Journal Of Investment Management, Vol. 12, No. 1, (2014), pp. 103–106

www.joim.com

BOOK REVIEW

Mark Kritzman, Senior Editor

RISK-RETURN ANALYSIS

Harry M. Markowitz with Kenneth A. Blay, McGraw-Hill, 2014 (*Reviewed by Mark Kritzman*)

Harry Markowitz is deservedly the most well-known financial economist. In 1952 he instigated a virtual renaissance in the theory and practice of finance with his landmark publication, "Portfolio Selection," which introduced the concept of mean-variance analysis. Although Markowitz introduced mean-variance analysis in 1952, it was not until the mid-1970s that investors seriously began to embrace this approach to efficient diversification, largely in response to the epic stock market selloff and the enactment of ERISA. Since then mean-variance analysis has emerged as the dominant approach to portfolio construction among informed investors, but it has also engendered a fair amount of criticism among those who are less informed. In "Risk-Return Analysis" Markowitz and Blay present a robust defense of the broad applicability of mean-variance analysis and decisively dispel what they term "the Great Confusion."

This book is Part I of a four part project. It deals with the theory of rational decision making in a single-period setting in which the relevant odds are known. Part II is intended to address a multi-period setting, again with known odds. Part III will address multi-period decision making with unknown odds. Finally, Part IV is intended to address the division of labor among data, theory, and computation.

Chapter 1 of this book is mainly concerned with demonstrating why the expected utility maxim is appropriate for choosing probability distributions. The authors distinguish rational decision makers from human decision makers and address the observation by Maurice Allais that humans select alternatives that contradict the expected utility maxim. The authors demonstrate that human decision makers are imprecise in assessing small probability events; hence the apparent paradox.

In Chapter 2 the authors begin with the proposition demonstrated in Chapter 1 that concave utility implies risk aversion. As noted by the authors, this proposition originated with Daniel Bernoulli (1954), who introduced the notion of utility in 1738. A translation of his original work was published in *Econometrica* in 1954.

The authors then go on to demonstrate that mean and variance can be used to approximate concave utility functions within non-extreme ranges. This demonstration dispels the Great Confusion, which is the belief by many, if not most students and practitioners of portfolio selection, that the validity of mean-variance analysis rests on the assumptions that investors have quadratic utility and returns are normally distributed.

The assumption of quadratic utility is troubling because it implies that at certain wealth levels investors would prefer less wealth to more wealth. As much as I would like to, I have never met anyone with such a preference. It is more plausible to assume that investors have power utility functions, such as a logarithmic function, which never imply a preference for less wealth. Drawing upon and augmenting earlier work by Levy and Markowitz (1979), the authors show that within a return range of −30% to +40%, power utility is well approximated as a function of mean and variance; to wit, mean-variance approximations are 99.7% correlated with power utility.

Paul A. Samuelson once argued that a better measure of the efficacy of mean-variance approximations to power utility maximizing portfolios would be the difference in the certainty equivalent of the mean-variance approximated portfolio and the certainty equivalent of the true utility maximizing portfolio. Samuelson referred to the difference in these certainty equivalents as "gratuitous dead weight loss." In response to Samuelson's request Cremers, Kritzman, and Page (2003) set out to calculate the gratuitous dead weight loss associated with three variations of power utility, as shown in the table.

Power Utility	Mean-Variance Approximation	Certainty Equivalent
$U = \ln(1+r)$	$\hat{U} = \ln(1+\mu) - \frac{1/2\sigma^2}{(1+\mu)^2}$	e^U
$U = \sqrt{1+r}$	$\hat{U} = \sqrt{1+\mu} - \frac{1/8\sigma^2}{(1+\mu)^{3/2}}$	U^2
$U = 1-(1+r)^{-1}$	$\hat{U} = 1 - \frac{1}{1+\mu} - \frac{\sigma^2}{(1+\mu)^3}$	$\frac{1}{1-U}$

$U = \ln(1+r)$ is the logarithmic utility function, $U = \sqrt{1+r}$ is a less conservative power utility function, $U = 1-(1+r)^{-1}$ is a more conservative power utility function. To address Samuelson's concern, the authors formed utility approximating portfolios of U.S. stocks, foreign stocks, U.S. bonds, private equity, and real estate, as prescribed by Markowitz (1952), and they calculated their utility. Three of these asset classes had significantly non-normal distributions. They then identified the true utility-maximizing portfolios of the same assets and calculated their utility. The gratuitous dead weight loss assuming a $100 million portfolio was $50 per month for an investor with logarithmic utility. For the more conservative investor it was $19 per month, and for the more aggressive investor it was $222. In all cases, the annual gratuitous dead weight loss was less than a single basis point. As Markowitz and Blay point out at the conclusion of Chapter 2, the stubborn adherence to the belief that normality and quadratic utility are necessary conditions to legitimize mean-variance analysis is cognitively equivalent to maintaining the view that the world is flat 60 years after Columbus disappeared over the horizon.

Having dispelled the Great Confusion in Chapter 2, the authors next present several functions for converting arithmetic means to geometric means in Chapter 3. This conversion is necessary because mean-variance analysis requires arithmetic averages as estimates of return, whereas the resultant portfolios grow at a rate equal to their geometric average returns. The reason mean-variance analysis requires returns to be expressed as arithmetic averages is that the arithmetic average return of a portfolio is the weighted sum of the arithmetic average returns of its components, whereas the geometric average return of a portfolio is not the weighted sum of the geometric average returns of its components.

The authors consider several techniques for estimating the geometric mean from the arithmetic mean and variance, and they apply these techniques to a variety of return series. They are able to dismiss half of those they consider, and they provide guidance about which of the surviving techniques or combinations thereof are most suitable given an investor's knowledge of return distributions.

In Chapter 4 the authors turn to alternative measures of risk including variance, mean absolute deviation, semi-variance, value at risk, and conditional value at risk. Based on a thorough analysis of a wide range of asset class returns they offer persuasive evidence that non-normality is not sufficient justification for discarding variance in favor of any of the other risk measures they consider.

Finally, in Chapter 5, Markowitz along with Anthony Tessitore, Ansel Tessitore, and Nilufer Usmen, conduct an empirical study of the return distributions of a wide set of country equity markets. They seek to determine which distribution had the greatest likelihood of generating the observed data. They advise investors to consider the historical distributions of assets in order to assess a portfolio's likelihood of generating a certain level of wealth or of experiencing a certain loss. But they caution investors to assess whether past events that contributed to non-normal higher moments are more or less likely to recur. They also advise investors to take into account current conditions that may influence future distributions, such as the prevailing interest rate environment.

Suffice it to say that Markowitz and Blay convincingly vanquish the Great Confusion. But I have touched upon only a few highlights of the wisdom and careful analysis contained within this excellent book. There is much, much more, even for the seasoned professional and accomplished scholar. Read this book and not only will you be convinced of the near universal applicability of mean-variance analysis. You will learn so much more about how best to put into practice this extraordinary innovation.

References

Bernoulli, D., "Exposition of a New Theory on the Measurement of Risk," *Econometrica*, January 1954 (translation from 1738 version).

Cremers, J.-H., Kritzman, M., and Page, S., Portfolio Formation with Higher Moments and Plausible Utility," Revere Street Working Paper Series, November 2003.

Levy, H. and Markowitz, H. M. (1979). "Approximating Expected Utility by a Function of Mean and Variance," *American Economic Review* **69**(3).

Markowitz, H. M., "Portfolio Selection," *Journal of Finance*, March 1952.