default parameters, it is important to document the performance of the simple Fama–French four-factor model with no default. Table 2 contains the estimates for the coefficients of the Fama–French four-factor model with and without a P/E ratio (models 1 and 2, respectively) and their t-scores. The F-test for model 1 in Table 2 tests the hypothesis that the model is significant (i.e. $\beta_{0t} = \beta_{1t} = \beta_{2t} = \beta_{3t} = \beta_{4t} = 0$). For every firm, the average p-value for this F-test is 0.0000, strongly rejecting the hypothesis of no significance. This test confirms the need to include the Fama–French four –Factor model to explain stock risk premiums. The average R^2 for model 1 is 0.5018.

8 Analysis of a bubble component (P/E ratio) in stock prices

This section tests for the significance of a bubble component in equity returns by testing the null hypothesis that the P/E ratio is insignificant, that is, $\beta_{5t} = 0$. The F-test for model 2 in Table 2 also tests this hypothesis. For models 1 and 2 (not including default), the average p-values for three firms (mer, amr, and ek) are significantly different from zero. The individual t-scores for β_{5t} show significance for four firms (mer, amr, wmt, and ek), confirming this conclusion. This represents 20 (=3/15) to 26% (=4/15) of our firms. For models 3–8 (including default), the average F-test gives significance for only one of these three firms (amr). The average t-scores for β_{5t} confirm this reduced significance. For amr alone (among the three: mer, amr, and ek), the estimated coefficient for the constant in the regression (λ_0) is zero.

It appears that for models with no default (models 1 and 2), the P/E ratio proxies for a bubble

Table 3 Summary Statistics for Model Performance

	Avg GCV	Avg RMSE	λ	se(λ)	1ydf	Avg Y values	R^2
Financials							
1—Bankers Trust NY (bt)							
Model 1	0.0027	0.0488				0.0048	0.4873
Model 2	0.0027	0.0490				0.0048	0.4901
Model 3	0.0027	0.0482	0.1851	0.0178	0.1686	0.0048	0.4967
Model 4	0.0028	0.0485	0.3244	0.0736	0.2683	0.0048	0.5028
Model 5	0.0028	0.0485	0.1929	0.0183	0.1758	0.0048	0.5025
Model 6	0.0029	0.0487	0.3531	0.0775	0.2920	0.0048	0.5105
Model 7	0.0028	0.0482	0.1400	0.0217	0.1316	0.0048	0.5208
Model 8	0.0029	0.0484	0.2968	0.0811	0.2519	0.0048	0.5284
2—Merrill Lynch & Co (mer)							
Model 1	0.0038	0.0585				0.0027	0.8292
Model 2	0.0037	0.0572				0.0027	0.8379
Model 3	0.0036	0.0564	0.2643	0.0166	0.2319	0.0027	0.8412
Model 4	0.0038	0.0568	0.3161	0.0929	0.2604	0.0027	0.8426
Model 5	0.0038	0.0567	0.2605	0.0171	0.2290	0.0027	0.8432
Model 6	0.0039	0.0573	0.2985	0.1001	0.2496	0.0027	0.8441
Model 7	0.0039	0.0566	0.2148	0.0217	0.1925	0.0027	0.8477
Model 8	0.0040	0.0572	0.2322	0.1062	0.1963	0.0027	0.8486
Food & Beverages							
3—Pepsico Inc (pep)							
Model 1	0.0026	0.0483				0.0018	0.5877
Model 2	0.0027	0.0487				0.0018	0.5889
Model 3	0.0027	0.0486	0.0841	0.0240	0.0799	0.0018	0.5887
Model 4	0.0028	0.0488	0.3186	0.1220	0.2696	0.0018	0.5936
Model 5	0.0028	0.0491	0.0850	0.0251	0.0805	0.0018	0.5894
Model 6	0.0029	0.0493	0.3349	0.1309	0.2812	0.0018	0.5944
Model 7	0.0029	0.0494	0.0742	0.0288	0.0696	0.0018	0.5956
Model 8	0.0030	0.0495	0.3506	0.1404	0.2930	0.0018	0.6023
4—Coca-Cola (cce)							
Model 1	0.0054	0.0693				0.0156	0.1675
Model 2	0.0056	0.0697				0.0156	0.1764
Model 3	0.0054	0.0689	0.2255	0.0640	0.1851	0.0156	0.1581
Model 4	0.0057	0.0694	0.2226	0.0660	0.1838	0.0156	0.1667
Model 5	0.0054	0.0679	0.2246	0.0631	0.1828	0.0156	0.2043
Model 6	0.0056	0.0684	0.2213	0.0651	0.1811	0.0156	0.2115
Model 7	0.0055	0.0674	0.3234	0.0697	0.2530	0.0156	0.2368
Model 8	0.0057	0.0678	0.3252	0.0719	0.2547	0.0156	0.2455

Table 3 Continued

	Avg GCV	Avg RMSE	λ	se(λ)	1ydf	Avg *Y* values	R^2
Airlines							
5—AMR Corporation (amr)							
Model 1	0.0044	0.0627				0.0015	0.3543
Model 2	0.0042	0.0607				0.0015	0.4100
Model 3	0.0046	0.0635	0.0000	0.0631	0.0000	0.0015	0.3543
Model 4	0.0044	0.0614	0.0000	0.0597	0.0000	0.0015	0.4100
Model 5	0.0047	0.0636	−0.0112	0.0654	−0.0131	0.0015	0.3669
Model 6	0.0045	0.0610	−0.0159	0.0610	−0.0186	0.0015	0.4331
Model 7	0.0047	0.0624	−0.1212	0.0717	−0.1407	0.0015	0.4082
Model 8	0.0043	0.0592	−0.1345	0.0657	−0.1595	0.0015	0.4797
6—Southwest Airlines Co (luv)							
Model 1	0.0113	0.1005				−0.0069	0.4663
Model 2	0.0116	0.1007				−0.0069	0.4796
Model 3	0.0114	0.0998	0.2486	0.0513	0.2160	−0.0069	0.4909
Model 4	0.0119	0.1005	0.4029	0.2547	0.3219	−0.0069	0.4947
Model 5	0.0111	0.0972	0.1798	0.0494	0.1524	−0.0069	0.5301
Model 6	0.0116	0.0981	0.1642	0.2591	0.1244	−0.0069	0.5329
Model 7	0.0116	0.0980	0.1468	0.0671	0.1176	−0.0069	0.5338
Model 8	0.0121	0.0989	0.1246	0.2835	0.0817	−0.0069	0.5370
Utilities							
7—Carolina Power + Light (cpl)							
Model 1	0.0017	0.0386				−0.0055	0.8275
Model 2	0.0017	0.0389				−0.0055	0.8294
Model 3	0.0017	0.0390	0.0015	0.0078	0.0015	−0.0055	0.8276
Model 4	0.0018	0.0393	0.0204	0.0233	0.0197	−0.0055	0.8301
Model 5	0.0017	0.0386	0.0121	0.0078	0.0133	−0.0055	0.8348
Model 6	0.0018	0.0388	0.0354	0.0231	0.0356	−0.0055	0.8367
Model 7	0.0018	0.0385	−0.0289	0.0100	−0.0286	−0.0055	0.8406
Model 8	0.0019	0.0388	−0.0040	0.0254	−0.0024	−0.0055	0.8416
8—Texas Utilities Ele Co (txu)							
Model 1	0.0017	0.0393				0.0024	0.2101
Model 2	0.0018	0.0393				0.0024	0.2278
Model 3	0.0016	0.0378	0.3137	0.0227	0.2688	0.0024	0.2715
Model 4	0.0017	0.0379	0.3038	0.0231	0.2615	0.0024	0.2847
Model 5	0.0015	0.0361	0.3096	0.0211	0.2673	0.0024	0.3513
Model 6	0.0016	0.0363	0.3021	0.0215	0.2618	0.0024	0.3594
Model 7	0.0015	0.0358	0.3406	0.0232	0.2888	0.0024	0.3783
Model 8	0.0016	0.0361	0.3336	0.0238	0.2837	0.0024	0.3835

Table 3 Continued

	Avg GCV	Avg RMSE	λ	se(λ)	1ydf	Avg Y values	R^2
Petroleum							
9—Mobil Corp (mob)							
Model 1	0.0010	0.0302				0.0115	0.3943
Model 2	0.0011	0.0305				0.0115	0.4077
Model 3	0.0011	0.0305	0.0208	0.0135	0.0201	0.0115	0.3918
Model 4	0.0011	0.0305	0.1189	0.0370	0.1028	0.0115	0.4104
Model 5	0.0011	0.0305	0.0212	0.0143	0.0216	0.0115	0.3991
Model 6	0.0011	0.0306	0.1043	0.0442	0.0910	0.0115	0.4165
Model 7	0.0011	0.0303	−0.0208	0.0158	−0.0212	0.0115	0.4294
Model 8	0.0011	0.0304	0.0577	0.0462	0.0457	0.0115	0.4420
Department Stores							
10—Sears Roebuck + Co (s)							
Model 1	0.0036	0.0571				0.0007	0.7250
Model 2	0.0038	0.0577				0.0007	0.7251
Model 3	0.0035	0.0556	0.2045	0.0132	0.1840	0.0007	0.7414
Model 4	0.0037	0.0562	0.2060	0.0137	0.1851	0.0007	0.7417
Model 5	0.0037	0.0559	0.1973	0.0136	0.1775	0.0007	0.7445
Model 6	0.0038	0.0565	0.1989	0.0141	0.1787	0.0007	0.7449
Model 7	0.0038	0.0560	0.1511	0.0183	0.1381	0.0007	0.7497
Model 8	0.0040	0.0567	0.1528	0.0190	0.1394	0.0007	0.7500
11—Wal-Mart Stores, Inc (wmt)							
Model 1	0.0036	0.0564				−0.0180	0.7309
Model 2	0.0034	0.0546				−0.0180	0.7770
Model 3	0.0035	0.0555	0.1722	0.0167	0.1577	−0.0180	0.7688
Model 4	0.0036	0.0551	0.0849	0.1055	0.0668	−0.0180	0.7805
Model 5	0.0037	0.0558	0.1772	0.0174	0.1625	−0.0180	0.7713
Model 6	0.0037	0.0555	0.0918	0.1082	0.0740	−0.0180	0.7824
Model 7	0.0038	0.0560	0.1878	0.0223	0.1700	−0.0180	0.7748
Model 8	0.0038	0.0557	0.0969	0.1158	0.0793	−0.0180	0.7851
Technology							
12—Eastman Kodak Company (ek)							
Model 1	0.0052	0.0683				0.0099	0.2610
Model 2	0.0050	0.0662				0.0099	0.3036
Model 3	0.0045	0.0627	0.4802	0.0262	0.3799	0.0099	0.3342
Model 4	0.0047	0.0630	0.4450	0.0365	0.3557	0.0099	0.3436
Model 5	0.0046	0.0626	0.5023	0.0265	0.3945	0.0099	0.3502
Model 6	0.0048	0.0629	0.4620	0.0367	0.3675	0.0099	0.3613

Table 3 Continued

	Avg GCV	Avg RMSE	λ	se(λ)	1ydf	Avg Y values	R^2
Model 7	0.0048	0.0630	0.5324	0.0328	0.4112	0.0099	0.3589
Model 8	0.0049	0.0631	0.4931	0.0431	0.3851	0.0099	0.3721
13—Xerox Corp (xrx)							
Model 1	0.0027	0.0486				0.0042	0.6048
Model 2	0.0028	0.0489				0.0042	0.6125
Model 3	0.0025	0.0467	0.2090	0.0199	0.1836	0.0042	0.6130
Model 4	0.0026	0.0468	0.2316	0.0214	0.2021	0.0042	0.6224
Model 5	0.0026	0.0469	0.2106	0.0205	0.1848	0.0042	0.6210
Model 6	0.0026	0.0468	0.2385	0.0221	0.2072	0.0042	0.6336
Model 7	0.0027	0.0471	0.1953	0.0244	0.1725	0.0042	0.6248
Model 8	0.0027	0.0471	0.2259	0.0261	0.1965	0.0042	0.6372
14—Texas Instruments (txn)							
Model 1	0.0096	0.0926				0.0089	0.5997
Model 2	0.0098	0.0923				0.0089	0.6083
Model 3	0.0088	0.0875	0.4585	0.0437	0.3635	0.0089	0.6291
Model 4	0.0090	0.0877	0.6052	0.0836	0.4322	0.0089	0.6350
Model 5	0.0091	0.0881	0.4489	0.0462	0.3579	0.0089	0.6331
Model 6	0.0094	0.0882	0.6048	0.0866	0.4332	0.0089	0.6397
Model 7	0.0091	0.0869	0.5750	0.0570	0.4300	0.0089	0.6487
Model 8	0.0094	0.0874	0.6790	0.0993	0.4727	0.0089	0.6528
15—International Business Machines (ibm)							
Model 1	0.0066	0.0769				0.0059	0.2812
Model 2	0.0069	0.0775				0.0059	0.2867
Model 3	0.0066	0.0757	0.3041	0.0505	0.2548	0.0059	0.3125
Model 4	0.0069	0.0764	0.2940	0.0522	0.2464	0.0059	0.3173
Model 5	0.0062	0.0725	0.3086	0.0468	0.2543	0.0059	0.3841
Model 6	0.0065	0.0732	0.2996	0.0484	0.2467	0.0059	0.3877
Model 7	0.0064	0.0727	0.3349	0.0551	0.2775	0.0059	0.3956
Model 8	0.0067	0.0734	0.3268	0.0570	0.2715	0.0059	0.3996
Average							
Model 1	0.0044	0.0597				0.0026	0.5018
Model 2	0.0045	0.0595				0.0026	0.5174
Model 3	0.0043	0.0584	0.2115	0.0301	0.1797	0.0026	0.5213
Model 4	0.0044	0.0586	0.2596	0.0710	0.2118	0.0026	0.5317
Model 5	0.0043	0.0580	0.2080	0.0302	0.1761	0.0026	0.5417
Model 6	0.0044	0.0581	0.2462	0.0732	0.2004	0.0026	0.5526

Table 3 Continued

	Avg GCV	Avg RMSE	λ	se(λ)	1ydf	Avg Y values	R^2
Model 7	0.0044	0.0579	0.2030	0.0360	0.1641	0.0026	0.5562
Model 8	0.0045	0.0580	0.2371	0.0803	0.1860	0.0026	0.5670

Given are the average generalized cross-validation statistics (GCV), the average root mean squared error (RMSE), the average default intensity (λ), the average standard error of the default intensity (se(λ)), the average 1-year default probability (1 ydf), the average value for the dependent variable in the equity model regression, and the average R^2. The number of observations per regression is 48. The averages are taken across all months from the equity model regressions. The number of regressions in the average is 23.

Table 4 Unit root test performance across all companies

	λ_0	λ_1	λ_2	β_0	β_1	β_2	β_3	β_4	β_5
Model 1				2/15	0/15	1/15	2/15	4/15	
Model 2				3/15	1/15	1/15	2/15	3/15	2/15
Model 3	3/15			2/15	1/15	0/15	1/15	4/15	
Model 4	2/15			3/15	1/15	0/15	0/15	3/15	2/15
Model 5	2/15	0/15		4/15	1/15	1/15	3/15	2/15	
Model 6	3/15	2/15		1/15	3/15	1/15	2/15	4/15	3/15
Model 7	1/15	0/15	2/15	3/15	4/15	1/15	1/15	2/15	
Model 8	3/15	1/15	2/15	2/15	4/15	1/15	2/15	2/15	3/15

The entries under the columns correspond to the number of companies for the relevant coefficient where the null hypothesis of a unit root is rejected at the 90% level. There are 15 total companies—tests for a unit root.

component in stock prices not contained in the four factors of Fama–French. But, for the models with default (models 3–8), the P/E ratio becomes insignificant. The inclusion of a constant in the regression model (λ_0) appears to confound the bubble component in stock prices.[15]

An additional test for a possible model misspecification with respect to the bubble component is provided by the t-score for the stock's own variance (β_4). This t-score tests for the null hypothesis that $\beta_4 = -1/2$, the theoretical value as given in expression (8). As indicated, for all models and for all but four firms (cce, amr, mob, and ibm), one can reject the null hypothesis that $\beta_4 = -1/2$. This rejection is strong evidence consistent with the stock's own variance proxying for stock price bubbles.

In summary, this section provides evidence consistent with the existence of an equity price bubble not captured in the Fama–French four-factor model. Both the P/E ratio and the stock's own variance appear to be significant explanatory variables in the equity return regression model.

9 Analysis of the default intensity

As mentioned earlier, the average default intensity parameters and t-scores are contained in Table 2. The firms' estimates are presented in industry groupings for easy comparison. First to be noticed is that the fit of the linear regressions are quite good. The average R^2 varies between 0.1581 (for cce, model 3) to 0.8486 (for mer, model 8). From Table 3, the average R^2 across all models varies from 0.5018 for model 1 to 0.5670 for model 8. R^2 uniformly increases across firms with increasing model complexity up to model 8. This is to be expected because R^2 is a measure of the in-sample fit, and as we progress from model 1 to model 8, more independent variables are added to the linear regression.

Second, it is interesting to examine the signs of the coefficients for the default intensity parameters. The signs of λ_1 and λ_2 indicate the sensitivity of the firm's default likelihood to changes in the spot rate and the cumulative excess return on the equity market index, respectively. For example, for wmt (Wal–Mart Stores) the sign of λ_1 is positive indicating that as interest rates rise, the likelihood of default increases. Continuing, the sign of λ_2 is negative, indicating that as the market index rises, the likelihood of default decreases. The signs of these coefficients differ across firms and between firms within an industry. An example of different signs within an industry is for the department stores grouping, where Sears Roebuck and Company (s) and Wal–Mart Stores, Inc. (wmt) have contrasting signs for both the interest rate and market index variables.

Next, we discuss the statistical significance of these point estimates. For λ_0, the point estimate is significantly different from zero in model 3 for seven firms (mer, txu, s, wmt, ek, xrx, txn, and ibm). The F-test for model 3 also tests the hypothesis that $\lambda_{0t} = 0$. This test confirms the average t-score results because the average p-scores are low (below 15% for five firms). Including the P/E ratio in model 4 eliminates the significance of the default parameter λ_0 for two of the seven firms (mer and wmt), indicating a possible confounding of the default parameter estimate with equity price bubbles. This suspicion is confirmed for more complex models 5–8. The pattern with respect to the significance of the default parameter λ_0 is similar to that previously discussed for models 3 and 4.

With respect to the spot rate coefficient, λ_1, the significance of its t-scores varies across firms and model types. For four out of the 15 firms, the average t-score is significantly different from zero for at least one of models 5–8. This observation is also supported by the F-test for model 5 (the joint hypothesis $\lambda_{0t} = \lambda_{1t} = 0$). The average p-scores for this F-test are less than 20% for five firms (luv, txu, ek, txn, and ibm). This evidence strongly supports the inclusion of the interest rate variable in the default intensity model.

Finally, with respect to the market index coefficient, λ_2, the average t-scores indicate that it is insignificant from zero for all firms except one (amr). Unfortunately, the introduction of the independent variable $(Z(t)(T_L - t) - Z(t - \Delta)(T_L - (t - \Delta)))$ into this regression causes a severe multi-collinearity problem with another independent variable in the equity risk premium, the excess return on the market portfolio. As seen in models 7 and 8, for all firms the introduction of $(Z(t)(T_L - t) - Z(t - \Delta)(T_L - (t - \Delta)))$ causes the point estimates of β_0 to change dramatically from their values in models 1–6 and β_0 becomes insignificantly different from zero. The correlation matrix in Table 5 confirms this multi-collinearity problem. The correlation between these two variables is 0.9934. This implies that the estimates of the coefficients λ_2 and β_0 cannot be separated using this regression model. This evidence is consistent with a confounding of the default probability estimates with those of the equity's risk premium.

As seen from Table 3, the impact of these different default intensity models on the point estimates of the default intensities can be dramatic. For example, for mer the average default intensity varies from -0.0569 in model 7 to 0.3534 for model 6. Negative default intensities should be interpreted as being a point estimate of zero. Similar patterns can also be observed for the other firms in our sample. This sensitivity is consistent with a confounding of

Table 5 Correlation matrix for selected independent variables in the equity model regression

	λ_1	λ_2	β_0	β_1	β_2	β_3
λ_1	1.0000	0.4685	0.4580	−0.4072	0.2607	0.2896
λ_2	0.4685	1.0000	0.9934	−0.1277	−0.3539	0.1141
β_0	0.4580	0.9934	1.0000	−0.1449	−0.3765	0.1060
β_1	−0.4072	−0.1277	−0.1449	1.0000	−0.5153	0.0850
β_2	0.2607	−0.3539	−0.3765	−0.5153	1.0000	−0.0267
β_3	0.2896	0.1141	0.1060	0.0850	−0.0267	1.0000

the default probability model with the equity risk premium and bubble component.

Also documented in Table 3 are the magnitudes of these default probability estimates. As indicated, they are quite high relative to historical default frequencies. Indeed, the average default intensity across all firms and all models exceeds 0.20, whereas the magnitude of the average historical default intensities observed from bankruptcy data is less than 0.01 (see Chava and Jarrow, 2002). Although this difference could be due the fact that these estimates obtained are the risk-neutral probabilities, as opposed to the statistical probabilities, more likely the difference is due to a confounding of the default intensity model with both the equity risk premium and price bubble. As previously highlighted, many of the preceding test results are consistent with this second interpretation.

Also to be noticed at this juncture are the magnitudes of the RMSE of the regression model in comparison to the average magnitude of the dependent variable. The RMSE is an estimate of the standard error of the unpredictable component of equity returns. The average magnitude of the dependent variable is the average equity return over this period. As indicated in Table 3, the standard error of the unpredictable component of the equity's return is over 10 times the magnitude of its average value. This is true for all firms in our sample. This illustrates the magnitude of the "noise" in equity prices relative to our predictive ability using the Fama–French four-factor model. This noise inhibits our ability to estimate the default intensities with a great deal of precision.

In summary, an analysis of the default intensity parameters in the equity return regression model shows that although it is feasible to estimate the likelihood of default, the estimated intensity process parameters are confounded by the equity risk premium and price bubble component. Both default risk and the equity's risk premium appear to be positively correlated. Including both variables in a regression yields an upward bias in the estimated default probability (relative to historical default frequencies).

10 Relative performance of the equity return models

This section studies the relative performance of the eight equity return models. For each firm and for each model's regression, both a root mean squared error statistic (RMSE) and a generalized cross-validation statistic (GCV) are computed. The RMSE statistic measures the "average" pricing error between the model and the market price. It is an in-sample goodness-of-fit measure. As with all in-sample goodness of fit measures, a potential problem with RMSE is that it may provide a biased picture of the quality of model performance due to a model over fitting the noise in the data. The second GCV test statistic is designed to partially overcome this problem, as an out-of-sample goodness-of-fit measure that is predictive in nature.[16] The lower the GCV statistic, the better the out-of-sample model fit.

The average RMSE and GCV statistics for each firm and model are contained in Table 6. For 13 of the 15 firms, the RMSE statistic is smallest (or within 0.0001 of the smallest value) for models 3–6. For all 15 firms the GCV statistic is smallest (or within 0.0001 of the smallest value) for models 3–6. This is strong evidence consistent with the importance of including the default parameters into the equity return model and the feasibility of using equity returns to infer default intensity estimates.

Surprisingly, for no firms except one (txu) do models 7 and 8 have the smallest GCV statistics. Despite the multi-collinearity problem present when including $Z(t)$ into the default intensity process, this is strong evidence consistent with the insignificance of the λ_2 coefficient. This relative performance analysis confirms the insignificance of the λ_2 coefficient documented in Janosi *et al.* (2002) for the same firms, but using debt prices.

In summary, in terms of the GCV statistic, the best-fitting models are 3–6. Given the previous evidence with respect to the significance of the interest rate variable in the default intensity process, the preferred equity return models are probably 5 and 6.

11 Comparison of default intensities based on debt versus equity

This section investigates the equality between the default intensities estimated using the equity returns with the default intensities as estimated in Janosi *et al.* (2002). Using the identical structure, the identical data, and the same time period as employed above, Janosi *et al.* (2002) estimate the expected loss per unit time $\lambda(t)(1-\delta)$ implicit in debt prices, using a reduced-form credit risk model. Here, $0 \leq \delta \leq 1$ corresponds to the recovery rate on defaulted debt.

They selected 20 different firms, 15 of which overlap with this study (see Table 1). Janosi *et al.* (2002) estimated five different liquidity premium models. We use only the best-fitting model, the constant liquidity premium. To match the best-fitting equity pricing model (model 6), we use the intensity function from Janosi *et al.* (2002) without the market index included. For this credit risk model, we have the monthly time series of the estimated expected loss per unit time $\lambda(t)(1-\delta)$ for each of the 15 companies from Janosi *et al.* (2002) and their standard errors. Unfortunately, due to non-overlapping periods of observations in the two studies, only 10 firms are included in this comparison. The five firms omitted are: amr, bt, cce, cpl, and ek.

Available are estimated intensities $\lambda_e^i(t)$ from the equity returns for firm i in month t, and estimated expected losses (per unit time) $a_d^i(t) = \lambda_d^i(t)(1-\delta^i)$ from Janosi *et al.* (2002) for firm i in month t. The null hypothesis to be tested is

$$\text{(debt)}\ \ a_d^i(t)/(1-\delta^i) = \lambda_e^i(t)\ \ \text{(equity)}$$
$$\text{for all firms } i \text{ and all months } t.$$

Given an assumed value for the recovery rate δ^i, we can do a pair-wise t-test (using the standard errors of the estimates) of the difference $[a_d^i(t)/(1-\delta^i)] - \lambda_e^i(t)$ for a given firm i for each month t. Under the null hypothesis this difference is zero. Unfortunately, this is a joint test of the null hypothesis and the assumed value of δ^i. To eliminate this joint hypothesis, we test for equality of these differences

Table 6 Test for the equivalence between the default intensities based on debt prices versus equity prices

	η_0	η_1	Recovery range	Avg($\lambda_d(t)$)	Avg($\lambda_e(t)$)
amr	N/A	N/A	N/A	N/A	N/A
	N/A	N/A			
bt	N/A	N/A	N/A	N/A	N/A
	N/A	N/A			
Cce	N/A	N/A	N/A	N/A	N/A
	N/A	N/A			
cpl	N/A	N/A	N/A	N/A	N/A
	N/A	N/A			
Ek	N/A	N/A	N/A	N/A	N/A
	N/A	N/A			
Ibm	−1.3253	30.6004	None	0.0103	0.2996
	−1.8008	2.1955			
luv	−4.3092	84.5898	None	0.0104	0.1642
	−5.0623	5.2471			
mer	−2.5844	35.6537	[0.99, 1.00]	0.0100	0.2985
	−2.0105	1.4645			
mob	−1.7100	29.2491	[0.99, 1.00]	0.0036	0.2311
	−1.8682	1.7266			
pep	0.5209	−8.6911	[0.95, 1.00]	0.0082	0.3349
	0.8286	−0.7300			
S	0.4483	−7.0506	[0.88, 1.00]	0.0086	0.2058
	0.6984	−0.5852			
txn	3.0442	−50.7298	[0.94, 1.00]	0.0083	0.6048
	1.7638	−1.5520			
txu	0.5267	−7.1674	[0.93, 1.00]	0.0061	0.3021
	0.9931	−0.7136			
wmt	0.5775	−9.3437	[0.00, 1.00]	0.0036	0.0918
	0.5621	−0.4802			
xrx	1.4597	−27.3956	[0.96, 1.00]	0.0056	0.2385
	1.1763	−1.1658			

In the fourth and fifth columns, the average default intensities are provided. The debt estimates are for a zero recovery rate. The equity estimates are for the midpoint of the recovery rate from column 3. The equity estimation is from May 31, 1995 to March 31, 1997 for all the companies. The debt estimation time period is contained in Table 1. This table represents the intersecting dates from both experiments. For the first three columns, we estimate $\lambda^i_{\text{equity}}(t) - \lambda^i_{\text{debt}}(t) = \eta^i_0 + \eta^i_1 r(t) + \varepsilon_t$. We report the range of recoveries that makes $H_0 : \eta^i_0 = 0$ and $\eta^i_1 = 0$ insignificant at the 95% level. The first number is the point estimate, the second is the t-statistic. Only ibm and luv are rejected for all possible δ.

over a range of different values for δ^i from 0 to 1. If there is a value for δ^i where the null hypothesis $[\tilde{a}_d^i(t)/(1-\delta^i)] - \lambda_e^i(t) = 0$ is not rejected, then this value for δ^i is an estimate of the recovery rate. The relevant summary statistics for these tests are contained in Table 6.

For all firms but two (ibm and luv), there is some recovery rate such that the two estimates can be viewed as equivalent. This is strong evidence consistent with equality of the implicit estimation procedures across both equity and debt prices. However, the recovery rate needed to obtain equality of the two estimates is in excess of 88%, for all firms except one (wmt). This recovery rate is higher than the average recovery rate of 67% contained in Moody's (1992) for senior secured debt over the time period 1974–1991. This overestimate of the recovery rate suggests an upward bias in the estimated default rates obtained from equity returns, confirming the conclusions from the previous analyses.

12 Conclusions

This paper uses a reduced-form model to estimate default probabilities implicit in equity returns. The model implemented is a generalization of the model contained in Jarrow (2001). The time period covered is May 1991–March 1997. Monthly equity returns on 15 different firms are studied. The firms are chosen to provide a stratified sample across various industry groupings.

Three general conclusions can be drawn from this investigation. First, equity returns can be used to infer a firm's default intensities. This is a feasibility result. Two, equity returns appear to contain a bubble component, as proxied by the firm's P/E ratio. Bubbles in equity returns are not completely captured by the four-factor model of Fama and French (1993, 1996). Third, due to this imprecision in modeling equity risk premia, the point estimates of the default intensities confound with the equity risk premium. Estimated default probabilities using equity returns are larger than those obtained based on either historical bankruptcy data or implicitly using debt prices. This conclusion casts doubts upon the reliability of the default probability estimates obtained from equity prices using structural models as in Delianedis and Geske (1998) (confirming the previous conclusions of Jarrow and van Deventer, 1998, 1999, and Jarrow *et al.*, 2002, in this regard). This is also consistent with the inability of structural models using equity price information to explain credit spreads in corporate debt (see Collin-Dufresne *et al.*, 2001; Huang and Huang, 2002; Eom *et al.*, 2002).

Notes

1 The intensity process is defined under the risk neutral probability.
2 This assumption is easily relaxed, see Jarrow (2001).
3 One could assume that these dividends could be defaulted on as well, see Jarrow (2001).
4 See *Money Magazine* April 1999, p. 169 for Yahoo's P/E ratio of 1176.6.
5 This is a simple no arbitrage restriction that the present value of the sum of multiple cash flows equals the sum of the present values of the cash flows.
6 For the explicit equations, see Jarrow (2001).
7 This is in contrast to the typical situation where there are multiple debt issues outstanding that can be utilized to implicitly estimate default intensities when using debt prices.
8 The appendix contains a minor correction to the formula contained in Jarrow (2001).
9 The address is: http://web.mit.edu/kfrenc/www/data_library.html.
10 We perform a Dickey–Fuller (DF) test. The DF test statistic is the t-statistic for the ρ coefficient in the regression: $y_t - y_{t-1} = \mu + \rho y_{t-1} + \varepsilon_t$ where μ, ρ are constants and ε_t is an error term. The null hypothesis of a unit root for y_t is $\rho = 0$. A rejection of the null hypothesis implies that there is no unit root. The unit root test statistics are: $\sigma_r(-2.6348)$ and $a_r(-1.1632)$.

[11] The unit root test statistics are: σ_m (−3.9407) and φ (−1.3479).

[12] The exact formula for this computation is in Jarrow (2001).

[13] The t-score is adjusted to reflect the fact that the regressions contain overlapping time intervals, see Janosi *et al.* (2002) for more details on the adjustment.

[14] This is not to be confused with the number of observations used in the time t regression for a particular firm. At the time t regression, we use 48 months of data.

[15] This should not be surprising. In the standard implementation of the CAPM, the constant term in the regression equation is called the "alpha" and it is used to represent abnormal returns. If the bubble component is not adequately modeled, its time series variation would appear in the estimate of this coefficient.

[16] Roughly speaking, the GCV statistics measures the average predictive error obtained by systematically eliminating each data point from the time series regression, predicting that point's value with the regression, and then measuring the "average" predictive error that results, after adjusting for degrees of freedom (see Wahba, 1985).

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Appendix

From the appendix in Jarrow (2001), we have that:

$$\begin{aligned}
\log\left(\frac{\xi_t - \sum_{j\geq t}^{T_D} D_j v(t,j\colon e)}{\xi_{t-\Delta} - \sum_{j\geq t-\Delta}^{T_D^*} D_j v(t-\Delta,j\colon e)}\right) &= \log\left(\frac{L_t}{L_{t-\Delta}}\right) + \int_{t-\Delta}^{t} \mu_\theta(u)\,\mathrm{d}u + \lambda_0\Delta \\
&\quad - \lambda_1\left[-\log\left(\frac{p(t,T_L)}{p(t-\Delta,T_L)}\right) - b(t-\Delta,T_L)^2\Delta/2\right] \\
&\quad - \lambda_2[Z(t)(T_L-t) - Z(t-\Delta)(T_L-t+\Delta)] \\
&\quad - \lambda_1^2 b(t-\Delta,T_L)^2\Delta/2 - \lambda_2^2(T_L-t)^2\Delta/2 \\
&\qquad + \lambda_1\sigma_L\varphi_{rL}b(t-\Delta,T_L)\Delta \\
&\quad + \lambda_2\sigma_L\varphi_{mL}(T_L-t)\Delta
\end{aligned}$$

In the previous expression, the following quantities are unobservable: $\varphi_{rL}, \varphi_{mL}$. To eliminate these quantities from this expression, we compute the variance of the preceding expression.

$$\begin{aligned}
\sigma_\xi^2(t)\Delta &\equiv \mathrm{var}_t\left[\log\left(\frac{\left[\xi(t) - \sum_{j\geq t}^{T_D} D_j v(t,j\colon e)\right]}{\left[\xi(t-\Delta) - \sum_{j\geq t-\Delta}^{T_D} D_j v(t-\Delta,j\colon e)\right]}\right)\right] \\
&= \mathrm{var}_t\left[\log\left(\frac{L_t}{L_{t-\Delta}}\right) - \lambda_1\left[-\log\left(\frac{p(t,T_L)}{p(t-\Delta,T_L)}\right) - b(t-\Delta,T_L)^2\Delta/2\right]\right. \\
&\qquad \left. -\lambda_2[Z(t)(T_L-t) - Z(t-\Delta)(T_L-t+\Delta)]\right]
\end{aligned}$$

But, we have that:

$$\begin{aligned}
\left(-\left(\frac{b(t-\Delta,T_L)^2}{2}\right)\Delta - \log\left(\frac{p(t,T_L)}{p(t-\Delta,T_L)}\right)\right) &= \mu_1(t,T_L) - \mu_1(t-\Delta,T_L) + \mathrm{DET} \\
&= [b(t,T_L)r(t) - b(t-\Delta,T_L)r(t-\Delta)]/\sigma_r + \mathrm{DET} \\
&= b(t,T_L)[r(t) - r(t-\Delta)]/\sigma_r + \mathrm{DET}
\end{aligned}$$

where DET indicates non-random terms. Also note that

$$r(t) - r(t-\Delta) = \sigma_r \int_{t-\Delta}^{t} \mathrm{e}^{-a(\nu-t)}\,\mathrm{d}W(\nu) + \mathrm{DET}$$

Substitution gives

$$\left(-\left(\frac{b(t-\Delta, T_L)^2}{2}\right)\Delta - \log\left(\frac{p(t, T_L)}{p(t-\Delta, T_L)}\right)\right) = \left(b(t, T_L)\int_{t-\Delta}^{t} \mathrm{e}^{-a(\nu-t)}\,\mathrm{d}W(\nu)\right) + \mathrm{DET}$$

Combined we get that:

$$\sigma_\xi^2(t)\Delta = \mathrm{var}_t\left[\sigma_L[w_L(t) - w_L(t-\Delta)] - \lambda_1\left(b(t, T_L)\int_{t-\Delta}^{t} \mathrm{e}^{-a(\nu-t)}\,\mathrm{d}W(\nu)\right)\right.$$
$$\left. -\lambda_2(T_L - t)[Z(t) - Z(t-\Delta)] + \mathrm{DET}\right]$$

Computing this variance yields:

$$\sigma_\xi^2(t)\Delta \approx \lambda_1^2 b(t, T_L)^2\Delta + \lambda_2^2(T_L - t)^2\Delta + \sigma_L^2\Delta$$
$$- 2\lambda_1 b(t, T_L)\sigma_L\varphi_{rL}\Delta - 2\lambda_2\sigma_L\varphi_{mL}(T_L - t)\Delta + 2\lambda_1\lambda_2\varphi_{rm} b(t, T_L)(T_L - t)\Delta$$

where we have used the facts that:

$$\mathrm{var}_t\left(\int_{t-\Delta}^{t} \mathrm{e}^{-a(\nu-t)}\,\mathrm{d}W(\nu)\right) = \int_{t-\Delta}^{t} \mathrm{e}^{-2a(\nu-t)}\,\mathrm{d}\nu = (1 - \mathrm{e}^{2a\Delta})/2a \approx \Delta$$

and

$$\int_{t-\Delta}^{t} \mathrm{e}^{-a(\nu-t)}\,\mathrm{d}\nu = (1 - \mathrm{e}^{a\Delta})/a \approx \Delta.$$

Rearranging the terms gives:

$$-\sigma_\xi^2(t)\Delta/2 + \sigma_L^2\Delta/2 + \lambda_1\lambda_2\varphi_{rm} b(t, T_L)(T_L - t)\Delta \approx -\lambda_1^2 b(t, T_L)^2\Delta/2 - \lambda_2^2(T_L - t)^2\Delta/2$$
$$+ \lambda_1 b(t, T_L)\sigma_L\varphi_{rL}\Delta + \lambda_2\sigma_L\varphi_{mL}(T_L - t)\Delta$$

Substitution gives the result:

$$\log\left(\frac{\xi_t - \sum_{j\geq t}^{T_D} D_j\nu(t, j\colon e)}{\xi_{t-\Delta} - \sum_{j\geq t-\Delta}^{T_D} D_j\nu(t-\Delta, j\colon e)}\right) = \log\left(\frac{L_t}{L_{t-\Delta}}\right) + \int_{t-\Delta}^{t} \mu_\theta(u)\,\mathrm{d}u + \lambda_0\Delta$$
$$+ \lambda_1\left[\log\left(\frac{p(t, T_L)}{p(t-\Delta, T_L)}\right) + b(t-\Delta, T_L)^2\Delta/2\right]$$
$$- \lambda_2[Z(t)(T_L - t) - Z(t-\Delta)(T_L - t + \Delta)]$$
$$- \sigma_\xi^2(t)\Delta/2 + \sigma_L^2\Delta/2 + \lambda_1\lambda_2\varphi_{rm} b(t, T_L)(T_L - t)\Delta$$

Using Girsanov's theorem, we have that the original Brownian motion: $w_L(t) = \hat{w}_L(t) + \int_0^t \Theta_L(u)\,\mathrm{d}u$ where $\hat{w}_L(t)$ is a Brownian motion under the statistical measure and $\Theta_L(u)$ is the liquidation value's risk premium. Hence,

$$\log(L_t/L_{t-\Delta}) = r(t-\Delta)\Delta + \sigma_L\Theta_L(t-\Delta)\Delta - (1/2)\sigma_L^2\Delta + \sigma_L[w_L(t) - w_L(t-\Delta)]$$

Thus, we have the final result:

$$\begin{aligned}
&\log\left(\frac{\xi_t - \sum_{j\geq t}^{T_D^*} D_j\nu(t, j\colon e)}{\xi_{t-\Delta} - \sum_{j\geq t-\Delta}^{T_D^*} D_j\nu(t-\Delta, j\colon e)}\right) - r(t-\Delta)\Delta \\
&= \sigma_L\Theta_L(t-\Delta)\Delta + \mu_\theta(t-\Delta)\Delta - \sigma_\xi^2(t)\Delta/2 \\
&\quad + \lambda_0\Delta + \lambda_1\left[\log\left(\frac{p(t, T_L)}{p(t-\Delta, T_L)}\right) + b(t-\Delta, T_L)^2\Delta/2\right] \\
&\quad - \lambda_2[Z(t)(T_L - t) - Z(t-\Delta)(T_L - t + \Delta)] \\
&\quad + \lambda_1\lambda_2\varphi_{rm}b(t, T_L)(T_L - t)\Delta \\
&\quad + \sigma_L[w_L(t) - w_L(t-\Delta)]
\end{aligned}$$