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THE FULLY-ANTICIPATED *P/E* PROMISE AND ITS REALIZATION*

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In this paper, time paths of P/Es are projected, by applying a theoretical model in which the totality of "fully-anticipated" future "franchise" investments serve as the source of higher P/Es. At the outset, the P/E path slowly ascends until the first franchise opportunity is reached and funded. The actual act of funding transforms the "anticipated" Franchise Value potential into "realized" value-equivalent earnings. This equivalence does not change the price in the P/E numerator but, the P/E ratio declines because the new earnings flow into the ratio's denominator. The P/E decline bottoms out at a level based on new going-forward franchise investment potential and then rises until the next franchise event. In summary, this saw tooth pattern should not be surprising. If we view a firm's Franchise Value as a promise of future productive growth, then realization should lead to on-going Franchise Value declines—unless previously unexpected sources of future growth are uncovered. And, in actual markets, it is just such "unexpected" events and prospects—of both a positive and a negative nature—that intermittently bubble to the surface. While admittedly based on a highly theoretical model with the limiting assumption of perfect foresight, this specter of a downward gravitational P/E pull should act as a cautionary note for analyses that project an existing high P/E forward onto a higher earnings level assumed to be reached at some future point in time.

The Price/Earnings (P/E) ratio can be viewed as an indication of the extent to which a firm's estimated value is based on future business/investment prospects versus current earnings.

In this paper, P/Es and their time paths are studied from the vantage point of franchise investments. A potential future investment is termed a "franchise investment" if its expected return (%) exceeds the cost of capital. Such franchise opportunities reflect the firm's competitive

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advantages—patents, licenses, market presence, branding, etc.—that enable it to obtain excess returns.

A firm's Franchise Value (FV) is the discounted value of all future franchise investment opportunities. The present study is based on a theoretical model in which all such future investments are assumed to be "fully anticipated" and thus can serve as the source of higher P/Es. While no simple model can capture all the complexities of actual P/Es, this FV approach can be used to explore some of the theoretical determinants of the P/E time path over the long term.

At the outset, the initial P/E climbs as the first franchise opportunity is approached. At a franchise point, the associated investment generates an incremental stream of earnings having the same net PV as the franchise opportunity itself. Thus, the firm's total value is maintained because the franchise investment's depleted value is exactly replaced by the incremental earnings.

This act of funding can be viewed as a transformation of a franchise opportunity's potential value into a realization comprising value-equivalent earnings. Because of this equivalence, the price in the P/E numerator will not be changed, but the P/E ratio will decline as new earnings flow into the ratio's denominator. This P/E decline will then bottom out at a level based on the new going-forward FV. The P/E will then once again rise until the next franchise point.

Thus, given the model's fundamental assumption of a fully-anticipated finite sequence of franchise opportunities, this franchise conversion process will lead the P/Es to follow a generally descending saw tooth pattern.

It should be noted that, even as the P/E declines, the ever-higher earnings will support the firm's price, enabling it to follow a more continuous—and generally higher—path than the P/E ratio. In the context of this framework, a general formula is presented in the Appendix that relates each year's P/E change to a specific investment event and the "consumption" of the associated franchise.

The P/E's descending saw tooth pattern over the long term should not be surprising if we think of the FV as a promise of future productive growth. As these promises are realized, they are consumed, and the on-going FV declines—unless previously unexpected sources of future growth are uncovered. (And of course, in the actual markets, it is just such "unexpected" events and prospects—of both a positive and a negative nature—that continually bubble to the surface.)

While admittedly based on a highly theoretical model with the limiting assumption of full anticipation, the specter of such in a downward gravitational pull should act as a cautionary note for analyses that project an existing high P/E forward onto a higher earnings level assumed to be reached at some future point in time.

1 Literature review

There is a huge literature on equity valuation that ranges across many authors from the early works of John Burr Williams (1938), Molodovsky (1953), Miller and Modigliani (1961), Gordon (1962), Modigliani and Cohen (1979), Jacobs and Levy (1988), Asness (1997), Ohlson and Zhang (1998), and Campbell and Shiller (1998), to the more recent studies by Straehl and Ibbotson (2017), Siegel (2017), Damodaran (2017), Fabozzi *et al.* (2017), and Leibowitz and Kogelman (2019).

Most of this valuation literature has tended to focus on ratios based on the book value. There has been relatively little discussion of how the key market metric—the P/E ratio—behaves over time in the context of a specific valuation model.

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In the present paper we make use of a "Franchise Value" model to address more generalized patterns of earnings growth. In this model, the P/E is derived from a fully-anticipated sequence of franchise investments.

2 Growth models for a hypothetical firm

The goal of this paper is to better understand the relationship between earnings growth projections and the corresponding implied P/E path. Typically, such projections are accompanied by the simplifying assumption of a stable P/E, i.e., the terminal P/E and initial P/E are assumed equal. But, this P/E constancy is unlikely to be realized unless earnings, dividends and price all grow at the same rate.

In order to provide illustrative examples of the theoretical relationship between earnings growth and P/E growth, we first focus on the hypothetical growth firm delineated in Exhibit 1.

If Firm A retains 64% of annual earnings and then reinvests at r, the classic Dividend Discount Model (DDM) implies that earnings will grow at a steady rate g_E. The DDM assumes that b percent of earnings are retained and reinvested at r, thereby producing an additional stream of earnings $r(bE)$. From these assumptions, earnings growth is $r(bE)/E$ and

$$g_E = br = 64\% \quad \text{of } 8.18\% = 5.24\%.$$

From the above assumptions, it follows that dividends and price also grow at the earnings rate. The initial dividend is $(1-b)E$ and the DDM price is the present value of the growing stream of dividends. According to the DDM price formula (illustrated below), Firm A's price is \$100.

$$\begin{aligned} P &= (1-b)E/(k-g_E) \\ &= (1-64\%) \times \$4.90/(7\%-5.24\%) \\ &= \$100. \end{aligned}$$

The P/E is found by dividing the price by the earnings:

$$\begin{aligned} P/E &= (1-b)/(k-g_E) \\ &= (1-64\%)/(7\%-5.24\%) = 20.4. \end{aligned}$$

In subsequent years, the P/E remains at 20.4 because both price and earnings grow at 5.24%. In fact, as shown in Exhibit 2, the P/E is very sensitive to the earnings' growth assumption.

In the case when the investment return r equals the market rate k, there are no premium investments.

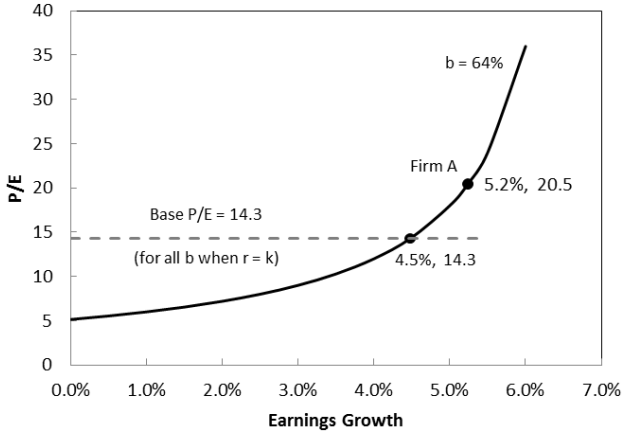
$$\begin{aligned} g_E &= bk = 64\% \text{ of } 7.0\% = 4.48\%. \\ P/E &= (1-b)/(k-g_E) \\ &= (1-64\%)/(7\%-4.48\%) = 14.3. \end{aligned}$$

In this special case, substitution of bk for rg_E in the above P/E formula shows that P/E is

Exhibit 1. Hypothetical growth Firm A.

Current annual earnings	E	\$4.90
Earnings retention	b	64%
Equity capitalization rate	k	7%
Price/Earnings ratio	P/E	20.4
Return on investment	r	8.18%

Exhibit 2. P/E versus earnings growth rate ($b = 64\%$, $k = 7\%$).



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independent of b and

$$P/E = 1/k = 1/7\% = 14.3.$$

We refer to $1/k$ as the base P/E against which the full P/E can be compared. For example, Firm A with a P/E of 20.4 is said to be valued at a P/E premium of 6.1 (20.4–14.3) relative to the base P/E.

3 Earnings surge and *P/E* cliff

The stable P/E assumption discussed in the previous section applies only to retention-based growth. However, the premium P/E of 20.4, 6.1 units higher than the base P/E, can be attained without the smooth, retention-based growth of Firm A.

Consider, for example, Firm B which has the same initial earnings, retention and capitalization rate as Firm A, but with varying growth rates. In years 1–4, Firm B's earnings grow at only 4.48% because the return on retained earnings is 7% ($g = bk =$ 64% of 7% $=$ 4.48%). In year 5, Firm B experiences a 53.7% earnings surge (Exhibits 3 and 4) before returning to perpetual 4.48% growth.

Exhibit 3. Firm B: Projected 53.7% earnings growth surge in year 5.

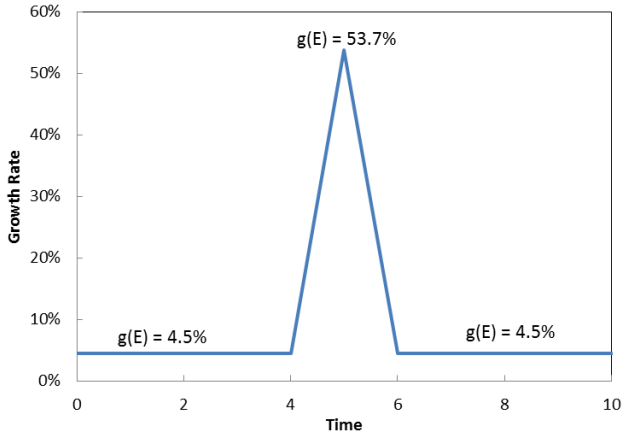


Exhibit 4. Firm B: Earnings and price paths with growth surge in year 5.

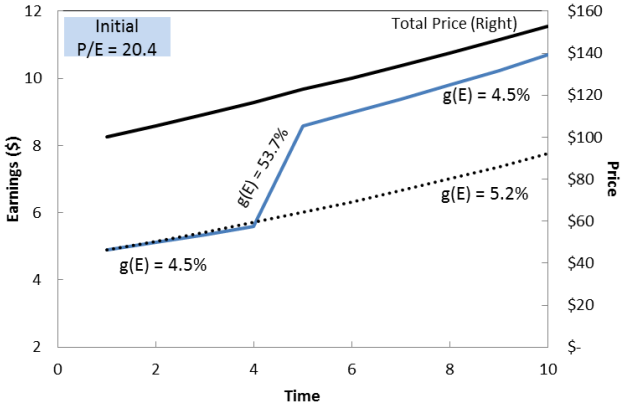
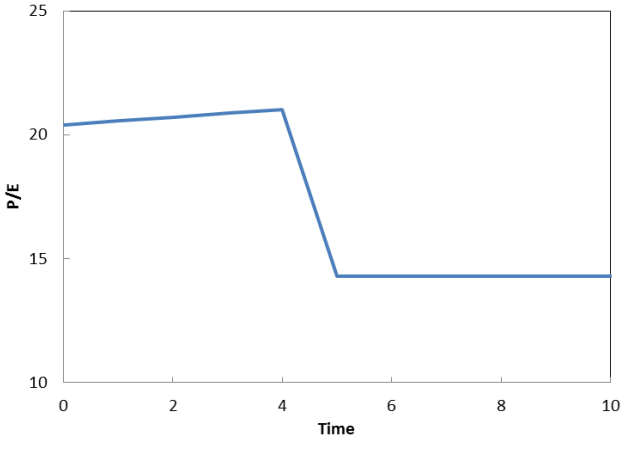


Exhibit 4 shows Firm B's earnings path. If the growth surge had been anticipated at the outset, then the value of that surge would have been embedded in the initial price. Consequently, both before and after the surge, the price rises at a fairly steady rate. For reference, the exhibit also shows the steadily rising earnings associated with the constant 5.2% growth of Firm A.

Exhibit 5 shows that the P/E grows slowly from its initial 20.4 until the growth surge point at which earnings jump and the P/E falls to the base P/E. In contrast, Exhibit 4 shows that the

Exhibit 5. Firm B: A P/E cliff follows the growth surge.



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price continues to grow at the same rate as before the surge. This relative price stability in the face of the earnings surge results in a precipitous P/E decline. The details of the driving forces of the earnings growth and corresponding P/E cliff are provided in the following sections of this paper.

4 Franchise Value

We now turn to understanding the drivers that might have fueled the projected earnings growth surge in Exhibit 3. While there are many possibilities, one simple explanation is that Firm B is expected to make a substantial new investment in year 4 that, in years 5 and beyond, will produce returns in excess of the cost of capital. Such an investment might have been made possible by the firm's unique market position, pricing power, patents, licenses, etc., that is, because of the firm's "Franchise Value."

More generally, Franchise Value (FV) is based on positive net earnings from anticipated but as yet unrealized investments. In Exhibit 3, for example, the growth spurt was driven by an investment returning 11.33%, 4.33% higher than the 7% cost of capital.

Exhibit 6 presents the present values of a hypothetical set of discrete year 4 investments possibilities. While high return opportunities are naturally preferred, they tend to be scarcer. This is reflected in the size of the opportunity decreasing as the return increases. Investments with returns less than the cost of capital do not add to the value of the firm, so the corresponding bars in Exhibit 6 are shaded in gray.

The funding for future investments with returns in excess of the cost of capital may derive from a variety of sources such as retained earnings or equity or bond issuance. If there is ample funding, all investments implied in Exhibit 6 would be pursued. However, if funding is limited, higher return investments would always be chosen first.

Exhibit 6. Present values of a hypothetical set of year 4 investments (with $k = 7\%$).

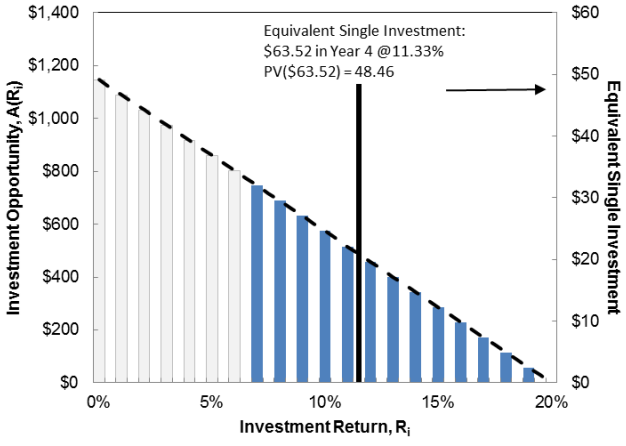

Exhibit 6 also shows that a hypothetical single investment of $63.52 in year 4 with a gross return of 11.33% in years 5 and beyond will result in precisely the same initial firm value (and P/E) as the entire set of investments in Exhibit 6. And, this $63.52 investment would drive the growth surge in Exhibit 3.

If the earnings generated by this investment begin in year 5 and continue indefinitely, then

$$\begin{aligned}\text{Gross revenue} &= \$63.52 \times 11.33\% \\ &= \$7.20 \text{ in years } 5, 6, \ldots\end{aligned}$$

The net revenue is found by deducting the cost of capital:

$$\begin{aligned}\text{Net revenue} &= \text{gross revenue} - \text{cost of capital} \\ &= \$7.20 - 7\% \text{ of } \$63.52 = \$2.75.\end{aligned}$$

These gross and net revenue streams are illustrated in Exhibit 7.

At year 4, just a moment before the investment is made, FV(4) is the net present value (NPV) of the $2.75 perpetual net revenue (earnings), calculated

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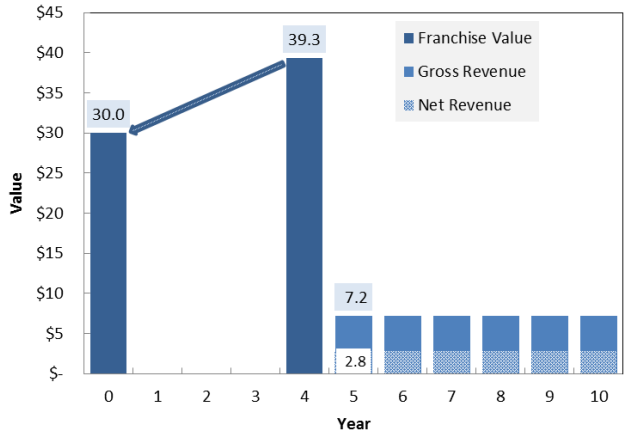
Exhibit 7. A \$63.52 year 4 investment yields \$2.75 in net revenue & \$30.0 in FV (or NPV).



by dividing \$2.75 by k.

$$\text{FV}(4) = \$2.75/7\% = \$39.28.$$

The contribution of FV(4) to the firm's current value is the present value of FV(4), calculated by discounting FV(4) back 4 years to time 0:

$$\text{PV of } \$39.28 = \$39.28/(1+7\%)^4 = \$30.00.$$

5 Core value, Franchise Value and the *P/E* orbit

To this point, we have focused only on investment-driven future earnings and total FV. In addition to FV, the total price must include the value of current earnings, assuming that those earnings continue in perpetuity. This Core Value (CV) is the present value of that earnings stream, calculated as E/k.

Since Firms A and B both have \$4.90 in current earnings, they have the same CV:

$$\text{CV} = E/k = \$4.90/7\% = \$70.$$

Since Firm B's FV is \$30, its

$$\text{Total value} = \text{CV} + \text{FV} = \$70 + \$30 = \$100.$$

In the case of Firm A with 5.2% earnings growth, the price was shown to be \$100. Thus, with a CV of \$70, the implicit FV embedded in the DDM's retention/reinvestment model must also equal \$30.

We now focus on Firm B and the implicit time path of its CV, FV, Price, and P/E. Since no investments are made in the first 4 years, there is no franchise consumption and FV increases at the capitalization rate of 7%/year, until the point at which the first investment is made (Exhibit 8).

Exhibit 9 illustrates how the \$63.52 investment in year 4 (discussed earlier) effectively "consumes" the firm's Franchise Value so, in years 5 and beyond, net future earnings are converted into actual earnings which become part of the CV. Exhibit 10 shows that, since price is the sum of CV and FV, the price and CV coincide once all FV is consumed.

Figure 11 shows how FV consumption and increased earnings lead to a lower P/E—the P/E cliff—depicted earlier in Exhibit 5. FV consumption implies that the proportion of FV in the price declines and, as a consequence, the P/E premium decreases.

$$P = \text{CV} + \text{FV} = E/k + \text{FV}, \quad \text{and}$$

$$P/E = \text{CV}/E + \text{FV}/E = \text{Base}P/E + \text{FV}/E$$

Exhibit 8. Firm B Franchise Value grows at 7% until first franchise investment.

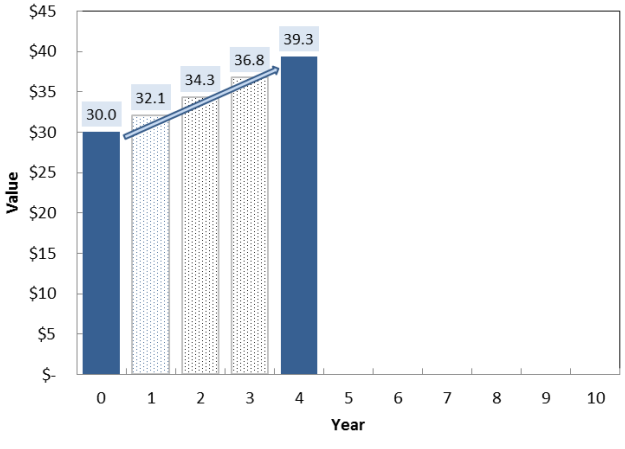


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Exhibit 9. Time paths of total value (price), Core Value, and Franchise Value.

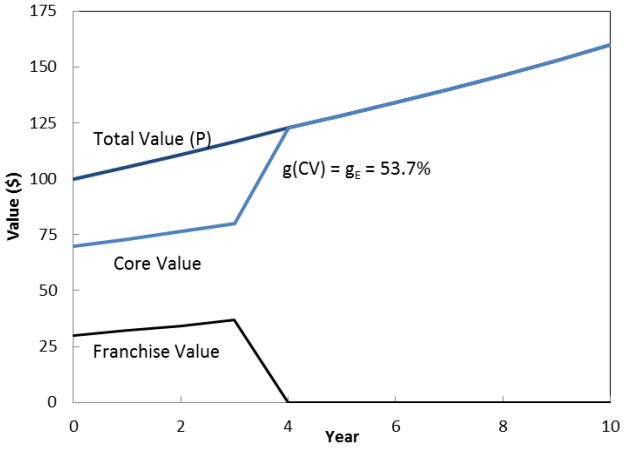


Exhibit 10. Conversion of Franchise Value into Core Value.

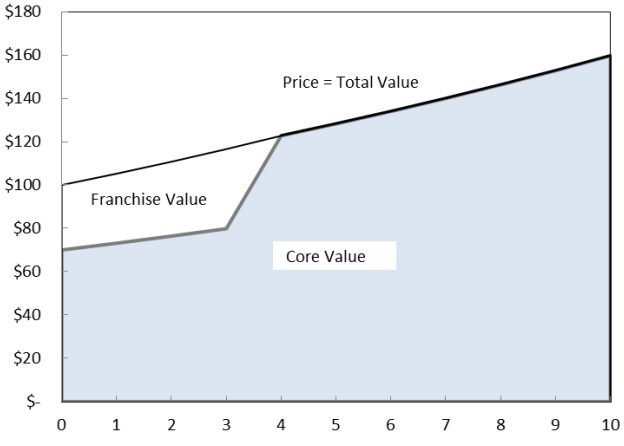
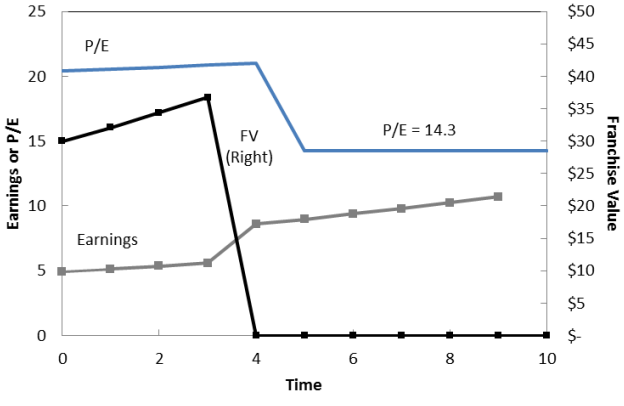


Exhibit 11. Franchise consumption and P/E decline.



Since the Base P/E is always $1/k$, when FV goes to zero the P/E approaches $1/k = 14.3$.

6 Growth surges and *P/E* paths

Earnings growth forecasts typically represent "headline numbers" but provide limited details behind that forecast. In this section, we study the investments, returns and P/E implications from a third firm, C, with the hypothetical 3-year growth surge illustrated in Exhibit 12 and the corresponding earnings growth in Exhibit 13.

At the outset, Firm C has the same characteristics as Firm A (Exhibit 1). However, instead of earnings growing at 5.2% forever, C makes premium

Exhibit 12. Firm C: Projected 15.4% rate of earnings growth in years 5–8.

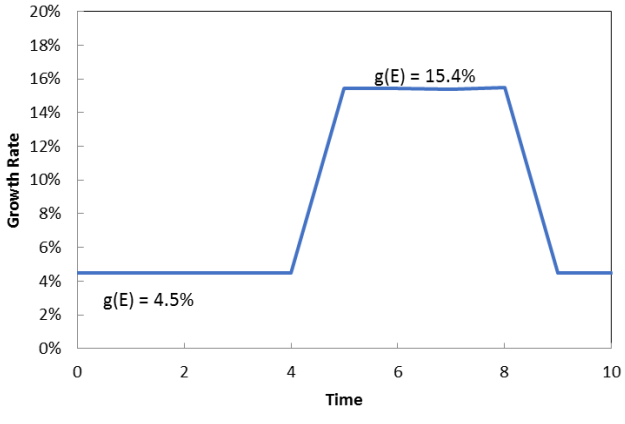


Exhibit 13. The earnings growth path of Firm C.

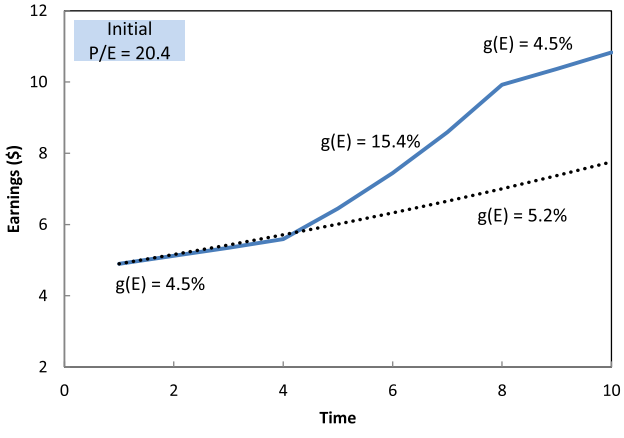


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Exhibit 14. Annual investments leading to Firm C's 15.4% earnings growth in years 5–8.

Time	$l(t)$	Gross $E(t+1)$ $l(t) \times r(\$)$	Cost of capital $l(t) \times k$	Net $E(t+1)$	FV(t) = NPV(t) Net $E(t+1)/k$	FV(0) PV(FV(t))
0	0.00					
1	0.00	–	–	–		
2	0.00	–	–	–	–	
3	0.00	–	–	–	–	
4	51.72	–	–	–	8.73	6.66
5	59.83	4.23	3.62	0.61	10.10	7.20
6	68.83	4.89	4.19	0.71	11.61	7.74
7	79.96	5.63	4.82	0.81	13.49	8.40
8	0.00	6.54	5.60	0.94		
					Total FV(0)	$ 30.00

investments in years 5–8 which earn 8.2% and fuel 15.4% earnings growth. These four investments consume all of C's FV, so in years 8 and beyond new investments are only able to earn k, and baseline 4.5% growth continues thereafter.

Exhibit 14 shows the series of investments that comprise the FV. For example, an investment of \$51.72 in year 4 provides gross earnings of \$4.23 in years 5 and beyond. The cost of capital for these earnings is \$3.62, so the net earnings is \$0.61/year. The NPV of these perpetual earnings at time 4 (which is the same as FV(4)), is $\$0.61/7\% = \$8.73$. The firm's total FV of \$30 is the sum of the present values of the Franchise Values in years 5–7.

Turning to the franchise component of the P/E, we note that, at the outset, the Franchise P/E is

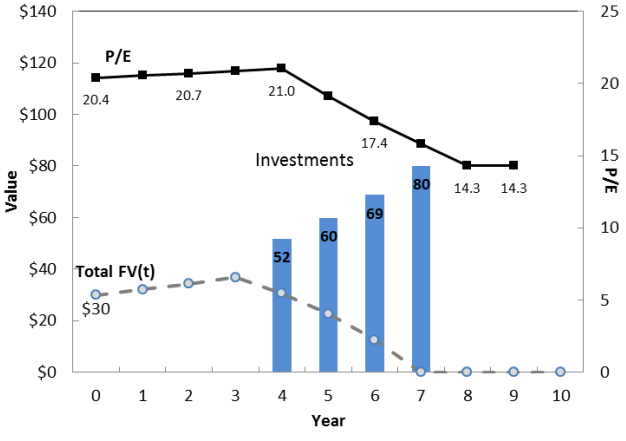
$$\text{FV}/E = \$30/\$4.90 = 6.1.$$

Since the Base P/E is always 14.3,

$$P/E = 14.3 + 6.1 = 20.4.$$

In the following years, when no FV is consumed, Exhibit 15 shows the P/E rising along with FV. As anticipated investments are made, both FV and P/E (with a 1-year delay) fall. Finally, when FV is fully consumed, there is only CV and the P/E remains at base P/E.

Exhibit 15. The P/E time path.



7 Two-step franchise consumption

In previous sections, the focus was on specific earnings growth rates (in the case of Firms B and C) and the sequence of investments required to achieve such rates. We now turn to Firm D which has the same initial characteristics as Firm A but with an FV that is based on investments in years 4 and 7. Earnings grow at the base rate of 4.5% and FV grows at 7% until year 4 when a \$30

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Exhibit 16. Earnings and Franchise Value with two discrete investments.

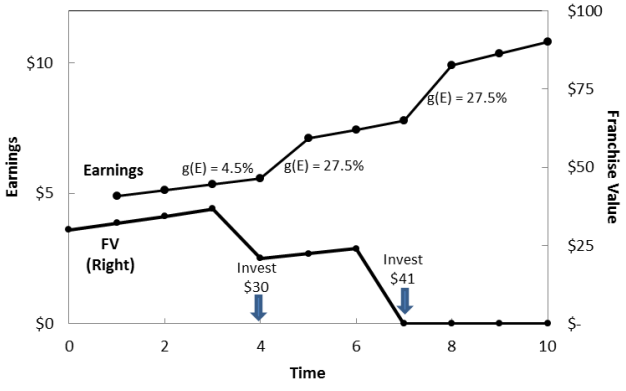
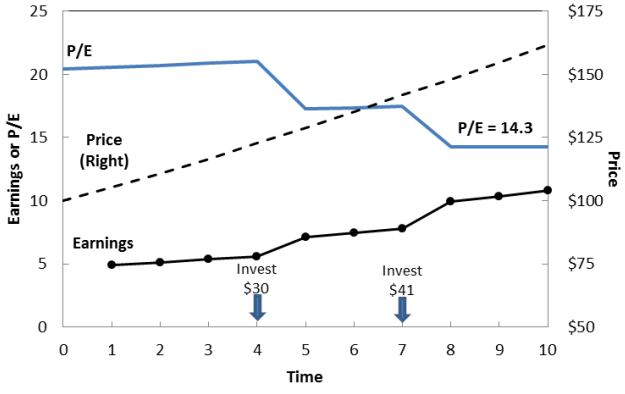


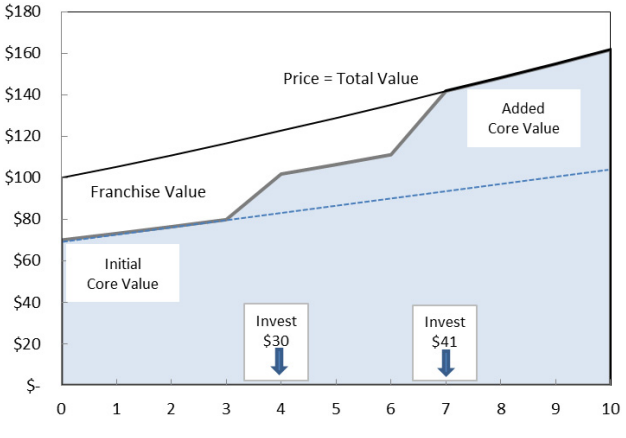
Exhibit 18. Earnings growth and *P/E* decline with two franchise investments.



investment is projected to provide a gross return of 11.3%. Exhibit 16 shows the resulting 27.5% earnings growth surge and a corresponding 43% drop in FV.

In years 5–7, earnings and FV again grow at 4.5% and 7%, respectively. Then, one final $41 investment in year 7 with an 11.3% gross return produces another one-time 27.5% earnings growth spike. However, in comparison to year 4, a much larger investment ($41 versus $30) is required to produce the same growth surge because the earnings base is higher. This final investment converts all the remaining FV to CV, as shown in Exhibit 17.

Exhibit 17. Conversion of Core Value into Franchise Value.



After year 5, since there is no remaining FV, earnings, CV, and price grow forever at the 4.5% rate.

Exhibit 18 shows the stepwise decline in *P/E* that follows Franchise consumption and earnings growth.

8 Conclusions

The theoretical Franchise Value model used in this paper provides a theoretical perspective on how high *P/E*s might evolve over the long term.

The model assumes that the current price reflects both the value of current earnings and the fully-anticipated net present value of all value-additive franchise investments. At the outset, this anticipation enables the *P/E* to rise until the time when the first franchise opportunity becomes available. At this point, the franchise investment is implemented, and what had previously been a franchise opportunity is now transformed into incremental earnings.

This new stream of earnings has the same net present value as the franchise opportunity itself. Thus, the firm's total value is maintained since the franchise opportunity's depleted value is exactly replaced by the new value-equivalent earnings. Because of this equivalence, the price in the *P/E*

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numerator will not change, but the P/E ratio will decline as the new earnings flow into the P/E ratio's denominator.

This P/E decline will bottom out at a level based on the new going-forward investment opportunities, and the P/E will then once again begin to rise towards the next franchise event.

As more time passes and more investments are realized, there may be fewer remaining franchise opportunities, even as the earnings level continues to grow. Thus, ever larger and more productive new investments would be needed to lift or even just maintain the P/E level.

Given the model's limiting assumption of a fully-anticipated finite sequence of franchise opportunities, this franchise conversion process will lead to a P/E time path that follows a generally descending saw tooth pattern.

On the one hand, this model suggests that fully-anticipated P/Es are vulnerable to a gravitational downward pull over the long term.

On the other hand, this gravitational pull can be countered by previously-unanticipated sources of earnings growth. And, in fact, the market actually does seem to provide ample new "beyond-model" opportunities for such earnings growth. After all, this is how high P/Es are formed in the first place.

Appendix

The Franchise Value model

The FV model distinguishes between Core Value, derived from a firm's perpetualized current earnings, E, and Franchise Value, based on future investment-driven earnings. The firm's total value, P, is the sum of these two value factors.

$$P = \text{Core Value} + \text{Franchise Value}.$$

The present value of the perpetual earnings, E, is E/k where k is the capitalization rate. So, $\text{CV} = E/k$ and

$$P = E/k + \text{FV}.$$

The FV is based on the present value of projected net earnings from future investments. If G is the present value of all future investments with average return r, the net annual Franchise Earnings,

$$\text{FE} = (r - k)G.$$

Since FV is the present value of a stream of constant net earnings:

$$\text{FV} = \text{FE}/k.$$

Price earnings ratio and base *P/E*

$$P/E = \text{CV}/E + \text{FV}/E$$
$$= 1/k + FV/E.$$

The CV-based P/E of $1/k$, is referred to as the Base P/E.

$$\text{Base } P/E \equiv 1/k$$

If $\text{FV} > 0$,

$$P/E \text{ Premium} = P/E - 1/k$$
$$P/E \text{ Premium} = \text{FV}/E.$$

Earnings yield and return

In theory, all stocks with a given risk profile should be priced so that their annual return is equal to the market rate k. The price formula in the previous section now can be used to derive an expression for the annual return based on the Earnings Yield (E/P) and an incremental growth factor g^*.

$$P = E/k + \text{FV}$$
$$= (1/k)(E + k\text{FV})$$
$$k = E/P + k\text{FV}/P$$

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If we define

$$g^* \equiv k\frac{FV}{P}, \quad \text{then}$$
$$k = E/P + g^*.$$

The above formula shows that the earnings yield (EY) alone is likely to underestimate the return unless the firm is viewed as having absolutely no future premium investment opportunities. In that "no-investment" case, $g^* = 0$.

For firms with projected franchise investments, the return estimate can be improved by adding the growth correction g^* to the EY. Similarly, estimates of the Equity Risk Premium based solely on the spread between EY and the applicable interest rate may be improved by applying the same g^* correction.

The Dividend Discount Model and g^*

In the special case of the Dividend Discount Model (DDM), we can derive a more explicit formula for g^*.

The DDM-based price is the present value of an infinite dividend stream, assuming b percent of annual earnings (bE dollars) are always retained and reinvested at a return $r \geq k$. The corresponding incremental earnings each year are $r(bE)$ so that $g = r(bE)/E = rb$. The initial dividend is $(1-b)E$ and subsequent dividend payouts also grow at g.

The DDM price is given by,

$$P = \frac{(1-b)E}{k-g}, \quad \textit{and}$$
$$P/E = \frac{(1-b)}{k-g}.$$
$$EY = \frac{k-g}{1-b}.$$

In the previous section, we showed that

$$g^* = k - E/P, \quad \textit{so}$$
$$g^* = k - \frac{k-g}{1-b}$$
$$= \frac{k(1-b)-(k-g)}{1-b}$$
$$= \frac{-bk+br}{1-b} = \frac{b(r-k)}{1-b}.$$

The above expression implies that $g^* = 0$ if and only if there is either no retention ($b = 0$) or there is no excess return from new investments so $r = k$.

The *P/E* growth approximation

In this section, we focus on estimating the path of P/E growth or decay. To do so, we utilize a simple P/E growth approximation. The first step in deriving this approximation is to represent the price as the product of Earnings and P/E,

$$P = E \times P/E.$$

Then, by taking the differential of both sides of the above equation and dividing by P, we find that

$$g_{P/E} \cong g_P - g_E.$$

In the special case of the DDM, as shown in the last section, both price and earnings grow $g = br$, so

$$g_{P/E} = 0.$$

In the general FV model, price growth generally does not equal earnings growth. From the price formula developed earlier in this Appendix, it is easily shown that

$$g_P = (CV/P)g_E + (FV/P)g_{FV}$$
$$g_{P/E} \cong g_P - g_E$$
$$= (CV/P - 1)g_E + (FV/P)g_{FV}$$

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$$g_{P/E} = (FV/P)(g_{FV} - g_E)$$
$$= k(FV/P)\left(\frac{g_{FV}}{k} - \frac{g_E}{k}\right)$$
$$= g^*\left(\frac{g_{FV} - g_E}{k}\right).$$

Franchise consumption and the *P/E* Path

In the FV model, the first franchise investment is assumed to occur at least 1 year from the initial valuation date. More generally, if the first franchise investment is delayed t years, FV grows at k as the discounting period decreases.

$g_{FV} = k$ over any year in which no investment is made.

In "delayed franchise" years, there are no premium investments and the initial earnings grow at rate

$$g = bk.$$

On substituting the above growth rate in the P/E growth formula (from the previous section), we obtain a simple formula:

$$g_{P/E} = g^*\left(\frac{g_{FV} - g_E}{k}\right)$$
$$= \frac{g^*(k - bk)}{k} = g^*(1 - b).$$

We now turn to the active franchise investment years, beginning in year t. Let

$$\delta = \textit{franchise value of investment at time } t,$$

Then, FV, E, CV, P at time $t+1$ are computed as follows:

$$\mathrm{FV}(t+1) = \mathrm{FV}(t)(1+k) - \delta$$
$$= \mathrm{FV}(t) + k\mathrm{FV}(t) - \delta$$
$$E(t+1) = E(t)(1+g) + k\delta$$
$$\mathrm{CV}(t+1) = \frac{E(t+1)}{k} = (1+g)\mathrm{CV}(t) + \delta$$
$$P(t+1) = \mathrm{CV}(t+1) + \mathrm{FV}(t+1)$$
$$= (1+g)CV(t) + \delta + \mathrm{FV}(t)$$
$$+ k\mathrm{FV}(t) - \delta$$
$$P(t+1) = P(t) + k\mathrm{FV}(t) + g\mathrm{CV}(t).$$

From the above formulas, we can compute earnings growth g_E, and price growth g_P.

$$g_E = \frac{E(t+1)}{E(t)} - 1$$
$$= \frac{k\delta}{E(t)} + g = \frac{k\delta}{P(t)} \times P/E(t) + g$$
$$g_P = \frac{P(t+1)}{P(t)} - 1 = \frac{k\mathrm{FV}(t)}{P(t)} + \frac{g\mathrm{CV}(t)}{P(t)}$$
$$= g^* + g\left\{1 - \frac{\mathrm{FV}(t)}{P(t)}\right\} = g^* + g\left\{1 - \frac{g^*}{k}\right\}$$
$$= g^*\left\{1 - \frac{g}{k}\right\} + g.$$

Now, by using the above formulas for g_E and g_P, we obtain a formula for the change in the P/E ratio that accompanies a franchise investment.

$$g_{P/E} \cong g_P - g_E$$
$$= g^*\left\{1 - \frac{g}{k}\right\} + g - \frac{k\delta}{P(t)} \times P/E(t) - g$$
$$= g^*(1 - b) - \frac{k\delta}{P(t)} \times P/E(t).$$

Since $\delta = 0$ at any time when there is no investment, the preceding formula can be viewed as generally applicable for any period.

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References

Asness, C. S. (1997). "The Interaction of Value and Momentum Strategies," *Financial Analysts Journal* **53**(2).

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Campbell, J. Y. and Shiller, R. J. "Valuation Ratios and the Long-Run Stock Market Outlook," *The Journal of Portfolio Management Winter* **24**(2), 1998.

Damodaran, A. *Equity Risk Premiums (ERP): Determinants, Estimation and Implications—The 2017 Edition*. Stern School of Business, 2017.

Fabozzi, F. J., Focardi, S. M. and Jonas, C. (2017). "Equity Valuation: Science, Art, or Craft?" *CFA Research Foundation Books* **2017**(4).

Gordon, M. (1962). *The Investment, Financing, and Valuation of the Corporation* (Homewood, IL: Irwin).

Jacobs, B. I. and Levy, K. N. (1988). "On the Value of 'Value'," *Financial Analysts Journal* **44**(4).

Leibowitz, M. L. and Kogelman, S. (2019). "P/E Ratios, Risk Premiums, and the g* Adjustment," *The Journal of Portfolio Management* **45**(4).

Miller, M. H. and Modigliani, F. (1961). "Dividend Policy, Growth, and the Valuation of Shares," *The Journal of Business* **34**(4).

Modigliani, F. and Cohn, R. (1979). "Inflation, Rational Valuation, and the Market," *Financial Analysts Journal* **35**(2).

Molodovsky, N. (1953). "A Theory of Price-Earnings Ratios," *The Analysts Journal*.

Ohlson, J. A. and Zhang, X. (1998). "Accrual Accounting and Equity Valuation," *Journal of Accounting Research* **36**.

Straehl, P. U. and Ibbotson, R. G. (2017). "The Long-Run Drivers of Stock Returns: Total Payouts and the Real Economy," *Financial Analysts Journal* **73**(3).

Siegel, L. B. (2017). "The Equity Risk Premium: A Contextual Literature Review," *Research Foundation Literature Reviews* **12**(1).

Williams, J. B. (1938). *The Theory of Investment Value* (Harvard University Press).

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