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SOVEREIGN WEALTH AND RISK MANAGEMENT: A FRAMEWORK FOR OPTIMAL ASSET ALLOCATION OF SOVEREIGN WEALTH

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This paper sets out an analytical framework for optimal asset allocation of sovereign wealth, based on the theory of contingent claims analysis applied to the sovereign's economic balance sheet. A country solves an asset-liability management problem involving its sources of income and its expenditures. We derive analytically the optimal asset allocation of sovereign wealth, taking explicit account of all sources of risks affecting the sovereign's balance sheet. The optimal composition of sovereign wealth should involve a performance-seeking portfolio and three hedging demand terms for the variability of the fiscal surplus and external and domestic debt. A real-life application of our model in the case of Chile shows that its sovereign investment is under-diversified.

1 Introduction

How countries should manage their resources and wealth? The issue has come under the spotlight due to the recent expansion of sovereign balance sheets and risks. Over the past 20 years, sovereign wealth funds (SWFs) have been set up by governments with budgetary surpluses from either the rising price of natural resources (Norway, the United Arab Emirates, etc.) or persistently large trade surpluses (China, etc.). In both cases, the objective is to transform these transitory resources into a lasting source of national income. SWFs serve various political and economic objectives, such as budget stabilization, diversification from commodities, saving for future generations, and political strategies. In consequence, they are managed very differently than private sector investments (Ang, 2012). One recent trend is to invest the reserves less and less in safe assets, notably US Treasury Bonds, and to diversify the funds' portfolios (Bernstein *et al.*, 2013;

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Bortolotti *et al.*, 2013). How to manage this wealth efficiently remains an open question.

This paper proposes an analytical framework for optimal asset allocation of sovereign wealth, taking explicit account of all sources of risk affecting the sovereign's balance sheet. We take the point of view of a central government allocating the wealth of public institutions (SWFs, public pension or social security funds, central banks, etc.), and we provide an approach for a country to perform asset-liability management (ALM) on its overall sovereign balance sheet. The paper works in two steps. It uses the Merton (1974) approach to estimate the process of the country's assets, and then optimizes the balance sheet using the ALM approach (Merton, 1993).

Previous literature typically viewed the management of public institution's wealth separately and independently from sovereign liabilities. Scherer (2009a, 2009b), Brown *et al.* (2010), and Martellini and Milhau (2010) have addressed the optimal allocation for an SWF by examining its "shadow" oil asset or exogenous liabilities, proxied by an inflation-linked benchmark. Bernardell *et al.* (2004) and Beck and Rahbari (2008) determine the optimal allocation of central banks' foreign exchange reserves. But the example of the recent crisis[1] clearly shows that when a government is short of liquidity to meet its debt repayments, all assets are automatically available to substitute for the resources initially earmarked for this purpose. Segregating the various items of the sovereign balance sheet according to the institution managing them is a delusion. An innovation of this paper is to consider joint management of sovereign assets and liabilities. We extend the theory and practice of modern contingent claim analysis (CCA) to risk management for sovereign wealth. The sovereign balance sheet is not analyzed solely to evaluate the country default risk, as in Gray *et al.* (2007) and Gray and Malone (2008), but also to determine the optimal sovereign asset allocation in an integrated ALM framework. We derive analytically the optimal sovereign allocation and show that it should involve a performance-seeking portfolio and three hedging demand terms for the variability of the fiscal surplus and external and domestic debt. To implement this allocation in practice, the central government would have to coordinate sovereign wealth management with fiscal policy, monetary policy, and public debt management.[2] Finally, we provide a real data application for Chile. We show that a substantial portion of the country's asset allocation ought to be dedicated to emerging bonds (which provide relatively good protection for liabilities), as well as to developed and emerging equities. This optimal asset allocation is very different to the one currently implemented by Chile in its SWF and the central bank's reserve management department.

The remainder of this paper is organized as follows. Section 2 describes the conceptual framework. Section 3 presents Chile as a practical example of sovereign balance sheet estimation and sovereign ALM. Section 4 concludes.

2 The conceptual framework

In this article, we consider a central government authority allocating the wealth of all public institutions. Its goal is to maximize expected utility, which is a function of its surplus measured as the discounted present value of assets minus liabilities. This supposes that all sovereign assets and liabilities are estimated at market value and their risk is estimated in the sovereign's economic balance sheet (Gray *et al.*, 2007).

2.1 The sovereign economic balance sheet

The ALM exercise involves estimating the sovereign's global economic balance sheet. Just as a company's balance sheet is regularly used to assess bankruptcy risk (Merton, 1974, 1977; KMV, 1999, 2001, 2002), the same analytical framework may be applied to a state. We follow Gray *et al.* (2007) and Gray and Malone (2008) to estimate the various balance sheet components.

The sovereign's assets break down as follows:

(1) International reserves: in general, foreign currency reserves held by the central bank, commodities (especially gold), and Special Drawing Rights with the International Monetary Fund. These reserves often have multiple objectives, including to maintain currency stability or at least avoid excessive appreciation caused by export-linked inflows of currencies; and to serve as a reserve asset in the event of a liquidity crisis.
(2) One or more SWFs, managed by the finance ministry or central bank, also with multiple objectives, which may include savings, macroeconomic stabilization, and even political goals.
(3) Pension fund assets.
(4) Other public-sector assets (property, state-owned enterprises, etc.).
(5) Fiscal assets: taxes and revenues (fees, etc.) collected as tax receipts.

The sovereign's liabilities include the following:

(1) The monetary base (currency in circulation, banks' reserves with the central bank).
(2) Local debt: debt denominated in the local currency of the monetary authorities, and mainly held by domestic agents.
(3) Foreign debt: debt denominated in foreign currency (frequently US dollars), mainly held by foreigners.
(4) Pension fund liabilities.
(5) Present value of expenditures on economic and social development, security, government administration, benefits to other sectors.
(6) Present value of target wealth to be left to future generations.
(7) Financial guarantees to the private sector, notably too-big-to-fail institutions in the banking sector.

The present value of future income and expense flows can be estimated by discounting all forecasted financial flows. However, this would require precise information on the sovereign's expected budget inflows and outflows linked to economic policy decisions on taxation levels, spending targets, implicit liabilities,[3] and so on. An alternative method is to estimate the balance sheet on an integrated basis using market data only (Merton, 1974, 1977; Gray *et al.*, 2007). Table 1 presents this aggregated sovereign balance sheet. In this case, the entries are rearranged, subtracting the present value of expenses and guarantees to the private sector (too-important-to-fail institutions) from the present value of income.

The total value of sovereign assets at any date t is equal to the market value of the claims on these assets: domestic liabilities and foreign debt, all expressed in foreign currency. Following Merton (1974, 1977), the two liabilities can be valued as contingent claims on sovereign assets and be modeled as options on the total value of the assets. Gray *et al.* (2007) consider foreign currency debt as a senior claim ("debt") and local currency debt plus base money as a junior claim ("equity"). In practice, this ranking will depend on governments' objectives and priorities in reimbursing their debt,[4] but historical evidence has shown that many countries prefer to inflate local currency debt instead of defaulting on foreign currency debt. For simplicity, we consider a basic debt structure with two types of debt in zero-coupon form. In what follows, we omit subscript t at $t = 0$.[5]

Table 1 Aggregated sovereign balance sheet.

Assets	Liabilities
Foreign reserves, gold, and Special Drawing Rights	Domestic liabilities: base money + local currency debt ("equity")
Pension fund assets-liabilities	Foreign currency debt ("debt")
SWF	
Other public-sector assets (state-owned enterprises, real estate)	
Present value of future income (taxes, fees, seigniorage) minus present value of future expenditures on economic and social development minus present value of target wealth to be left to future generations minus financial guarantees to too-important-to-fail institutions	

Domestic liabilities expressed in foreign currency, DL_f, is the sum of the monetary base and local debt expressed in foreign currency:

$$DL_f = \frac{(M_d e^{r_d T} + B_d)e^{-r_f T}}{X_F} \quad (1)$$

where r_d is the domestic interest rate, r_f the foreign interest rate, B_d the value of local debt, M_d the monetary base in local currency, and X_F the forward exchange rate.

Domestic liabilities can also be seen as a call option[6] on the value of sovereign assets V_{A_f}, also expressed in foreign currency, with a strike price equal to the default barrier B_f derived from payments promised in foreign currency until time *T*. The question of identifying the default point is key. Contrary to corporates, legal enforcement is irrelevant for sovereigns. Thus, the decision to default largely depends on the trade-off between the short-run benefits and longer-run costs stemming from a loss of reputation for repayment (Eaton and Gersovitz, 1981; Eaton and Fernandez, 1995). As stressed by Rogoff (2011), examining the behavior of governments during periods of stress[7] can be a reliable guide for identifying default barriers. When historical evidence is scarce, a "theoretical barrier" may be estimated. Gray *et al.* (2007), KMV (1999, 2001, 2002) measure medium-term default risk, and define the distress barrier as short-term foreign debt plus one-half of long-term foreign debt. Our objective here is different. We are not attempting to estimate the probability of a sovereign's defaulting in the medium run but to measure the best way for it to manage its wealth. As a consequence, total short- and long-term foreign debt is taken into account.

Following Gray *et al.* (2007), we assume that the value of sovereign assets and liabilities follows a lognormal diffusion process[8] with constant volatility and risk-free rate. We denote A_f and DL_f the sovereign's assets and domestic liabilities, respectively, with expected returns μ_A, μ_{DL} and variance Ω_A, Ω_{DL}.

Under these assumptions, the value of domestic liabilities can be computed using the Black and Scholes (1973) formula (Merton, 1974, 1977):

$$DL_f = A_f N(d_1) - B_f e^{-r_f T} N(d_2) \quad (2)$$

with $N(\cdot)$ the cumulative standard normal distribution.

$$d_1 = \frac{\ln\left(\frac{A}{B_f}\right) + \left(\mu_{\$A} + \frac{\Omega_A}{2}\right)}{\sqrt{\Omega_A}}$$

$$d_2 = d_1 - \sqrt{\Omega_A T}$$

The real-world asset drift is related to the foreign interest rate according to the following relationship:

$$\mu_A = r_f + \lambda\sqrt{\Omega_A}$$

where λ is the market price of risk reflecting the risk aversion of the investor (here, the sovereign entity).[9]

To find the values of the two unknowns V_A and Ω_A, we use a second equation linking the volatility of the sovereign asset to that of the junior claim. Under the Black and Scholes (1973) assumptions,[10] the following relationship holds:

$$DL_f \sigma_{LCL} = A_f \sqrt{\Omega_A}(d_1) \tag{3}$$

Having estimated the value of domestic liabilities from Equation (1), Equations (2) and (3) can be solved to determine the value of the sovereign's assets V_A and their variance, Ω_A, as a function of the foreign debt default barrier B_f.

2.2 *Portfolio choice for sovereign wealth*

Managing sovereign wealth is similar to managing the wealth of an individual (Merton, 1969; Bodie *et al.*, 1992; Bodie *et al.*, 2008), pension fund (Bodie *et al.*, 2009), or foundation (Merton, 1993). The sovereign receives tax revenues each year. Part of this income can be spent, with the residual being saved in the SWF, central bank reserves, or public pension fund. How much should be saved and how it should be invested is a classic ALM problem. The optimal allocation and expenditures of the sovereign will crucially depend on the nature and size of its assets and liabilities, and their sources of uncertainty. Merton (1993) solved a similar problem analytically in a dynamic case for a university endowment fund. The optimal portfolio can be decomposed into speculative demand (the result of the standard mean variance optimal portfolio), and hedging demand components intended to cover unanticipated changes in assets and liabilities.

In the sovereign case, applying a dynamic approach would entail defining the macroeconomic dynamics of sovereign balance sheet variables, notably fiscal surplus, and foreign debt, and domestic debt, which would prove complex and is beyond the scope of this paper. We propose to approach the ALM exercise from the perspective of optimizing a surplus (or funding gap), as is traditionally done in the ALM literature for pension funds (van Binsbergen and Brandt, 2006; Hoevenaars *et al.*, 2008; Amenc *et al.*, 2009). The sovereign's objective is to maximize its expected utility, which is a function of its Global Sovereign Surplus (GSS) measured as the value of assets minus liabilities (all expressed in foreign currency). In all that follows, we omit subscript t:

$$GSS_f = A_f - FL_f - DL_f$$

with A_f, FL_f, and DL_f the sovereign's assets, foreign and domestic liabilities, respectively.

We can further decompose sovereign assets into the sum of financial assets held by the public institutions and the "fiscal surplus"[11]:

$$GSS_f = FA_f + FS_f - FL_f - DL_f$$

with FA_f and FS_f the sovereign's financial assets and fiscal surplus, respectively.

Basically, the sovereign will try to maximize the value of the GSS (the target wealth to be left to future generations) for a given amount of risk. In practice, it will choose the most appropriate risk measure for its situation, such as volatility, probability of shortfall, or expected shortfall. In what follows, we consider volatility.

The return of the GSS can be written as:

$$r_{GSS} = \alpha w r_{FA} + (1-\alpha) r_{FS} - \beta r_{FL} - (1-\beta) r_{DL}$$

where r_{FA} is the annualized vector of returns of the n financial assets in the portfolio over the investment horizon, $w = (w_1, w_2, \ldots, w_n)$ the fraction of capital invested in the asset i, r_{FS}, r_{FL}, and r_{DL} the fiscal surplus, foreign debt, and domestic debt returns, respectively, α the fraction of total sovereign assets dedicated to financial wealth (the remainder is the fiscal surplus), and β the fraction of total sovereign liabilities dedicated to foreign debt (the remainder is domestic debt). We assume that asset returns, fiscal surplus, and foreign debt, and domestic debt have lognormal distributions.

We define their annualized expected returns and second-order moments as:

$$\begin{aligned}
E(r_{FA}) &= \mu_{FA}, \quad E(r_{FS}) = \mu_{FS},\\
E(r_{FL}) &= \mu_{FL}, \quad E(r_{DL}) = \mu_{DL}\\
E(r_{FA}^t r_{FA}) &= \Omega_{FA}, \quad E(r_{FS}^t r_{FS}) = \Omega_{FS},\\
E(r_{FL}^t r_{FL}) &= \Omega_{FL}, \quad E(r_{DL}^t r_{DL}) = \Omega_{DL}\\
E(r_{FA}^t r_{FS}) &= \Omega_{FA,FS}, \quad E(r_{FA}^t r_{FL}) = \Omega_{FA,FL},\\
E(r_{FA}^t r_{DL}) &= \Omega_{FA,DL}, \quad E(r_{FS} r_{FL}) = \Omega_{FS,FL},\\
E(r_{FS} r_{DL}) &= \Omega_{FS,DL}, \quad E(r_{FL} r_{DL}) = \Omega_{FL,DL}
\end{aligned}$$

The expected return and variance of the GSS can be written as:

$$\begin{aligned}
\mu_{GSS} &= \alpha w \mu_{FA} + (1-\alpha)\mu_{FS} - \beta\mu_{FL}\\
&\quad - (1-\beta)\mu_{DL}\\
\sigma_{GSS}^2 &= \alpha^2 w \Omega_{FA}^t w + (1-\alpha)^2\Omega_{FS} + \beta^2\Omega_{FL}\\
&\quad + (1-\beta)^2\Omega_{DL} + 2\alpha(1-\alpha)w\Omega_{FA,FS}\\
&\quad - 2\alpha\beta w\Omega_{FA,FL} - 2\alpha(1-\beta)w\Omega_{FA,DL}\\
&\quad - 2(1-\alpha)\beta\Omega_{FS,FL}\\
&\quad - 2(1-\alpha)(1-\beta)\Omega_{FS,DL}\\
&\quad + 2\beta(1-\beta)\Omega_{FL,DL}
\end{aligned}$$

Like van Binsbergen and Brandt (2007) and Hoevenaars *et al.* (2008), we assume Constant Relative Risk Aversion preferences on the GSS:

$$U(GSS) = \frac{1}{1-\rho} GSS^{1-\rho}$$

with $\rho > 0$ the relative risk aversion.

The sovereign chooses the weights assigned to the financial assets. Its optimization problem is:

$$\mathrm{Max}_w \left[\mu_{GSS} + \frac{1}{2}(1-\rho)\sigma_{GSS}^2\right] \tag{4}$$

Differentiating this expression with respect to w, we obtain the first-order condition:

$$\begin{aligned}
&\alpha\mu_{FA,t} + \frac{1}{2}(1-\rho)[2\alpha^2 w\Omega_{FA}\\
&\quad + 2\alpha(1-\alpha)\Omega_{FA,FS} - 2\alpha\beta\Omega_{FA,FL}\\
&\quad - 2\alpha(1-\beta)\Omega_{FA,DL}] = 0
\end{aligned} \tag{5}$$

The solution to this asset allocation problem is:

$$\begin{aligned}
w^* &= \frac{1}{(\rho-1)\alpha}\Omega_{FA}^{-1}\mu_{FA,t} - \frac{(1-\alpha)}{\alpha}\Omega_{FA}^{-1}\Omega_{FA,FS}\\
&\quad + \frac{\beta}{\alpha}\Omega_{FA}^{-1}\Omega_{FA,FL} + \frac{(1-\beta)}{\alpha}\Omega_{FA}^{-1}\Omega_{FA,DL}
\end{aligned} \tag{6}$$

The portfolio has four components: a speculative portfolio (w_{spec}) that depends only on expected asset returns and is the solution of the Markowitz (1952) problem, and three hedging portfolios that protect against intertemporal variations in the fiscal surplus ($w_{hedge,FS}$), foreign liabilities ($w_{hedge,FL}$), and domestic liabilities ($w_{hedge,DL}$), respectively:

$$\begin{aligned}
w_{spec} &= \frac{1}{(\rho-1)\alpha}\Omega_{FA}^{-1}\mu_{FA,t}\\
w_{hedge,FS} &= -\frac{(1-\alpha)}{\alpha}\Omega_{FA}^{-1}\Omega_{FA,FS}\\
w_{hedge,FL} &= \frac{\beta}{\alpha}\Omega_{FA}^{-1}\Omega_{FA,FL}\\
w_{hedge,DL} &= \frac{(1-\beta)}{\alpha}\Omega_{FA}^{-1}\Omega_{FA,DL}
\end{aligned}$$

When $\rho \rightarrow \infty$, speculative demand disappears and the optimal portfolio shrinks to the one that minimizes the mismatch risk between assets and liabilities. For $\alpha = 1$, the first hedging demand term disappears. This is the optimal solution for an investor having only financial wealth and no fiscal surplus. For $\beta = 1$, the third hedging demand term disappears. This is the optimal solution for an investor with only foreign debt. The relative size of the hedging demand terms depends not only on the relative risk of the fiscal surplus, domestic debt, and foreign debt, but also on the respective size of these items (α and β) in the total balance sheet.

These results shed new light on the sovereign's optimal allocation of its wealth. Previous results on SWFs' asset allocations ignored the three sources of risk affecting the sovereign balance sheet. Martellini and Milhau (2010) express the SWF's preference in real terms and observe a hedging demand against realized inflation. Scherer (2009a, 2009b) identifies the optimal asset allocation of an SWF with non-tradable wealth and observe a hedging demand against oil price variations. We show that in a more general framework, taking explicit account of all sources of risk affecting the sovereign balance sheet, three hedging demand terms should be added to the classic speculative portfolio. It is necessary to take into account not only the risks from inflation and fluctuations in natural resources prices, which both influence the variability of the fiscal surplus, but also all the risks stemming from the fiscal surplus, and from foreign and domestic liabilities. Moreover, fiscal surplus variability is influenced not just by commodity price and inflation volatility, but also by the sovereign's policies on natural resource extraction, taxation, and so on.[12]

Our theoretical results have important practical implications for countries where macroeconomic variables are highly volatile and produce considerable uncertainty. Countries that are natural resource-rich or those exporting manufacturing goods are particularly exposed to fluctuations in commodity or goods prices, which affect their revenues. Moreover, the uncertainty surrounding the level of production or the speed of extraction of natural resources may also make the sovereign's revenues more volatile. These countries can use their SWF to partly hedge fluctuations in the prices and production levels that affect their balance sheet. This stabilizing role is one of the missions of stabilization funds.

In the next section, we assess the quantitative implications of the model by taking the example of a natural resource exporting country. Chile is a particularly interesting case study since it is natural resource-rich, with revenues strongly dependent on copper. The central bank and finance ministry publish comprehensive information that is used to estimate the sovereign balance sheet. The country's two SWFs and its central bank are extremely transparent, making it possible to compare the optimal portfolio composition of Chile's wealth in our framework with the actual composition.

3 The case of Chile

Chile has two SWFs. The Economic and Social Stabilization Fund (ESSF) was launched in early 2007 with the goal of stabilizing fiscal spending. It aims to reduce budget dependency on global business cycles and the volatility of revenues derived from price fluctuations for copper, among other commodities. The Pension Reserve Fund (PRF) was set up at the end of 2006 in response to Chile's new demographic situation. The PRF serves as a supplementary source of income for funding future pension contingencies. The market values of both funds as well as details of their benchmark portfolio composition (at the end of

2010) are provided by the Chilean ministry of finance.

Chile's central bank reserves are intended to guarantee secure and efficient access to international liquidity and to permit intervention in the foreign exchange market in times of crisis. They are invested in liquid foreign assets through two main portfolios: the Investment Portfolio (84.3% of total reserves) and the Liquidity Portfolio (10.2%). Reserves also contain 1.2% of other assets: gold, Special Drawing Rights, IMF reserve positions, and reciprocal credit agreements. Information on the stock and composition of reserves at end-2010 are provided by the Chilean central bank.

The asset allocations of the two SWFs and the central bank reserve portfolios are provided in Table A1 in Appendix A; the currency composition of the ESSF[13] and central bank reserves are given in Table A2. Chile is heavily invested in money market instruments (46.3%) and bonds (52.4%). Among the latter, sovereign nominal bonds account for the bulk, with a small share invested in inflation-linked and corporate bonds. Equities, introduced recently into the PRF, make up a tiny portion of the total allocation (1.3%). In terms of currency composition, both the ESSF and central bank reserves are mainly invested in US dollars and euro, with a small segment of the portfolio diversified into other currencies: the Australian dollar, Canadian dollar, pound sterling, and Japanese yen.

For the sovereign wealth allocation exercise, we allow for a broader investment universe than the one used by today's Chilean government via its currency reserves and SWF. We also assume eight investable asset classes: emerging equities, emerging bonds, developed equities, developed bonds, world inflation-linked bonds, as well as three short-term bond investments in US dollars, euro, and yen. The indices used for data are the MSCI Emerging Equity, MSCI World (developed countries), JP Morgan EMBI + (external debt), JP Morgan ELMI + (local debt), and JP Morgan GBI Broad All-Maturities (developed countries' government debt, all maturities). The short-term debt indices are the Merrill Lynch BOFA Corporate and Government 1-3Y in the US and EMU, and the Merrill Lynch BOFA 1-3Y Broad in Japan. We use monthly data for the period August 2000–December 2010. All data come from Datastream, except the Merrill Lynch indices, taken from Bloomberg.

Chile's central bank provides data on the stock of domestic and foreign debt[14] and the amount of base money in circulation. They represent respectively \$14.9 billion, \$3.5 billion, and \$17.5 billion at the end of 2010. To proxy the volatility of domestic liabilities (domestic debt plus base money) and foreign liabilities, we take the JPMorgan Chile Local Government Bond index (EMBI+)[15] and the JPMorgan Chile Local Government Bond index (ELMI+), respectively.[16]

Based on the analytical framework developed in Section 3, we estimate the sovereign default barrier, the stock of domestic (junior) debt, and the sovereign entity's total assets and their volatility. As previously discussed, the distress barrier for foreign currency debt is taken as the entire stock of foreign debt. By resolving the system of Equations (1) and (2), we can evaluate total sovereign assets in US dollars, as well as their volatility.[17] We use a long time horizon for this analysis, with $T = 50$ years.[18] Table A3 in Appendix A presents a simplified estimated economic balance sheet for the sovereign entity. The various assets and liabilities are shown, along with their volatilities.[19] The estimated balance of total assets is \$64.5 billion, with 11.4% volatility. Subtracting financial wealth (SWFs and central bank reserves) from total assets, the present value of the fiscal surplus is estimated at \$20.1 billion.

To estimate the monthly volatility of the fiscal surplus, we proxy the latter by the discounted sum of all the sovereign's revenue flows less the discounted sum of all its expenditures.[20]

$$P_{\text{fiscal surplus}} = \left[\sum_{i=1}^{\infty} \frac{R_i}{(1+r)^i}\right] - \left[\sum_{i=1}^{\infty} \frac{E_i}{(1+r)^i}\right]$$

where R_i and E_i are the revenues and expenditures expected for year i, r is the discount rate.

We further assume that receipts are indexed to Chilean inflation, copper prices, and equities and that spending is indexed to inflation.[21] This is a reasonable assumption since the sensitivity of inflation to the annual expenditure growth rate is close to 1 over the period 1991–2009 (see Figure B1 and Table B3 in Appendix B).[22] According to Ffrench-Davis (2010), copper accounts for around 15% of fiscal income, including taxes on and all profits from CODELCO, the state-owned copper company, and taxation of private mining companies. We estimate the sensitivities of the expenditure growth rate to inflation, copper and Chile's equity index over the period 1991–2009 at respectively 1.05, 0.15, and 0.10.[23] (see Figure B2 and Table B3 in Appendix B). We thus confirm Ffrench-Davis' estimation.

The present value of government expenses is proxied by the price of an inflation-linked bond with a real coupon equal to the real-term expenditures that the government expects to make in the future. The nominal coupon paid by the government at a future date i is:

$$E_i = RR_i(1 + \pi_i)$$

with RR_i the real coupon, and π_i the annual inflation rate until year i.[24]

In the same vein, the present value of fiscal revenues is estimated as a bond with a real coupon indexed on inflation, copper, and equities. The nominal coupon received by the government at year i can thus be written as:

$$R_i = RR_i(1 + \beta_{\text{inflation}} * \pi_i + \beta_{\text{copper}} * r_i^{\text{copper}} + \beta_{\text{equity}} * r_i^{\text{equity}})$$

with RR_i the real coupon, π_i the annual inflation rate until year i, r_i^{copper}, and r_i^{equity} the copper and equity returns over year i, $\beta_{\text{inflation}}$, β_{copper}, and β_{equity} the sensitivities to copper and equity prices, respectively (assumed constant over time).

We consider a 50-year bond. This maturity is consistent with the assumed sovereign horizon in the balance sheet estimation.[25] Considering the market quotes for real interest rates (based on 20-year Chile Inflation Adjusted Notes 20, data available since 1994 on Datastream), inflation, and copper prices, we estimate the monthly change in the present value of the fiscal surplus.

Table B4 in Appendix B presents descriptive statistics for each balance sheet variable, as well as the asset classes considered for the sovereign allocation. Table B5 presents their correlations. Among the possible investments, emerging equities have the most attractive returns (15.41%) over the study period, but also the highest risk (25% volatility, with high extreme risks), followed by emerging bonds (10.33% return, 10.39% volatility). Unsurprisingly, USD short-term bonds are the least risky investment (1.65% volatility) since the balance sheet is expressed here in USD, but they also offer relatively low returns (4.40%) over the period compared with EUR and JPY short-term bonds (8.69% and 3.76%, respectively). Estimated liabilities have an annualized return of 8.08% for external debt and 6.60% for internal debt, with a volatility of 6.64% and 11.89%, respectively. EUR short-term bonds and developed equities are slightly negatively correlated with the fiscal surplus (−9% and −7%, respectively)—see the correlation matrix in Table B5 in Appendix B. Chile's local and external debt is positively correlated with all markets. External debt has the highest correlation with world inflation-linked bonds (65%), whereas local debt presents the highest correlation with emerging government bonds (57%).

We determine the sovereign's optimal asset allocation (i.e. allocation of its currency reserves and SWF) that: (1) minimizes overall volatility of the GSS, (2) minimizes the volatility of the GSS for a 5% target rate of return, and (3) for a 8.1% target rate of return—the maximum achievable without shorting[26] some asset classes. Figure B6 in Appendix B shows the efficient frontier obtained. Table C7 presents the results of three optimal allocations. To achieve the lowest risk, a substantial weight has to be given to equities: more than 34% of the allocation. Among these assets, 28% should be dedicated to developed markets, and 6% to emerging markets. In addition, 37% needs to be invested in short-term bonds, especially in EUR (negatively correlated with the fiscal surplus and offering attractive returns over the period), and 27% in emerging bonds, which are closely linked to Chile's local and external debt and therefore provide relatively good protection for the liabilities. Only 2% should be allocated to inflation-linked bonds. This overall allocation achieves a 2.83% return on the GSS. When the required return on the surplus is increased to 5%, the proportion allocated to emerging bonds and equities also increases (to 42% and 24%, respectively), whereas the allocation to developed equities declines to zero. A significant proportion (31%) is still allocated to EUR-denominated short-term bonds. Lastly, the maximum return on the surplus is reached with a portfolio allocated 100% to emerging equities. Note that the volatility of the GSS rises from 11.01% to 16.07% due to allocation changes. This is because 69% of sovereign assets can be exposed to investment, and we have excluded shorting and derivatives trading, which would have significantly modified the overall exposure of the sovereign balance sheet.

Our results show that a substantial portion of the asset allocation should be dedicated to emerging bonds, which provide relatively good protection for Chile's liabilities, and to developed and emerging equities. These optimal allocations are very different from those currently implemented by Chile in its SWF and by the central bank's reserve management department. Chile has invested mostly in developed countries through short-term bonds issued by banks or sovereigns and through long-term sovereign bonds. We compare the investor's certainty equivalent return (CER)[27] for the three optimal allocations to Chile's real allocation implemented in its sovereign funds. In all cases, certainty equivalent return differences are substantial and range between 3% and 11%. Ignoring sovereign ALM leads to economically significant losses for Chile.

4 Conclusion

This paper extends the theory of CCA to propose an analytical framework for sovereign wealth and risk management. It derives the optimal sovereign asset allocation in an integrated ALM framework, taking explicitly into account all sources of risk affecting the sovereign balance sheet. Our theoretical results show that the optimal composition of sovereign wealth should involve a performance-seeking portfolio as well as three demand hedging terms for the variability of the fiscal surplus, and external and domestic debt. Our real data application shows that Chile's optimal asset allocation should be much more diversified than at present. A substantial portion should be dedicated to emerging bonds, which provide relatively good protection for Chile's liabilities, and to developed and emerging equities.

Our work has several limitations. Introducing a dynamic model for the balance sheet variables would make it more realistic, but also much more complex. The existence of a "default" boundary is also a strong assumption for countries, subject to sovereign immunity. Repudiation or financial repression (Landon-Lane and Oosterlinck, 2006; Reinhart and Sbrancia, 2011) is an option for a sovereign but unavailable to an individual or a firm. Finally, intangible assets such as human and natural capital should be taken into account. The initiative was taken by the World Bank (2006, 2011) to measure these components of national wealth, if further developed, could open new possibilities.

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Appendix A

Table A1 Asset allocation of the two SWFs (ESSF and PRF) and central bank reserve portfolios (% of total assets).

	ESSF	PRF	Central bank reserves	Total
Money market				
Bank	4.3	0.0	6.3	10.6
Sovereign	4.3	0.0	31.4	35.7
Bonds				
Sovereign nominal	19.0	4.1	21.4	44.5
Sovereign inflation-linked	1.0	1.5	3.8	6.2
Corporate	0.0	1.7	0.0	1.7
Equities	0.0	1.3	0.0	1.3
Total	28.6	8.6	62.8	100.0

Data provided by the Chilean ministry of finance and the Central Bank of Chile, 2010.

Table A2 Currency composition of the ESSF and central bank reserve portfolios (% of total assets).

	USD	EUR	AUD	CAD	GBP	JPY
Central bank reserves	50	40	3.5	3.5	3	0
ESSF	50	40	0	0	0	10

Data provided by the Chilean ministry of finance and the Central Bank of Chile, 2010.

Table A3 Estimation of Chile's balance sheet, 2010.

ASSETS (bn $)			LIABILITIES (bn $)		
	Assets	Volatility (%)		Liabilities	Volatility (%)
Balances of: STABILIZATION FUND (ESSF)	12.7	6.1	Balances of: FOREIGN CURRENCY GVT DEBT (senior claim)	3.5	6.5
PENSION RESERVE FUND (PRF)	3.8	6.1			
CURRENCY AND OTHER RESERVES	27.9	7.2	MONETARY BASE + LOCAL CURRENCY GVT DEBT (junior claim)	61.0	11.6
PRESENT VALUE OF FISCAL SURPLUS – GUARANTEES TO BANKS AND NONBANKS	20.1				
TOTAL ASSETS	64.5	11.4	TOTAL LIABILITIES	64.5	11.4

Appendix B

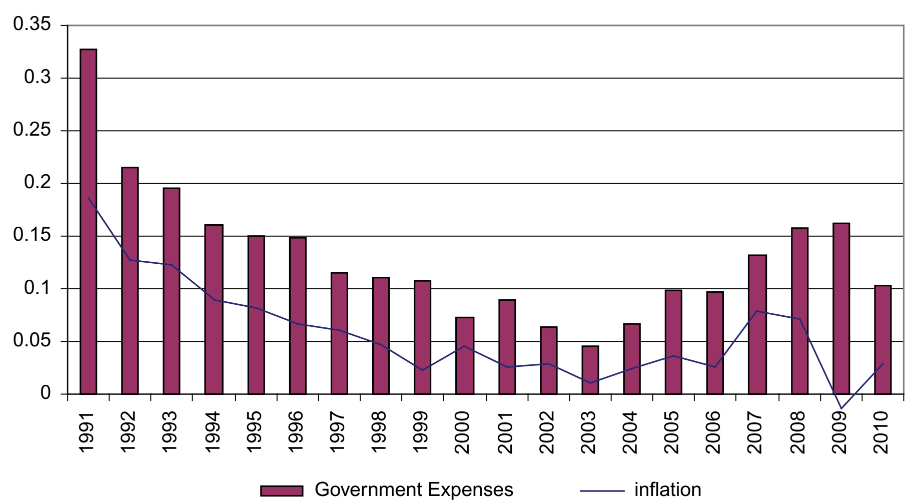

Figure B1 Annual growth of government expenditures and annual inflation in Chile, 1991–2010.

Government expenditures are provided by the Chilean ministry of finance; inflation is measured by the yearly change in headline CPI.

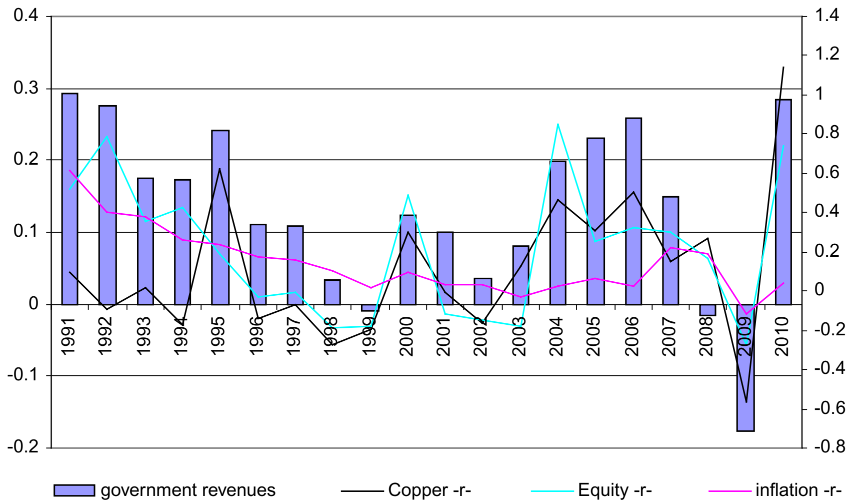

Figure B2 Annual growth of government revenues and annual inflation, copper and equity returns in Chile, 1991–2010.

Government revenues are provided by the Chilean ministry of finance; inflation is measured by the yearly change in headline CPI; the equity market is the MSCI Chile (total return including dividends).

Table B3 Results of annual regression of expenses and revenue growth rate on inflation, copper, and equity returns, Chile, 1991–2010.

	Expenses	Revenues
c	0.06*** (5.03)	0.03 (1.36)
$\beta_{\text{inflation}}$	1.16*** (6.86)	1.05*** (2.68)
β_{copper}	—	–0.15*** (2.86)
β_{equity}	—	–0.10 (1.53)
R^2	72.4%	75.2%
$AdjR^2$	70.8%	70.5%
SEE	0.035	0.065

***,**,*Significant respectively at the 1%, 5% and 10% levels.

Table B4 Descriptive statistics of monthly returns of assets and liabilities, Chile balance sheet, August 2000–December 2010.

	USD	EUR	JPY	Emg equity	Dvp equity	Emg bond	Dvp bond	World IL bonds	Fiscal surplus	External debt	Local debt
Mean	0.37%	0.72%	0.31%	1.28%	0.30%	0.86%	0.62%	0.67%	1.11%	0.67%	0.55%
Ann. Mean	4.40%	8.69%	3.76%	15.41%	3.63%	10.33%	7.44%	8.07%	13.27%	8.08%	6.60%
Median	0.36%	0.58%	0.02%	1.44%	0.94%	1.23%	0.46%	0.88%	1.37%	0.71%	0.59%
Maximum	1.72%	10.37%	8.47%	17.85%	11.91%	8.52%	7.33%	7.41%	19.91%	6.39%	7.79%
Minimum	–0.97%	–8.82%	–8.14%	–27.66%	–19.37%	–13.79%	–4.94%	–11.74%	–26.41%	–7.97%	–17.59%
Std. Dev.	0.48%	3.23%	2.85%	7.22%	5.01%	3.00%	2.19%	2.48%	7.11%	1.92%	3.43%
Volatility	1.65%	11.20%	9.87%	25.00%	17.34%	10.39%	7.60%	8.58%	24.62%	6.64%	11.89%
Skewness	0.11	0.07	–0.26	–0.67	–0.71	–1.01	0.01	–0.96	–0.48	–0.59	–1.22
Kurtosis	3.41	3.82	3.40	4.29	4.25	7.32	3.20	7.43	5.28	6.84	8.04

Table B5 Correlation matrix between assets and liabilities, Chile balance sheet, August 2000–December 2010.

	USD (%)	EUR (%)	JPY (%)	Emg Equity (%)	Dvp equity (%)	Emg bond (%)	Dvp bond (%)	World IL bonds (%)	Fiscal surplus (%)	External debt (%)	Local debt (%)
USD	100										
EUR	41	100									
JPY	33	30	100								
Emg Eqty	−13	35	−11	100							
Dvp Eqty	−19	39	−7	89	100						
Emg Bond	22	38	6	61	57	100					
Dvp Bond	61	85	65	15	16	37	100				
World IL Bonds	53	83	32	39	37	54	83	100			
Fiscal Surplus	9	−9	−1	2	−7	14	−1	4	100		
External Debt	57	49	7	18	13	54	57	65	8	100	
Local Debt	5	35	3	56	51	57	25	39	10	31	100

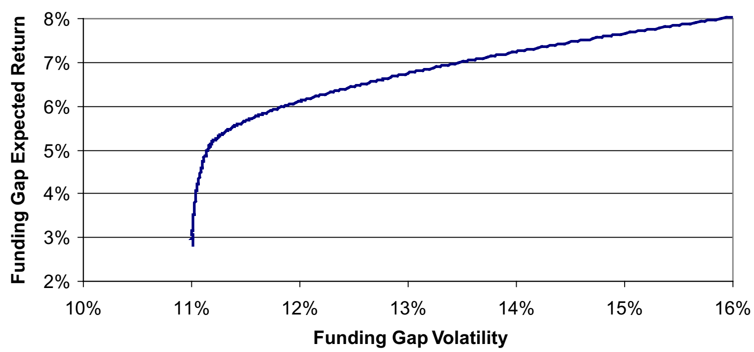

Figure B6 Efficient frontier, GSS expected return and volatility trade-off, August 2000–December 2010.

Table B7 Optimal portfolios for Chile, August 2000–December 2010.

	Min Vol	5% GSS return	8.1% GSS return
	Efficient Portfolios		
Mean	0.24%	0.42%	0.67%
Ann. Mean	2.83%	5.00%	8.07%
Median	0.32%	0.41%	0.81%
Maximum	9.01%	9.49%	13.32%
Minimum	–9.07%	–8.90%	–12.78%
Std. Dev.	3.18%	3.22%	4.64%
Volatility	11.01%	11.15%	16.07%
Skewness	–0.05	–0.01	–0.05
Kurtosis	3.41	3.53	3.29
CER difference*	2.78%	5.02%	10.55%

Table B7 (*Continued*)

	Min Vol	5% GSS return	8.1% GSS return
		Weights	
USD	7%	0%	0%
EUR	30%	31%	0%
JPY	0%	0%	0%
Emg Eqty	6%	24%	100%
Dvp Eqty	28%	0%	0%
Emg Bond	27%	42%	0%
Dvp Bond	0%	2%	0%
World IL Bonds	2%	0%	0%

*CER difference measures the difference between the certainty equivalent return of the optimal allocation and Chile's suboptimal allocation in its sovereign funds.

Notes

1 In 2010, Russia, Ireland, Kazakhastan, and Qatar used SWFs or public pension fund assets to invest in banks or shore up equity markets.

2 This approach may not be compatible with the organization of every country, or it may be subject to legal constraints. In some countries, such as Norway, it may be facilitated by the fact that the Finance Ministry is the central authority responsible for debt issuance, fiscal policy, and the definition of the strategic asset allocation of the SWF. In Turkey, fiscal policy and debt management are defined in an ALM framework by separate but closely connected institutions.

3 Expected losses of the financial and corporate sectors were implicitly guaranteed by the government. Arslanalp and Liao (2012), Gray and Jobst (2010), and Merton *et al.* (2013) have stressed the importance of contingent liabilities, especially from the banking sector.

4 A different debt hierarchy could be adopted without difficulty in our framework.

5 More complex capital structures could also be introduced (Geske, 1977) with multiple debt issues differing in maturity, coupon, seniority, etc.

6 Base money and local currency debt can be issued in very large amounts by governments (causing a potential dilution or reduction of value), and are thus similar to equities (Gray *et al.*, 2007).

7 Some countries are used to serial defaults, others are not.

8 Departing from this hypothesis would lead to a more complex estimation of the sovereign assets, since the Black–Scholes option pricing formula would no longer apply but would still be feasible (Christoffersen *et al.*, 2010).

9 For the empirical investigation, we follow KMV (2002) and set $\lambda = 0.45$.

10 Efficient markets, no dividend during the option's life, option exercised at maturity, no trading costs or taxes, constant volatility and interest rate, returns lognormally distributed.

11 In our simplified framework, this comprises the present value of future taxes, fees, seigniorage.

12 This leads to another important difference with previous literature. In our framework, the variability of the flow of revenues from the sale of natural resources needs to be hedged, not the fluctuations of commodity prices themselves (Scherer, 2009a, 2009b). This has important implications, as the fiscal surplus may not have a sensitivity of one to natural resource prices, as we will see in our estimation on Chile in Section 4.

13 The PRF is invested in international indices; details of its currency composition are not provided on the official web site.

14 Issued by the central government and the central bank of Chile.

15 This simplifying hypothesis overstates the volatility of domestic liabilities, as base money may have a lower duration and thus lower volatility than local currency debt.

[16] Implied volatility would have been more relevant, but futures markets are not available for government bonds in Chile.

[17] We use a Newton Raphson iteration technique with 1% tolerance interval.

[18] In theory, the sovereign horizon should be perpetuity. In practice, the government may have low visibility on the distant future. Fifty years seems a reasonable long-horizon assumption.

[19] The SWFs' volatilities are taken from the "Annual Report on Sovereign Wealth Funds", Ministry of Finance 2011. They have been estimated for both funds since inception. Similar information is not available for central bank foreign exchange reserves. The volatility of reserves is thus estimated as the volatility of the investment and liquidity portfolio over the period May 2008–December 2010, with the proportions invested remaining constant over the history (taken at the end of 2010).

[20] As discussed in the previous section, we assume that contingent liabilities are negligible. This assumption seems quite realistic even in the aftermath of the subprime crisis, as there has been no systemic banking crisis in Chile (Cowan and Marfan, 2010).

[21] In fact, the fiscal surplus is also affected by the government's political decisions, e.g. choice of future expenditures, and tax policy. It ought to depend on intangible assets such as human capital, and take account of the fact that natural resources are finite. Most of these data are not publicly available. We have taken a simplified case by way of illustration.

[22] This single factor explains more than 74% of the total volatility of government expenses.

[23] The three factors explain more than 73% of the total volatility of government revenues.

[24] The usual inflation indexation mechanism of inflation-indexed bonds uses lagged rather than current inflation. Since our purpose is to approximate the present value of inflation-indexed expenses, we consider no delay in indexation.

[25] In practice, the government may have low visibility on its expenditures and revenues in the distant future. Moreover, some of its resources, especially natural ones, may be exhaustible.

[26] A sovereign of this size would find it hard to short a large portion of the portfolio.

[27] The CER is the riskless return that an investor is willing to accept instead of investing in the risky strategy (Kandel and Stambaugh, 1996).

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