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GREAT MOMENTS IN FINANCIAL ECONOMICS: III. SHORT-SALES AND STOCK PRICES

*Mark Rubinstein**

This is the third in a series of articles on Great Moments in Financial Economics to appear in the Journal. For each, the purpose is to trace, as well as the author can, the history of the development of an important idea. In this case, the idea, usually associated with the work of Edward Miller, is that in the real-world of investors with (1) homogeneous beliefs and of markets with (2) significant barriers to short-sales, stocks for which these barriers are binding will have prices at least temporarily too high relative to an alternative economy in which these two features are not both simultaneously present. In the early twenty-first century this idea has experienced a sort of renaissance of research which has been used to explain a large number of phenomena which can otherwise appear anomalous, from the momentum factor in stock returns, to market crashes, to the recent Internet-based stock market "bubble," and to the growing popularity of certain hedge fund strategies.

Academic literature on short-sales and stock prices has had a renaissance in the early twenty-first century. This literature argues that several examples of apparently anomalous security price behavior may at their root simply be attributed to the failure to appreciate the full implications of constraints on short-sales. Here is a list of some of these apparent anomalies:

- overpricing of near-expiration warrants;
- initial overpricing of IPOs;
- "Monday effect";
- momentum factor observed in stock returns;
- evidence that stocks with high turnover have lower returns;
- high beta stocks seem to have lower returns than the original CAPM would predict;
- closed-end fund discounts;
- failure of realized returns to match aggregate investor expectations;
- predictions of differential stock returns as a function of their short-interest;
- ability of stock breadth of ownership to predict future stock returns;
- relationship between dispersion of analysts' earnings forecasts and future stock returns;
- excess volatility of stock returns;

*The author is the Paul Stephens Professor of Applied Investment Analysis at the University of California at Berkeley.

- post-WWII tendency of the US stock market to experience many more large single-day declines than single-day rises;
- tendency of market "crashes" to occur without sufficiently motivating fundamental news;
- tendency of stocks to become more highly correlated during significant declines than would be indicated from standard models of their joint probability distributions;
- tendency of realized stock index return distributions as well as implied index option distributions to be negatively skewed; and
- aspects of the recent Internet-based stock market "bubble."

To recapitulate these arguments with an eye to their historical development we start with the CAPM of Treynor (1962), Sharpe (1964), Lintner (1965), and Mossin (1966), the standard finance paradigm for market equilibrium under uncertainty. That model has one key assumption that will here concern us: beliefs about the future joint return distributions of all securities are the same for all investors. In this context, as is well known from the portfolio separation property of that model: if all investors have the same beliefs and a riskless security exists, then every investor divides his wealth between cash and a single mutual fund, the "market portfolio," which contains all securities in the market.

Arrow (1965) has proven, in a very general risk aversion setting, that every investor will want to invest at least part of his wealth in positive amounts in a favorable gamble. The market portfolio then being a favorable gamble (since its equilibrium expected return exceeds the riskless return), all investors will have a long position in the market. Restrictions on short-sales would then not be binding since no investor would go short the market even if he could (and, therefore, any risky security). Investors, lend, borrow, and buy risky securities, but none short. However, once one allows for different beliefs, then, intuitively, investors who are sufficiently pessimistic about the returns of certain securities, other things equal, may want to short these securities, with the other side of their transaction taken up by comparatively optimistic investors.

We can divide attempts to create a more realistic model, where investors end up choosing different portfolios of risky securities, into three categories: (1) *heterogeneous belief* theories without barriers like short-selling restrictions or trading costs to exchanging securities; (2) *market segmentation theories* with homogeneous beliefs but where at least some investors are simply precluded from holding, exchanging, or short-selling some securities, or can only do so at considerable cost; (3) *combined theories* with both heterogeneous beliefs and some form of market segmentation.[1]

A particular version of the third category will be our focus in which heterogeneous beliefs and short-selling restrictions will interact to create pricing effects that can potentially mimic a number of real life observations that might otherwise seem anomalous. However, it will be helpful if we first briefly look at results related to categories (1) and (2), where these two generalizations—heterogeneous beliefs and forms of market segmentation—are considered separately.

Rubinstein (1974) and Kraus and Litzenberger (1975) develop, perhaps, the simplest rational competitive equilibrium model embodying heterogeneous beliefs. They assume that each investor maximizes the expected logarithmic utility of his future wealth by selecting state-by-state claims to future wealth constrained only by his initial wealth. They show that if all investors start out with the same wealth, current prices are set as if the probability of occurrence for each state were a simple arithmetic average of the probabilities to that state across all investors. With this one can ask: to what extent does heterogeneous beliefs across investors affect equilibrium prices? In particular, does increasing dispersion of beliefs across different investors have any systematic effect on security prices? To isolate a pure dispersion effect, suppose for a given state we consider a "mean-preserving spread" of beliefs across different investors. Such a change leaves the arithmetic average (mean) belief the same. For example, suppose there are just two investors A and B, and for a given state they attach probabilities $p^A = 0.3$ and $p^B = 0.5$. The average belief $p^M = (0.3 + 0.5)/2 = 0.4$. With logarithmic utility, the only aspect of the dispersed beliefs that affects prices is this average. An example of an increase in belief dispersion with a mean-preserving spread would be a change in beliefs to $p^A = 0.1$ and $p^B = 0.7$, thus preserving the mean of $(0.1 + 0.7)/2 = 0.4$. Trivially, for our example, it is only the mean that matters; pure belief dispersion has no effect. *So we conclude that in the case of logarithmic utility, moving from homogeneous to heterogeneous beliefs has no effect on current prices since the only property of investor beliefs that affects prices is their mean across investors for each state.*

However, Varian (1985) shows that logarithmic utility is a knife-edge case (as it is in so many other situations). For example, if we suppose more generally that all investors have utility functions with constant proportional risk aversion (of which logarithmic utility is but a special case), for economies in which investors are more (less) risk averse than logarithmic utility, pure increases in the dispersion of beliefs will tend to reduce (increase) current security prices. Varian concludes that since several pieces of empirical evidence suggest investors are more risk averse than logarithmic utility, pure increases in dispersion of beliefs will tend to reduce current security prices.[2]

Turning to our second category of models, those with market segmentation but homogeneous beliefs, perhaps the most elaborately developed segmentation model is that of Merton (1987), in his presidential address to the American Finance Association. Merton uses a model very similar to Levy (1978), in which different investors are exogenously assumed to be restricted to just a few securities, with each investor potentially constrained to allocate his wealth among a different and possibly small subset of all available securities. Different investors know about different subsets of securities, but all investors who know about a security agree about the key parameters describing its return distribution. As a result, Merton's model fits into the second category of segmentation with homogeneous beliefs. Among his many results, Merton confirms the simpler earlier segmentation models that prices vary with the size of the market. In particular, other things equal, firms with larger exposure to the common factor (similar to beta), larger stock-specific residual variance, larger market value, and fewer investors will tend to have lower current prices and higher expected returns. Perhaps his most interesting result is to point out that it is not size measured by the proportion of the market value of the firm to total market wealth that counts, but rather *the market value of the firm relative to the aggregate wealth of the investors who consider investing in that firm* (p. 495). The simple intuition underlying these results is that if the typical investor must confine himself to just a few securities instead of the "market portfolio," the investor will benefit less from diversification and place a lower current value on the risky securities he owns. Similarly, if fewer investors must hold the stock of the same firm, then its risk is less easily reduced through diversification and, therefore, investors are willing to pay less for the stock of that firm.

So, to summarize the results from models in categories (1) and (2):

(a) increases in pure belief dispersion should tend to lower security prices;

(b) increases in pure market segmentation should tend to reduce the prices of stocks that have, as a result of the segmentation, a smaller number of investors relative to their market values.

As we shall see, once heterogeneous beliefs and segmentation created by short-selling constraints are combined together in the same model, these conclusions are reversed, giving rise to the next Great Moment in Financial Economics.

1 Edward Miller, John Burr Williams, and John Lintner

Perhaps the most intriguing form of segmentation derives from short-selling constraints. These constraints are particularly interesting because (1) they are bound up with heterogeneous beliefs (since as mentioned in the original CAPM, with homogeneous beliefs these constraints are not binding); (2) each investor chooses the extent to which he will be bound by them (unlike the pure segmentation models mentioned above); and (3) several features of real-world US markets erect barriers to short-selling:

1. To short-sell, a lender of the shares must be found and motivated to temporarily part with his shares. The proceeds of the short-sale are held as collateral to help protect the stock lender. The interest rate on that collateral that is returned to the short-seller is called the "rebate rate." To motivate the lender to loan his shares, the rebate rate is lower than the full interest rate that is earned on the collateral. So, the lender earns the difference between that interest rate and the rebate rate. In times of very restricted supply, the rebate rate can even be negative so the short-seller not only earns no interest on the short-sale proceeds but pays an additional fee to the lender.
2. To understand the lack of symmetry from loss of interest on short-sales, consider a "short-sale against the box": being long and short the same stock simultaneously. If the investor does not receive any interest on the proceeds of the short-sale, even though he must invest his own funds to buy the long side, this position has a zero payoff. If the short-sale were symmetric, the investor should be able to earn the riskless return on his investment (which would have occurred had he earned the interest on the proceeds of the short-sale).
3. If, after the short-sale occurs, the broker can no longer find a willing lender to continue the short-sale, the short-seller may be forced to cover prematurely.
4. Short-selling is potentially vulnerable to another investor who succeeds in intentionally monopolizing the floating supply of stock in a "short squeeze."
5. Investors cannot short-sell after a down-tick or after a zero-down tick.
6. If the stock price jumps up more than 100% before a short-seller can close out his position, the short-seller will experience a greater loss compared to the long-buyer who suddenly experiences the largest possible stock price decline (of 100% due to limited liability). However, shorting by buying put options does not have this drawback.[3]
7. All profits and losses from short-sales are treated relatively unfavorably as short-term capital gains irrespective of the short-sale holding period.
8. Many institutional investors, particularly mutual funds, are contractually precluded from short-selling and mutual fund shares themselves cannot be sold short.

For our next Great Moment in Financial Economics, asymmetric restrictions such as these led Edward M. Miller in his 1977 article, "Risk, Uncertainty, and Divergence of Opinion," to postulate that relatively pessimistic investors will often not register their opinions in the market since they will find short-selling quite costly. On the other hand, the same reservations do not stop an optimistic investor from going long. As a result, particularly when there is substantial divergence of opinion about a stock, and short-sales are difficult, Miller argues that current stock prices will tend to reflect only the more optimistic information since the negative opinions of would-be short-sellers never make it into the stock price. Hayek's (1945) model of information pooling fails, and the current price of such stocks becomes higher than the price they would have in a market in which all available information is reflected in the price.

A very similar observation was made about 40 years earlier by John Burr Williams (1938):

> In multiple stock markets, each stock will be held only by those who like that particular stock issue better than any other, and those who prefer some other stock will not be owners of that particular stock, even though they may entertain an opinion on that one along with opinions on all others. . . . In other words, in a multiple stock market there is a tendency for most people to think all stocks but their own too high. If most people are right in their opinion of the other fellow's investments, then it would follow that stocks in general have a tendency to sell too high, because almost every stock will enjoy some distinction of its own, and will tend to gather around itself its own special group of enthusiasts who will bid its price up too high. If every stock is somebody's favorite, then every price should be viewed with skepticism. (pp. 28–29)[4]

To some extent, the Williams–Miller argument can also be traced to Lintner (1969) who presents apparently the first formal analysis of the formation of equilibrium prices under binding short-selling constraints (short-selling is simply not permitted). Although Miller cites Lintner's paper, strangely he only mentions the portion of the paper which does not deal with short-sale constraints. But the clear connection between heterogeneous beliefs and short-selling, which was subsequently to be the heart of the short-selling literature, was forged by Lintner.

To derive more specific results, to the assumptions usually made for the CAPM, Lintner in addition assumes that all investors have exponential utility functions, known from Wilson (1968), which Lintner references, to produce aggregate closed-form results even when agents have differing beliefs. Lintner's first important conclusion is that, for a given security, its equilibrium price is determined only by those investors who hold that security. Those who in the absence of short-selling constraints would have tended to short it and, in the presence of these constraints, now have zero holdings, play no direct role in determining its equilibrium price. That is, only the preferences and beliefs of those investors that end up holding a security have a direct influence on its equilibrium price. The preferences and beliefs of those who do not hold the security only indirectly affect its price via their effect on the prices of other securities that are held in common. For example, suppose there are three securities 1, 2, and 3, and two investors, A and B. Say, in equilibrium, investor A holds securities 1 and 2, and investor B holds securities 2 and 3. Then, investor B only affects the price of security 1 indirectly though his influence on the price of security 2, which in turn affects the demand for security 1 by investor A. Surprisingly, despite what some seem to believe, Lintner does not draw the Williams–Miller conclusion that risky securities will tend to be priced higher because they will be held by relatively optimistic investors. Instead, Lintner simply says that the pricing of any security will only depend directly on the beliefs and risk aversions of the investors that hold it, and that in determining its price, greater weight is given to the risk aversions and beliefs of those investors who hold more of the security. Indeed, as Lintner points out (pp. 395–396), in his model, the fewer the investors in a security (in his terminology, "the smaller the market"), the higher the risk premium of that security and the lower, therefore, its market price, other things equal. In effect, because investors have available fewer desirable securities, they diversify less and tend to hold portfolios with higher variance. This, in turn, tends to reduce the desirability and, therefore, the prices of risky securities. So Lintner, since he only draws conclusions about pure segmentation issues, would have concluded the opposite of Williams and Miller: that short-selling constraints, by reducing the size of the market, should tend to reduce prices.

Jarrow (1980) refines the Williams–Miller hypothesis further. A disconcerting implication of the Markowitz (1952) and Roy (1952) mean–variance portfolio selection model in the case of unconstrained short-sales is that the optimal portfolio often includes very large long positions in some securities hedged and financed by very large short positions in other securities. In an extreme case, suppose securities 1 and 2 have almost perfectly positively correlated returns, but the investor believes that the expected return of security 1 is slightly higher than that of security 2. He can exploit this small difference in expected returns by shorting a large amount of security 2 and using the proceeds to fund correspondingly large purchases of security 1—almost a riskless arbitrage. In this case, imposing short-sale constraints not only eliminates the short-sales in security 2, tending to increase its price, but it can also significantly reduce the size of long positions in security 1, tending to *decrease* its price. Jarrow concludes that *a priori*, considering both these effects, means that short-sale constraints can drive the prices of some securities up and others down. However, he also proves that in the context of Lintner's heterogeneous expectations model, if the only source of disagreement among investors is their expected returns (in particular, investors have identical beliefs about covariances), then the imposition of short-selling constraints will only increase the prices of risky securities.

Although antecedents to Miller's paper can be found in Williams (1938), Lintner (1969), Sharpe (1970), and perhaps others, Miller is the first to emphasize several implications of the hypothesis and a surprising number of apparently anomalous observations that contradict standard models but can be potentially explained by asymmetric short-selling restrictions:

1. One should not be surprised to find that warrants, particularly near expiration warrants, tend to be overpriced (as claimed by Thorp and Kassouf (1967)) since it is precisely these securities which will be the most attractive to optimistic investors, as a result of their high implicit leverage.
2. Syllogistically, since divergence of opinion tends to be correlated with increased risk and since securities with the greatest divergence of opinion, by Miller's theory, tend to be the most overpriced, then ironically, increased risk and overpricing (hence lower expected returns) for securities with divergent opinions tend to go together. For example, overpricing of stocks at IPOs, another apparent anomaly, may then be explained by the fact that it is at the moment of the IPO when divergent opinion about the stock is often at its greatest.
3. High turnover and relatively divergent opinions tend to go together, therefore, high turnover is the market trace evidence of divergence. Again, one should not be surprised to find that stocks with high turnover tend to have lower returns, as claimed by Cooley and Roenfelt (1975).
4. Rosenberg and McKibben (1973) provide evidence that high turnover and high beta tend to be observed together. Therefore, one should not be surprised to find that high beta stocks have lower returns than the CAPM would predict (as found by Black *et al.* (1973), among many others).
5. Miller speculates that the closed-end fund discount anomaly may be at least partially due to short-sale restrictions. It is not that closed-end funds are selling at a discount, but rather that individual securities are selling at a premium. Investors who are optimistic about individual stocks cannot take optimal advantage of their beliefs by purchasing a managed portfolio

of stocks, but will instead prefer to purchase specific stocks.

The Williams–Miller argument also has an unfortunate side effect: Since prices will not reflect, as Hayek (1945) argued, the beliefs of the pool of all potential investors, realized returns cannot be used without bias to infer their *ex-ante* probability distribution, as is often assumed in empirical work.

Figlewski (1981) conducts an early direct test of the Williams–Miller hypothesis. The hypothesis implies that if one can identify stocks and times when dispersion of beliefs is unusually high, one should find those stocks at those times to be overpriced. Figlewski uses the percentage of outstanding stock held short as an indicator of belief dispersion. Considering the significant cost of short-selling, those that do may have very negative information compared to those that hold the stock. The opposite position has often been argued that stocks with high short-interest should be underpriced since at some point the short-sellers will need to cover by buying stock and this will force the price up in the future. Covering the period 1973–1979, Figlewski's empirical evidence, however, supports both the Williams–Miller hypothesis and his use of the short-interest percentage as an indicator of belief dispersion, but the identified stocks are not found to be sufficiently overpriced to compensate for the loss of interest on the proceeds of the short-sale likely to be experienced by most investors. His results also confirm what may be the earliest published test of the predictability of short-interest by Seneca (1967) covering the earlier 1946–1965 period using mid-monthly announcements of open interest.

Chen *et al.* (2002) test the Williams–Miller hypothesis by using breadth of stock ownership, defined as the number of investors with long positions in the stock, as a measure of pent-up, short-selling demand (that is, the fewer investors, the more pent-up demand since the harder it will be to locate share lenders). If this proxy works and the hypothesis is true, then reductions in breadth of ownership should forecast reduced future stock returns. Chen *et al.* (2002) confirm this prediction for their sample of mutual funds for which they have breadth information. They presume that all investments of these funds are long since very few mutual funds engage in short-sales. Moreover, they also show that breadth and momentum are positively correlated. Therefore, it is possible that part of the anomalous observation that momentum can predict returns (see Jegadeesh and Titman (1983) as well as their updated 2001 article) may be simply that momentum is a by-product of changes in breadth, which in turn influences stock returns due to the Williams–Miller hypothesis.

The Williams–Miller hypothesis can also potentially explain other, more recently observed, anomalous observations. Diether *et al.* (2002) attempt to explain why it appears that, compared to otherwise similar stocks, stocks with higher dispersion in analyst's earnings forecasts tend to have lower future stock market returns. Arguing that this dispersion proxies for differences in opinion about these returns, the Williams–Miller hypothesis can be used to explain the future lower stock market returns (since these securities tend to be the most overpriced). Supporting this explanation is the additional observation that the dispersion effect is greater for small stocks—stocks that are particularly difficult to short and have no exchange-traded options, where shorting can be easily accomplished by selling calls or buying puts. The dispersion effect also seems to be stronger for growth stocks where a given level of earnings estimate dispersion translates into a larger disagreement over current value.

2 Douglas Diamond and Robert Verrecchia

An important objection to the Williams–Miller theory, as mentioned in a footnote by Figlewski (1981), p. 465, is that it contradicts rational expectations. This idea is thoroughly developed by Douglas W. Diamond and Robert E. Verrecchia (1987) in "Constraints on Short-Selling and Asset Price Adjustment to Private Information," possibly the most interesting paper written on the Williams–Miller hypothesis since 1977. If investors know that divergence of opinion with asymmetric short-selling restrictions tends to leave only the most optimistic investors trading in the market, then those investors should temper their optimism, implicitly trying to incorporate in current prices the information held by the more pessimistic investors who fail to trade because of short-sale restrictions. For example, it is hard to believe that investors will persistently overprice IPOs and never learn from their experience. So in the Diamond–Verrecchia model, stocks are not overpriced. The authors distinguish carefully between two types of short-selling constraints: (1) "short-prohibition" constraints, such as an institutional ban against short-selling, which do not discriminate between optimistic or pessimistic investors, and (2) "short-restriction" constraints, such as a below market rebate rate, which do discriminate. Only the latter type of short-selling constraint can create the asymmetric pricing response, which is their chief innovative conclusion, to which I now turn.

Suppose, for some reason, perhaps not fully rational, some traders are optimistic and others pessimistic about individual stock returns. As in Williams–Miller, because of short-restriction constraints, the views of the pessimists are not clearly reflected in prices. However, because of rational expectations, all investors take into account the extent to which the bad news known by the pessimists is not reflected in prices, and correspondingly bid down prices to reflect an expectation of the unknown pessimistic information. So, unlike the hypothesis of Williams–Miller, prices *on average* are not too high (or too low). However, and this is key, only the expected and not the actually known negative information is reflected in prices. During periods when the optimists subsequently receive bad news and prices fall, the rational traders would now expect some of those who were formerly pessimistic to enter as buyers and cushion the fall. However, occasionally, there are periods when the negative information that was originally not reflected in prices is surprisingly bad. At these times, prices will fall much more than anticipated before the pessimists are ready to step in and buy—hence a stock market crash. Note that this effect is asymmetric since on the upside there is no hidden positive information not embedded in prices. So the Diamond–Verrecchia model contains the novel prediction that observed stock returns should be skewed to the left—that is, around the mean, large downward changes should be more frequent than correspondingly large upward changes.

The Diamond–Verrecchia model also provides a good example of a more general issue that surely must characterize realistic financial markets: with dispersed fundamental information but some constraint on trade, market prices, contrary to Hayek (1945), will not reflect pooled investor wisdom; rather, over time, trading itself will gradually reveal this dispersed fundamental information and only then will it find itself completely embedded in prices. In the meantime, prices can change rationally even in the absence of news concerning fundamentals. This view offers a rational way to explain the evidence of French and Roll (1986) that stock volatility is much higher per hour (13–100 times) when exchanges are open than when they are closed. For example, the three-day weekend variance is only slightly higher than the single trading day variance. Romer (1993) presents alternative ways that do not rely on constrained short-sales by which rationally set prices can move in the absence of fundamental news. These ideas suggest that much of the excessive volatility that behavioralists hold out as evidence of market-wide irrationality is actually required in a rational market where investors use price trends, volume, and other "technical" information, such as short-interest, to learn about dispersed fundamental information that is not yet embedded in current prices.

Of course, while rational expectations is often held out as an assumption that "good" finance models must not contradict, many intelligent observers believe that as a practical matter, rational expectations often fails to explain real world behavior and should not be taken too seriously. If, as Figlewski's (1981) evidence supports, the short-interest percentage is a good proxy for belief dispersion, then a clear test of the Williams–Miller theory versus the Diamond–Verrecchia rational expectations modification would be to examine whether announcements of significant increases in short-interest forecast lower future returns (Williams–Miller) or whether prices immediately adjust downward synchronous with the announcement (Diamond–Verrecchia). In Diamond–Verrecchia, since by observing the order flow, investors cannot distinguish ordinary sells from short-sales, news of increased short-interest, since it effectively reveals the pessimistic information, when it becomes available, should immediately lead to a downward revision in stock prices. In addition, days of relatively little trading volume may be another indicator of greater dispersion of beliefs than usual since inactivity may indicate that even with the further dispersal of the known positive news as time moves on, constrained short-sellers continue to stand pat since their information is particularly negative. Therefore, unusually low trading volume may play a similar informational role as announced increased short-interest in depressing prices.[5]

Senchack and Starks (1993) test just such an implication of the Diamond–Verecchia model: Since would-be short-sellers are assumed to have pessimistic information generally not accurately reflected in prices (that is, only an unbiased guess about this information is embedded in prices), news of increases in short-sales should reduce the previous relative optimism of the holders of these securities, and the prices of the securities should fall. Moreover, the significant cost short-sellers often face suggests that, to go short, one must typically have very negative information. As I have argued, this is not a prediction of the Williams–Miller hypothesis since in that context investors do not learn from the trading of other investors. The obvious test is to examine whether or not changes in recently announced short-interest are negatively correlated with contemporaneous returns. Senchack and Starks not only confirm this implication empirically, but they also show that the correlation is much less pronounced for stocks with exchange-traded options for which short-selling is effectively easier, again confirming another implication of the Diamond–Verecchia model.

A very similar model to that of Diamond–Verrecchia (1987) is contained in the recent article by Hong and Stein (2003). Hong and Stein argue that this model is capable of explaining three virtually defining features of crashes:

1. the post-WWII empirical fact that of the ten largest moves in stock prices, nine have been down (the one increase occurred two days after the October 19, 1987 crash);
2. that most of these moves have not seemed to be accompanied by sufficiently significant public news to justify the price movement (this has been particularly remarked about the largest of these crashes in 1987);
3. correlation across stocks, both domestically and internationally, seems to increase sharply and suddenly during a crash.

It is easy to see why 1 and 2 are consistent with their short-selling model, as they are with Diamond–Verrecchia. In turn, 3 is explained if declines in some stocks reveal the presence of unexpectedly negative systematic (or market-wide) information heretofore hidden by short-sale constraints. Related to 1 are the empirical observations, both from time series and from option-implied probability distributions, that stock index returns are negatively skewed, and that this negative skewness attenuates at longer time horizons. Both these observations would also be predicted from the Hong–Stein model.

A further prediction of their model is that abnormally high trading volume should accompany large negative stock market moves more than large positive stock market moves. This association between negative return skewness and trading volume is tested and confirmed in another paper by Chen *et al.* (2001). This result is also supported in the domain of option-implied stock distributions by Dennis and Mayhew (2002) who conclude that periods of larger than usual implied negative skewness in stock returns also tend to be periods with abnormally high market volatility. As Hong and Stein point out, other types of crash explanations either fail to explain the asymmetry, that is, why large post-WWII jumps in the US seem to occur primarily on the downside (behavioral theories), or why crashes tend to occur without apparent significant public news (volatility feedback theories). Only the short-selling explanation coupled with investor heterogeneity seems to deal with all three of the defining features of crashes that they identify.[6] At the conclusion of their paper, Hong and Stein write:

> This article can be seen as part of a recent resurgence of theoretical and empirical interest in the general topic of how short-sale constraints shape stock prices. . . . This work is also beginning to suggest that short-sales constraints may play a bigger role than one might have guessed based on just the direct transactions costs associated with shorting. . . . There remains much to be done, both in terms of developing a fuller understanding of why so many investors behave as if they were facing prohibitive shorting costs, and of exploring the consequences of such behavior for stock prices. (p. 516)

3 Short-selling puzzles

Perhaps the greatest puzzle surrounding the relationship between short-sales and stock prices, as Hong and Stein suggest, is one of demand: why is so little stock shorted? During 1976–1993, for example, more than 80% of all NYSE stocks had short-interest of less than 0.5% of their outstanding shares. Figlewski (1981) reports that even the top decile from a universe of over 400 S&P 500 companies from 1973 to 1979 had an average short-interest percentage of less than 1%. However, it appears from Ofek and Richardson (2003) that short-interest has dramatically increased in recent years. For their sample of about 4200 US stocks in February 2000, the average percentage of short-interest to shares outstanding was about 2%.[7]

How can this bias be explained? Even with symmetric short-selling conditions, there are many reasons to expect significantly less short- than long-interest:

1. Since a long position must pre-exist equal to the number of outstanding shares and a short-sale must give rise to an additional exactly offsetting long position, the number of shorted shares must necessarily be less than shares held long.
2. In a market with identical investors, since they would all hold the same market-wide index fund, there would be no desire to short. Even in the standard finance equilibrium model, the CAPM, no investor sells any stock short.
3. Starting from that default, an investor with somewhat relatively pessimistic beliefs can assert these by taking less of a long position without needing to go so far as to short.
4. For many securities short-selling can be accomplished by trading in their derivatives (selling futures, selling calls, or buying puts). Using back-of-the-scratch-pad estimates in October 2003, while the short-interest percentage (that is, the percentage of short to outstanding shares) is typically about 2% for stocks with listed options, the aggregate amount of open interest including exchange-traded, delta-adjusted calls and puts on stock indexes and individual stocks,

as well as stock index futures, adds about another 4% to the typical direct short-interest, bringing it to about 6% (if the implicit short positions of open derivative positions are included).

Detailed published estimates of the extent of the loss of the interest on the proceeds of short-sales is now finally available in a recent article by Jones and Lamont (2002). The estimates cover 1926–1933 when there was briefly a centralized market on the NYSE for borrowing stocks. These authors then use these estimates to test the Miller prediction (and I think implicitly Williams as well) that stocks with high barriers to short-sales should be overpriced and experience lower subsequent returns. They find that the overpricing of stocks that are expensive to short is sufficiently large to produce profits to short-sellers even after subtracting these costs.

Supplementing Jones and Lamont, D'Avolio (2002) also documents the costs of short-selling using 18 months during 2000–2001 of transactions obtained from a large stock lending intermediary. He again confirms the Williams–Miller hypothesis for a number of proxies for large differences of opinion—high share turnover, large dispersion of analysts forecasts, high visibility among unsophisticated investors, high P/E ratios and low cash flows relative to stock price (these latter two create increased uncertainty and room for differences of opinion). D'Avolio also reports that forced premature covering of short positions affects about 2% of loaned stocks each month across his sample. But the costs of shorting, inclusive of lost interest and potential forced covering, just do not seem significant enough to explain the low incidence of short-sales (even if indirect shorting through derivatives is included), particularly for large S&P 500 companies for which these costs are quite low. The missing short-sales is reminiscent of the cosmological puzzle of the whereabouts of the missing mass in the universe.

A related supply puzzle is why more investors do not take advantage of opportunities to lend their stock. In a 2002 working paper, "Why Constrain Your Mutual Fund Manager?", Almazan, Brown, Carlson, and Chapman report that 70% of institutional managers are precluded from short-selling by contract, and of those remaining only 10% actually short-sell. Despite this, on occasion, the payment for security lending becomes so high that it would seem that no one holding the stock long should not be lending it. For example, the recent spin off of Palm by 3Com implied significant returns to stock lending. At the beginning of March 2000, 3Com spun off 5% of a wholly owned subsidiary, Palm, Inc., stating (and there was virtually no doubt this would happen) that it would distribute the remaining 95% of Palm shares on July 27. At that time, for each 3Com share, a shareholder would receive 1.483 shares of Palm. This suggests that 3Com's shares should be worth at least 1.483 times the price of Palm's shares. To take a typical day, on April 18, the following prices were being quoted by Charles Schwab:

$$\text{3Com} \quad \$39\tfrac{3}{8} \qquad \text{Palm} \quad \$30\tfrac{1}{2}$$

Under these circumstances, an arbitrage opportunity would seem to exist: for each share of 3Com that you buy, short 1.483 shares of Palm:

Cash flow on April 18:

$$-\$39.375 + 1.483(\$30.50) = \$5.85 \text{ (received)}$$

Cash flow on July 27:

0 (since cover short with distribution of Palm)

However, Palm was very expensive to short. Indeed, instead of receiving interest on the proceeds of the short-sale, the short-seller got no interest and actually had to pay a fee on top. As it turned out, the fee demanded by lenders of Palm (about $5.85) was exactly enough to make the arbitrage opportunity disappear.

As an alternative, one could have tried to short indirectly using exchange-traded Palm options. From the put–call parity relation (Stoll, 1969),

$$-S_0 = P_0 - C_0 + Kr^{-t}$$

That is, a short position in a stock with current price S_0 and with no payouts (Palm did not pay dividends) can be replicated by buying a European put with current price P_0 and selling a European call with current price C_0 on the stock, both with the same strike price K and time-to-expiration t (r being the return on a riskless zero-coupon bond maturing after time t). On April 18, estimating $r = 1.06$ and using this equation to imply the stock value from option prices:

Option	t	K	C_0	P_0	Implied S_0
Aug 25	0.33425	25	7.125	5.375	26.27
Aug 30	0.33425	30	4.675	8.35	25.85
Aug 35	0.33425	35	3.125	3.125	25.70

Therefore, the average implied stock value is (\$26.27 + \$25.85 + \$25.70)/3 = \$25.94. Short-selling stock at this price indirectly through the options market, unfortunately, leads to a current cash flow from the above arbitrage of:

$$-39.38 + 1.483\ (\$25.94) = -\$0.91\ \text{(loss)}$$

So that would not work either. The options market had clearly caught on to the arbitrage and had aligned the prices of each put–call pair such that each implied about the same stock price for Palm, sufficiently low to eliminate the arbitrage. But the key puzzle remains: There must always be more shares held long than shorted. So there must be some who held Palm who did not lend it out. So one wonders why anyone who held Palm long did not try to lend his stock out and pick up a free \$5.85 per share!

Further reflection shows that in the presence of security lending costs, the equilibrium price of a security (measured before considering these costs) could be indeterminant. To repeat the example given by Duffie *et al.* (2002), suppose optimists believe the value of a stock is \$100 while pessimists believe its value is \$90. If short-selling were not possible, the optimists would determine the market price, which would then be \$100. But suppose instead there were a market for lending the stock to the pessimists for short-selling, and suppose the optimistic lenders had all the market power. In that case, since the pessimists would pay at most \$10 (\$100 − \$90) to borrow the stock, the lending fee would be \$10. But, then, optimists would be willing to pay \$110 for the stock since they would now get \$100 stock value plus the \$10 lending fee. So the stock price would rise to \$110. Now the pessimistic short-sellers would be willing to borrow the stock for at most a \$20 fee (\$110 − \$90), so the price could then rise to \$120, etc. This cycle would cease if eventually all the outstanding stock not yet lent out somehow ended up in the hands of investors that for some reason refuse to lend their stock. Ironically, as the authors point out, the price of the stock with possible short-selling can actually end up higher than if short-selling were prohibited, and even higher than the value placed on it by the most optimistic investor in the market!

Rational market theorists might hope that in this lies the key to explaining the Internet "bubble." In fact, as documented by Ofek and Richardson (2003), the rebate rates for shorting Internet bubble stocks averaged 1% to 1.5% per annum less than for non-Internet stocks. Moreover, other signs of constrained short-selling for Internet stocks included higher short-interest and frequent put–call parity violations. Also, supporting the hypothesis of high belief dispersion is evidence the authors present that retail investors, in contrast to institutions, played a greater role than usual as buyers of Internet stocks. Unfortunately, for rational market advocates, although providing some support for the Williams–Miller hypothesis, the magnitudes of these effects do not come close to explaining the rise phase of the bubble. Where were all the short-sellers who, even with somewhat higher costs, should have stepped in with so much that was apparently to gain?

However, the authors also emphasize that something quite similar to temporarily very strong shorting restrictions may explain the collapse. An extraordinarily large amount of insider IPO-related stock that was locked up during the rising phase of the bubble (early 1998 to February 2000) became unlocked just as the bubble was collapsing (March 2000 to December 2000). This meant that, for the first time, relatively well-informed shareholders whose relative pessimism may have been hidden during the rise now entered the market, and their selling may have at least precipitated the collapse, which then subsequently became fed by the previously optimistic investors who now began to factor the implied beliefs of the new traders into their calculations.[8]

The sudden rise (and success of some) hedge funds over the last 15 years may owe much of its impetus to the puzzle of the missing short-interest. Our discussion suggests that for investors who can find a way to overcome the normal reluctance to short, even after appropriate compensation for risk, abnormal profit may be theirs. But a careful analysis of this controversial claim would take us well beyond the intended scope of this paper.

4 Conclusion

If we perturb the standard model of security pricing under uncertainty either by (1) permitting heterogeneous beliefs or (2) imposing barriers to diversification such as constrained short-selling, but not both, as we have argued, if anything, the current prices of risky securities such as stocks will tend to fall. However, in a Great Moment in Financial Economics, Edward Miller (1977) observed that existing together in the same economy these two generalizations would tend to produce exactly the opposite result because those that end up holding a stock will tend to be more optimistic than the average investor in the economy. As we have seen in a previous article in this series about the Modigliani–Miller theorem, this idea can again be traced back to Williams (1938).

The next significant advance is very briefly mentioned in Figlewski (1981) but fully developed in Diamond and Verrecchia (1987)—if the constraint of "rational expectations" is imposed, to the contrary prices will, on average, not be too high (or too low). That is, if each investor considers that he/she only knows part of the pricing-relevant information, buyers of a short-sale constrained stock will downgrade their opinion of the fair market price in an attempt to take into account the negative information not in the market because of the trading constraints. However, since prices will then only reflect the buyer's best expectation, on average over time, of the negative information hidden by the short-sale constraints, future trading will occasionally reveal that this hidden information is exceptionally negative and the market will crash. Since this is a clear asymmetry, significant price decreases should be experienced more frequently than price increases of equivalent magnitude.

Again, as we have seen, implications of these extended models of equilibrium security pricing, Williams–Miller or Diamond–Verrecchia, can potentially explain a wide range of phenomena that might otherwise seem anomalous, from closed-end fund discounts, to the failure of realized returns to match aggregate investor expectations, to the momentum factor in stock returns, to market crashes, to the recent Internet-based stock market "bubble," and to the growing popularity of certain hedge fund strategies. But the match between theory and reality still leaves unresolved two significant short-selling puzzles: (1) the demand puzzle of why there seems to be so little short-selling, and (2) the supply puzzle of why investors seem so reluctant to lend their shares to those who would sell short.

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Notes

[1] There is a fourth category that need not concern us here: even if beliefs are homogeneous and with no barriers to exchange, without the availability of a complete securities market (in particular, without a riskless security) or with fairly general preferences (non-HARA) and beliefs (non-normal), different investors will want to choose different portfolios of risky securities. See Cass and Stiglitz (1970) for further elucidation of this point.

[2] If we move beyond the traditional rational paradigm and, following Knight (1921) and Ellsberg (1961), acknowledge that there is a difference between "risk" and "uncertainty" and that most investors have an extra aversion to uncertainty beyond their aversion to risk, then we can find yet another reason why increased dispersion of beliefs should be associated with lower prices. Epstein and Wang (1994) and Rigotti and Shannon (2003) provide examples of such an economy. Although, not fully drawn out, it would seem, at least to me, that uncertainty aversion should generally increase with belief dispersion; that is, the fact that different investors more strongly disagree about subjective probabilities attached to some states is indicative of a greater difficulty of estimating these probabilities, and hence of greater uncertainty. Then, because of uncertainty aversion, the market prices for claims to states should be lower with greater dispersion of beliefs.

[3] Indeed, short-selling by buying put options largely circumvents the first six of these barriers. However, while the loss of the interest on the proceeds of short-sales is not clearly visible in a put purchase, it can be inferred using the put–call parity relation from the price of the put relative to the price of an otherwise identical call. Several empirical studies suggest that the implicit rebate rate earned by a put buyer is often significantly less than the market riskless rate, for example, Figlewski and Webb (1993) and Ofek *et al.* (2003).

[4] Unfortunately Williams, our oft-touted unsung hero, having, in 1938, not yet read Markowitz (1952) or Roy (1952), did not appreciate the portfolio point of view. In his discussion of how shares of the same stock are allocated among different investors, he emphasizes that investors will have different beliefs about the value of that stock, but he believes investors with the highest valuations will end up owning all of the shares. He ignores the good sense of holding some stocks to take advantage of risk reduction through diversification, even if they are not your first choice and may even seem somewhat overpriced. As a result, he argues that the only investor that determines the price of a stock is the *marginal* or last investor who is the most relatively pessimistic among all the optimistic investors who own the stock. With the later perspective of Markowitz and Roy, in the absence of short-sales (implicitly assumed by Williams), the modern view is to see each investor who owns the stock as a candidate to purchase even more should its price fall, so that the price of the stock is not simply determined by the preferences and beliefs of the marginal investor, but rather the preferences and beliefs of the *average* investor who holds the stock.

[5] The familiar logical problem of the three men with the colored hats illustrates how the delayed decision of some conveys information to others. Three men A, B, and C each have a hat on their head that they know must either be red or black. While each can see the hats on the heads of the others, he is unaware of the color of his own hat. Any man who can see another who has a red hat is asked to raise his hand, and a prize will be given to the first man who can correctly identify the color of his own hat. All three hats happen to be red so all three raise their hands. For the first few moments, none of the men commit to an answer. C then reasons as follows: Suppose he C has a black hat, then since B raises his hand and can only see two hats, A will then know that he A must have a red hat since he knows that C does not have one. Similarly, B will then know that he B must have a red hat. But since neither B nor C has reached this conclusion, C concludes he must instead have a red hat. So the inaction of others has provided C with the needed information.

[6] However, to illustrate that the world is never so well behaved as one would like, the observation remains unexplained that skewness of returns, both time-series and option-implied, is negative for stock index returns while positive or at least much less negative for individual component stocks.

[7] Ofek and Richardson emphasize that Internet stocks had a significantly higher short-interest percentage, closer to 3%. The highest 5% of Internet stocks short-interest percentage was about 10.5%, with only about a 0.5% annualized rebate rate when the rebate rate to the short-seller for a normal stock was over 5.0%.

[8] Unrelated to the Williams–Miller hypothesis is a potential new explanation using short-sales for the "Monday effect"—that stock returns from Friday close to Monday close are on average quite consistently negative over many decades despite the fact that averaged annualized stock returns are clearly significantly positive. Chen and Singal (2003) contend that the well documented "Monday effect" can be largely explained by short-selling. They argue that short-sellers much prefer to maintain positions in continuous markets and, compared to buyers, are much more worried about the increased possibility of jumps in prices during extended periods of market closure, i.e. the weekend. Many will, therefore, tend to close out short positions near the Friday close and reopen those positions Monday. These transactions exert pressure on prices creating the Monday effect. Support for this contention comes from a variety of empirical observations: stocks that (1) have higher short-interest relative to the number of shares outstanding, (2) have less actively traded options, and (3) have greater volatility of returns, all tend to have a larger Monday effect. The short-selling hypothesis also fits with the recent attenuation of the Monday effect observed for larger stocks since those now have very well-developed markets for exchange-traded options.

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